ALGORITHM FOR DECOUPLING AND COMPLETE POLE ASSIGNMENT OF LINEAR MULTIVARIABLE SYSTEMS

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Abstract

The problem of decoupling and complete pole assignment of linear square, and controllable systems by static state feedback is addressed in this paper. Based on a characterization of the whole set of attainable finite pole-zero structures of a decouplable system, we present a reliable numerical algorithm which tests the conditions for decoupling and computes the state feedback which decouples the system with a particular pole-zero finite structure, avoiding unnecessary cancellations of invariant zeros. With the use of this algorithm, fixed decoupling poles are determined, non-fixed poles can be arbitrarily located, and no cancellation of system invariant zeros is produced if this is not necessary for decoupling.

1 Introduction

We are interested in this work in the row-by-row decoupling of linear multivariable systems with the same number of inputs and outputs (square systems) by static state feedback. The solution to this problem was obtained in [6], based on the nonsingularity of a matrix constructed from the system matrices; the solution in terms of the infinite structure of the system is presented in [5]. The decoupling problem with stability of square systems has been solved in [13] using a geometric approach, and in [14] using a polynomial equation approach. Even though there exist many results concerning this problem, most of the contributions in the literature about decoupling focus mainly on the necessary and sufficient conditions to solve the problem, but they usually do not consider neither the issue of what the structure of the decoupled closed-loop system may be nor the characteristics of the decoupling state feedback. Actually, in order to simplify the problem, a common consideration is that the entries of the closed-loop transfer function matrix are supposed to be of the form $1/s^j$, where j is a positive integer, which is also referred to as *integrator decoupling*. Of course, no pole locations to obtain adequate system dynamics are considered within this approach, not to speak of the problems which may be caused by possible pole-zero cancellations. Achieving first decoupling, for example in integrator decoupling form, and after that trying to assign the poles of the system can be a difficult problem since the state feedback designed to solve the pole-assignment will usually destroy the diagonality of the closed-loop transfer function matrix. In light of this, a more reasonable approach seems to be to achieve both objectives using the same state feedback. Then, a complete pole assignment for the decoupling problem should provide the whole set of finite pole-zero structures which can be obtained for the closed-loop system, avoiding unnecessary cancellations of invariant zeros.

As far as the structure of a decoupled closed-loop system is concerned, a first attempt to study this structure was presented in [6], where the authors characterized the class of all feedback matrices which decouple a system, and the number of closed-loop poles which can be assigned. Their conditions, however, are cumbersome and difficult to apply, there is no connection whatsoever of these conditions to the structure of the system, and they show how to assign only a number of poles equal to the sum of the system infinite zero orders, which is in general less than the true number of assignable poles. The problem of decoupling and pole assignment is tackled in [19] using a geometric approach, and the authors present necessary and sufficient conditions to solve this problem based on the concept of controllability subspaces and their properties. Fixed decoupling poles are proved in [12] to be equal to the interconnection transmission zeros, as defined in this reference. The characterization of the closed-loop structure of a decouplable system, and the properties of the decoupling state feedback are presented in [16]. In this reference, the family of all attainable transfer function matrices for the decoupled closed-loop system is determined, which also establishes all possible combinations of finite closed-loop pole and zero structures.

Concerning numerical algorithms related to decoupling, a reliable numerical algorithm for the computation of the interactor of a linear multivariable system is presented in [15]. Avoiding of use of elementary operations, the algorithm is further used to compute the state feedback which decouples a linear multivariable system, producing the inverse of the system interactor as closed-loop transfer function matrix. In [4], a numerical method for proportional and derivative state-feedback decoupling controller design is presented. Solvability conditions of all solutions for the triangular decoupling problem are presented in [3]. In [2] it is presented an algorithm for the decoupling problem with stability, based also on orthogonal transformations and condensed forms.

We present in this work a reliable numerical algorithm which tests the conditions for the decouplability of a linear, multivariable, square and controllable system, and computes the corresponding state feedback which decouples the system with a particular pole-zero finite structure. From the previously mentioned references on numerical algorithms related to decoupling, the closest to our present work is [2]. Making a comparison, it can be said that the problem we are considering in this work is more general than that of [2], in the sense that we provide a numerically reliable algorithm to compute a state feedback which not only decouples the system, but also determines the fixed decoupling poles and allows a complete pole assignment avoiding unnecessary cancellation of invariant zeros. Decoupling with stability is indeed a particular case of this more general setting.

In this work, we first extend the results presented in [16] to drop the observability assumption. From this result, fixed decoupling poles can be determined, as well as the complete finite pole-zero structure which can be obtained for the decoupled system. We then develop a numerical algorithm based on these results. The relevant information (global and row infinite zeros, and global and row finite zeros of the system) is obtained in a numerically reliable way from the Kronecker invariants of suitable matrix pencils. Given a particular attainable finite pole-zero structure for the closed-loop system, the algorithm computes the state feedback which decouples the system from the constant kernel of a polynomial matrix. The algorithm also determines the fixed decoupling poles of the system, which are fundamental, for instance, in the issue of internal stability. Cancellation of invariant zeros of the system is completely avoided if it is not necessary for decoupling. The algorithm described in the paper will be implemented in the new release of the Polynomial Toolbox for MATLAB, see `www.polyx.cz`.

2 Preliminaries

We consider in this work linear multivariable systems with the same number of inputs and outputs, described by

$$(A,B,C)\left\{\begin{array}{l}\dot{x}(t)=Ax(t)+Bu(t)\\ y(t)=Cx(t)\end{array}\right.$$

where $x \in \mathbb{R}^{n}$, $u \in \mathbb{R}^{m}$ and $y \in \mathbb{R}^{m}$ are, respectively, the state, input and output vectors of the system.

The system (A, B, C) is said to be row by row decouplable by static state feedback if there exists a state feedback

$$(F,G): \quad u(t)=Fx(t)+Gv(t),$$

where $F \in \mathbb{R}^{m\times n}$ and $G \in \mathbb{R}^{m\times m}$ are constant matrices, with G nonsingular, and $v(t)$ is a new input vector, such that the input $v_i(t)$ controls the output $y_i(t)$, $i = 1, \ldots, m$, without affecting the other outputs.

The previous formulation is equivalent to the existence of a state feedback (F, G) such that the transfer function matrix $T_{F,G}(s)$ of the closed-loop system $(A+BF, BG, C)$ is a nonsingular diagonal matrix, i.e.,

$$\begin{array}{rl}T_{F,G}(s) & = C(sI-A-BF)^{-1}BG\\ & = \text{ diag }\{w_1(s),\ldots,w_m(s)\} =: W(s)\end{array} \quad (1)$$

where $w_i(s) \neq 0$, $i = 1, \ldots, m$, are strictly proper rational functions.

If the stability issue is considered in the problem formulation, then the system (A, B, C) is said to be decouplable with stability if it is decouplable and the closed-loop system $(A + BF, BG, C)$ is internally stable.

The conditions for decoupling a linear multivariable system (A, B, C) are intimately connected to the structure of the system matrix

$$P(s)=\left[\begin{array}{cc} sI-A & B\\ C & 0\end{array}\right] \quad (2)$$

related to the structure of the matrices

$$P_i(s) = \begin{bmatrix} sI - A & B \\ c_i & 0 \end{bmatrix}, \quad i = 1, \ldots, m, \tag{3}$$

where c_i is the i-th row of matrix C, $i = 1, \ldots, m$.

Indeed, the system (A, B, C) is decouplable if and only if the infinite structure of $P(s)$ coincides with the infinite structure of the matrices $P_i(s)$, i.e., if and only if

$$\sum_{i=1}^{m} n'_i = \sum_{i=1}^{m} n_i \tag{4}$$

where $\{n'_1, \ldots, n'_m\}$ are the infinite zero orders of $P(s)$ (infinite zero orders of the system), and $\{n_1, \ldots, n_m\}$ are the infinite zero orders of $P_1(s), \ldots, P_m(s)$ (row infinite zero orders of the system) [6, 5].

If the system is decouplable, then it is decouplable with stability if and only if the number of unstable zeros of $P(s)$ (unstable zeros of the system), multiplicities included, is equal to the number of unstable zeros of $P_1(s), \ldots, P_m(s)$ (row unstable zeros of the system), taken all together [13, 14].

3 Characterization of the closed-loop structure

It is well known that in the process of decoupling a linear system, some of the finite zeros of the system may be cancelled by assigning closed-loop poles. It is important, however, to make the distinction between finite zeros that must be cancelled in order to achieve decoupling, and finite zeros which are not necessary to cancel. In practical designs, cancellation of finite zeros is usually avoided because of potential internal instability caused by hidden system dynamics and undesirable pole locations. Thus, if the main objective is to decouple the system, it is important at least to know the number of finite poles which can be freely assigned, and the number of poles which have to be cancelled with finite zeros in order to achieve decoupling, i.e., the so-called fixed decoupling poles.[1]

Concerning non-observable systems, instead of finite zeros it is necessary to consider the invariant zeros of the system. Results presented in [16] can be extended to consider this case as follows.

Lemma 1. Let (A, B, C) be a square controllable system, and let c_i be the i-th row of matrix C, $i = 1, \ldots, m$. Then, the matrix

$$P_i(s) = \begin{bmatrix} sI - A & B \\ c_i & 0 \end{bmatrix}$$

can have at most one non-unit invariant polynomial.

Proof. The invariant polynomials of $P_i(s)$ can be obtained as

$$\lambda_j(s) = \frac{\Delta_j(s)}{\Delta_{j-1}(s)}, \qquad j = 1, \ldots, n+1,$$

where

$$\Delta_0(s) := 0,$$
$$\Delta_j(s) = \text{ gcd of all } j \times j \text{ minors of } P_i(s),$$

are the determinantal divisors of $P_i(s)$ (see for instance [11]). Since the system is controllable, at least the first n determinantal divisors of $P_i(s)$ are all units. Then, the only possible non-unit invariant polynomial of $P_i(s)$ is the last one, which is equal to $\Delta_{n+1}(s)$. ∎

Let us denote by $z_i(s)$ the last invariant polynomial of matrix $P_i(s)$, $i = 1, \ldots, m$. The finite row zeros of the system is a subset of the roots of $z_i(s)$, and both sets coincide if (A, B, C) is observable. It can be seen that any finite zero of $P_i(s)$ is also a zero of the matrix $P(s)$ given by (2), but notice that a zero of $P(s)$ is not necessarily a zero of $P_i(s)$. Then the product of the polynomials $\prod_{i=1}^{m} z_i(s)$ divides exactly $\prod_{i=1}^{n+m} \epsilon_i(s)$, where $\epsilon_i(s)$ are the invariant polynomials of $P(s)$.

The family of all attainable transfer function matrices for the decoupled closed-loop system is characterized by the following result.

Theorem 1. Let (A, B, C) be a square, controllable, and decouplable system. Then, there exists a state feedback (F, G) which decouples the system, such that the transfer function of the decoupled closed-loop system is of the form

$$\begin{aligned} W(s) &= C(sI - A - BF)^{-1}BG \\ &= \begin{bmatrix} k_1 \frac{z_1(s)}{a_1(s)} & & \\ & \ddots & \\ & & k_p \frac{z_m(s)}{a_p(s)} \end{bmatrix} \end{aligned} \tag{5}$$

where $k_1, ..., k_m$ are real numbers, $z_i(s)$ is the last invariant polynomial of matrix $P_i(s)$, $i = 1, \ldots, m$, as introduced before, $a_1(s), ..., a_m(s)$ are monic polynomials with arbitrary roots, satisfying

$$\deg\, a_i(s) - \deg\, z_i(s) = n_i, \quad i = 1, \ldots, m, \tag{6}$$

and $n_1, ..., n_m$ are the row infinite zero orders of the system.

Proof. The proof is essentially the same as in Theorem 1 of [16]. Just observe that if a preliminary state feedback is applied to render the system observable, then the roots of $z_i(s)$ become the row finite zeros of the system. ∎

Theorem 1 completely characterizes the set of all matrices which can be obtained as closed-loop transfer function matrices of a decouplable system. Indeed, it can be seen

[1] Strictly speaking, cancelled frequency values are not system poles, since they do not appear in the system transfer function matrix. Then, it should be more appropriate to speak of fixed decoupling modes, where poles are a subset of the system modes, and both sets are equal if the system is controllable and observable.

that the finite zeros and poles of the closed-loop system are respectively given by the roots of numerator polynomials $z_i(s)$ and denominator polynomials $a_i(s)$ of $W(s)$.

Theorem 2. Fixed decoupling poles of the system correspond to the roots of the polynomial

$$\delta(s) := \frac{\prod_{i=1}^{n+m} \epsilon_i(s)}{\prod_{i=1}^{m} z_i(s)} \tag{7}$$

where $\epsilon_1(s), \ldots, \epsilon_{n+m}(s)$ are the invariant polynomials of $P(s)$, and $z_i(s)$ is the last invariant polynomial of $P_i(s)$, $i = 1, \ldots, m$.

Proof. The set of invariant zeros of (A, B, C) are the roots of the polynomials $\epsilon_i(s)$, and it is evident from (5) that the only frequency values that can be finite zeros of the decoupled closed-loop system are the roots of the polynomials $z_i(s)$. If $\delta(s)$ is not a unit, then some of the poles of the system (the fixed decoupling poles) must be located at the positions of the roots of $\delta(s)$ producing cancellation with finite zeros of the system. ■

Remark 1. From the previous result, it can be seen that the fixed decoupling poles correspond to invariant zeros which are not row invariant zeros of the system.

Remark 2. It follows from Theorem 2 that the number of poles which can be arbitrarily assigned while decoupling the system is equal to

$$n - \deg \delta(s), \tag{8}$$

where n is the order of the system and $\delta(s)$ is given by (7).

4 Algorithm for decoupling and pole assignment

The relevant information to solve the decoupling and complete pole assignment problem is the global and row infinite zeros, and global and row invariant finite zeros of the system. It is well known that computing the finite or infinite structure of a pencil from its Smith form is not numerically reliable. Instead of that, we will obtain this information in a numerically reliable way from the Kronecker invariants of suitable matrix pencils. The algorithm for that purpose is based on the results from [17, 18] and uses only numerically reliable operations such as Householder transformations or the singular value decomposition (SVD), see for instance [7, 8].

Let us consider an arbitrary $m \times n$ matrix A and compute its SVD,

$$P^T A Q = \Sigma$$

where Σ is an $m \times n$ matrix with singular values of A along its main diagonal. The rank r of A corresponds to the number of non-zero singular values, and we have that

$$P^T A = \left[\frac{A_r}{0} \right], \tag{9a}$$

$$AQ = \left[\begin{array}{c|c} A_c & 0 \end{array} \right] \tag{9b}$$

where A_r and A_c have r linearly independent rows and columns respectively. Operation (9a) is called row compression and operation (9b) column compression of A.

Consider an arbitrary $m \times n$ pencil $\mathcal{P}(s) = sE - L$. The algorithm to obtain the eigenstructure of $\mathcal{P}(s)$ is described as follows (see [17]).

- Let $E_1 = E$, $n_1 = n$, $m_1 = m$ and $L_1 = L$.
- Step k: Obtain the SVD $\Sigma = P^T E_k Q$ and the rank ρ_k of E_k. If $v_k = m_k - \rho_k$ is not zero, make the permuted row compressions.

  $$I_p P^T E_k = \begin{bmatrix} 0 \\ E_k \end{bmatrix}, \quad I_p P^T L_k = \begin{bmatrix} \bar{L}_k \\ L_k \end{bmatrix},$$

  where $I_p = \begin{bmatrix} 0 & I_{\rho_k} \\ I_{v_k} & 0 \end{bmatrix}$. Obtain the SVD $S = P^T \bar{L}_k Q$ and the rank r_k of $\bar{L}_k$, and make the column compressions

  $$\bar{L}_k Q = [\times|0], \quad E_k Q = [\times|E_{k+1}], \quad L_k Q = [\times|L_{k+1}].$$

  where irrelevant entries are denoted by $\times$.
- Update the dimensions

  $$m_{k+1} = m_k - v_k, \quad n_{k+1} = n_k - r_k$$

  and go to next step $k+1$ (notice that in each step matrices E_k and L_k are updated).

If v_k is zero, structural indices at infinity of $\mathcal{P}(s)$ can be recovered from vectors $v = [v_1, \ldots, v_k]$ and $r = [r_1, \ldots, r_{k-1}]$. For $i = 1, 2, \ldots, k-1$, $\mathcal{P}(s)$ has $r_i - v_{i+1}$ zeros at infinity of degree $i - 1$. In addition, it can be shown that the $m_k \times n_k$ pencil $sE_k - L_k$ contains only the finite structure and the right null space of $\mathcal{P}(s)$.

Now we take the reduced pencil $sE_k - L_k$ and apply a dual process, namely, we start with the column compressions

$$E_k Q = [E_k|0], \quad L_k Q = [L_k|\bar{L}_k],$$

and apply the row permuted compressions

$$I_p P^T \bar{L}_k = \begin{bmatrix} 0 \\ \times \end{bmatrix}, \quad I_p P^T E_k = \begin{bmatrix} \times \\ E_{k+1} \end{bmatrix},$$

$$I_p P^T L_k = \begin{bmatrix} \times \\ L_{k+1} \end{bmatrix},$$

update the dimensions

$$n_{k+1} = n_k - v_k, \quad m_{k+1} = m_k - r_k$$

and go to the next step.

When the dual process is finished, the new square reduced pencil $s\widehat{E} - \widehat{L}$ contains only the finite zeros of $\mathcal{P}(s)$.

Then, using the well known QZ factorization

$$Q\widehat{E}Z = \widetilde{E}, \;\; Q\widehat{L}Z = \widetilde{L},$$

the finite zeros can be obtained as ratios of diagonal elements $\alpha_i = \widetilde{l}_{ii}/\widetilde{e}_{ii}$. The QZ factorization is also based on Householder transformations, see [7].

The above method allows to obtain the eigenstructure of a given pencil in a numerically reliable way. Then, we can easily obtain the global and row, finite and infinite zeros of the system (A, B, C) from the eigenstructure of the pencils given by (2) and (3). In this way, the conditions (4) for decoupling can be tested, and the whole set of attainable finite pole-zero structures for the decoupled closed-loop system can be characterized.

To complete the algorithm, we show how to compute the state feedback which produces a decoupled closed-loop system with a particular finite pole-zero structure. It is shown in [16] that for a particular choice of closed-loop transfer function matrix from the set (5), say $W_1(s)$, the corresponding state feedback (F, G) producing $W_1(s)$ is unique if and only if the system is controllable. To obtain (F, G) we use the following method, which has the advantage in comparison to the one of [16] that it is not necessary to obtain a matrix fraction description $N(s)$, $D(s)$ of the system with $D(s)$ column reduced.

Let $W_1(s)$ be a particular transfer function matrix from the set (5), and define

$$Q(s) = T^{-1}(s)W(s) = Q_0 + \bar{Q}(s),$$

where $T(s)$ is the transfer function of the system, Q_0 is a constant matrix, and $\bar{Q}(s)$ is a strictly proper rational matrix.

Then, we seek matrices F and G such that

$$Q(s) = [I - F(sI - A)^{-1}B]^{-1}G.$$

From the last equation, it can be seen that matrix G is given by

$$G = \lim_{s\to\infty} Q(s) = Q_0. \tag{10}$$

To compute matrix F, let $\begin{bmatrix} L & E \end{bmatrix}$ be a basis for the left constant kernel of the matrix

$$\begin{bmatrix} (sI - A)^{-1}B \\ I - GQ^{-1}(s) \end{bmatrix}, \tag{11}$$

where $L \in \mathbb{R}^{m\times n}$, $E \in \mathbb{R}^{m\times m}$, and E is nonsingular (such matrices always exist since the system is decouplable). Observe that the left constant kernel is not modified if matrix (11) is transformed into

$$\begin{bmatrix} \det(Q)\ \mathrm{adj}(sI - A)B \\ \det(sI - A)(\det(Q)I - G\ \mathrm{adj}(Q)) \end{bmatrix}$$

using the reliable methods to compute the adjoint of a polynomial matrix presented in [9, 10]. Thus, this information can be obtained from the constant kernel of a polynomial matrix. To this end, we use numerical reliable routines based on Householder transformations, see [1].

Finally, the matrix F satisfying

$$C(sI - A - BF)^{-1}BG = W_1(s)$$

is given by

$$F = -LE^{-1}. \tag{12}$$

The previously described algorithm is applied to the following example, which illustrates also the issues of pole assignment and cancellation of invariant zeros.

Example 1. Let the controllable system (A, B, C) be given by

$$A = \begin{bmatrix} -2 & 3 & 0 & -1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ -2 & -1 & -1 & 3 & 5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ -1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

whose transfer function is

$$T(s) = \begin{bmatrix} \frac{1}{(s-2)(s+2)} & 0 \\ \frac{s-1}{(s-2)(s+2)^3} & \frac{s+1}{(s+2)^2} \end{bmatrix}.$$

Applying the reported algorithm, we obtain the following results, which can be easily checked: the system is decouplable, the set of matrices which can be obtained as transfer function matrices for the decoupled closed-loop system is given by

$$W(s) = \begin{bmatrix} \frac{k_1}{(s+\alpha_1)(s+\alpha_2)} & 0 \\ 0 & \frac{k_2(s-1)}{(s+\alpha_3)(s+\alpha_4)} \end{bmatrix}.$$

and there exists a fixed decoupling pole at $s = -1$. Notice that the system is decouplable with stability. Observe also that $s = 1$ is an invariant row and global zero of the system, which is not evident from the system transfer function, since the system is not observable.

Let us choose a pole-zero finite structure corresponding to the following matrix

$$W_1(s) = \begin{bmatrix} \frac{1}{(s+1)(s+2)} & 0 \\ 0 & \frac{s-1}{(s+2)^2} \end{bmatrix}$$

The unique state feedback producing $W_1(s)$ returned by the algorithm is given by

$$F = \begin{bmatrix} -3 & -6 & 3 & 9 & 6 \\ -2 & -7 & 0 & 1 & -1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

5 Conclusions

We presented in this paper a reliable numerical algorithm which tests the conditions for decoupling and computes the state feedback which decouples the system with a particular pole-zero finite structure, avoiding unnecessary cancellations of invariant zeros.

It is well known that small variations on the values of the system matrices of a perfectly decouplable system, due for instance to rounding errors, can lead to the wrong conclusion that the system is not decouplable, or not decouplable with a particular structure (see for instance Example 2 in [2]). This is a problem of the system representation, and not of the reliability or stability of the algorithm itself. Concepts of well-posedness, genericity, and "robust decoupling" have to be considered to give an answer to this problem.

Numerical testings, which are not included in this paper, demonstrated that the algorithm performs quite well. The stability and computational complexity of the algorithm remain to be studied.

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