H_∞ MODEL MATCHING IN TWO DEGREE OF FREEDOM CONTROL STRUCTURE

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Abstract
In this study, the model-matching problem (MMP) in two degree of freedom (2DOF) control structure is defined and solved in the sense of the H_∞ optimality criterion in the framework Linear Matrix Inequality (LMI) by using the results on the standard H_∞ OCP.

Keywords: Model Matching Problem; Linear Matrix Inequalities; H_∞ Optimal Control; 2DOF Control Structure.

1 Introduction

The standard H_∞ MMP is defined in [5] as to find a controller transfer matrix $\boldsymbol{R(s)}$ with property of stable and proper rational matrix, i.e., $\boldsymbol{R(s) \in RH_\infty}$, that minimizes the H_∞ norm of $\boldsymbol{T_m(s)-T_1(s)R(s)T_2(s)}$, such that the stable and proper rational matrices $\boldsymbol{T_m(s)}$ and $[\boldsymbol{T_1(s),T_2(s)}]$ are given as the model and the system transfer matrices, respectively. The H_∞ norm of a transfer matrix is defined as the maximum value over all frequencies of its largest singular value. This means that the performance of the system described by $\boldsymbol{T_1(s)R(s)T_2(s)}$ approximates the desired performance as given in $\boldsymbol{T_m(s)}$, in the sense of the following criterion,

$$\gamma_{opt} = \inf_{R(s) \in \Re H_\infty} \left\{ \left\| T_m(s) - T_1(s)R(s)T_2(s) \right\|_\infty \right\} \quad (1)$$

While this problem is also known as the bilateral H_∞ MMP, the unilateral H_∞ MMP is defined as to find a controller transfer matrix $\boldsymbol{R(s) \in RH_\infty}$ that minimizes the H_∞ norm of $\boldsymbol{T_m(s)-T(s)R(s)}$, such that the stable and proper rational matrices $\boldsymbol{T_m(s)}$ and $\boldsymbol{T(s)}$ are given as the model and the system transfer matrices respectively. In the literature, there are several results on the standard H_∞ MMP. Two of them are based on Nevanlinna-Pick Problem (NPP), [1], and Nehari Problem (NP) [4], [5]. In these studies, the H_∞ MMP has been reduced to the one of these problems and then by using the results on the solution of NPP or NP, first the value γ_{opt} defined in (1) is found and then the controller transfer matrix $\boldsymbol{R(s)}$ is obtained in the form of stable and proper rational matrix. Some studies concerning with H_∞ MMP are considered in the concept of the standard H_∞ Optimal Control Problem (OCP). A complete state space solution to the standard H_∞ OCP is given in [2], (DGKF, 1989). The relationship between model matching problem and DGKF solution for generalized plant setting has been investigated in [9] via J-spectral factorization theory. A state space solution of the unilateral H_∞ MMP is given in [10], such that this solution is based on canonical spectral factorizations and solutions of the Algebraic Riccati Equations (ARE). Gahinet & Apkarian re-derived the solution of the standard H_∞ OCP given by DGKF in the framework of LMI in [6], and by using these results an LMI-based solution of the unilateral H_∞ MMP is presented, also used to obtain a solution of a multiobjective H_∞ control problem in [8]. In all these studies on the standard H_∞ MMP, the controller structures that could be used in feedback configuration have not been considered in the formulation of the problem. However, one can say that the controller $\boldsymbol{R(s)}$ with property of stable and causal rational matrix, which is found in the form of a pre-compensator as a solution of the unilateral H_∞ MMP, can generally be established by dynamic state feedback in the feedback configuration [11], [8].

To consider the control structures used for the system to be controlled in the formulation of the problem, the H_∞ MMP is defined as to find a controller that minimizes the H_∞ norm of $\boldsymbol{T_m(s)-T_{cl}(s)}$ in the specific control structure, such that the rational matrices $\boldsymbol{T_m(s)}$ and $\boldsymbol{T_{cl}(s)}$ are the model and the closed loop system transfer matrices respectively.

In this paper, the 2DOF control structure is chosen as the structure that could be used in feedback configuration to study on the H_∞ MMP. To solve the H_∞ MMP in the 2DOF control structure, it is described as a standard H_∞ OCP and then the LMI-based solvability conditions are derived by using the results given in [6] and [8].

The following notation will be used throughout the paper: ***Ker M*** and ***Im M*** for the null space and range of the linear operator associated with M respectively and N^* for the transpose conjugate of N matrix. Finally, $P>0$ denotes that P matrix is positive definite.

2 Problem Formulation

Consider a realizations (A, B, C, D) of $T(s)$, namely the system to be controlled, and (F, G, H, J) of $T_m(s)$, namely model system, so the state space equations of these systems can be given as follows,

$$T(s);\quad \begin{aligned} \frac{dx(t)}{dt} &= A\,x(t)+B\,u(t) \\ y_s(t) &= C\,x(t)+D\,u(t) \end{aligned} \tag{2}$$

$$T_m(s);\quad \begin{aligned} \frac{dq(t)}{dt} &= F\,q(t)+G\,w(t) \\ y_m(t) &= H\,q(t)+J\,w(t) \end{aligned} \tag{3}$$

Also consider the dynamic 2DOF control structure with output feedback, the control input $u(t)$ is generated by the reference input $w(t)$ and the system output $y_s(t)$ such that,

$$U(s) = L(s)Y_s(s) + M(s)W(s) \tag{4a}$$

where $x(t) \in R^n$,$q \in R^{n_m}$, $u(t) \in R^m$, $w(t) \in R^{m_r}$, $y_s(t) \in R^p$,$y_m(t) \in R^p$. Figure 1 illustrates these considerations.

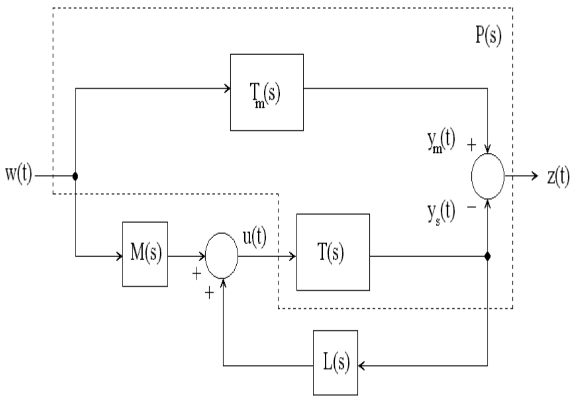

Figure 1

At this point, the following Definition can be given for the H_∞ MMP in 2DOF control structure.

Definition 1: The H_∞ MMP in 2DOF control structure is defined as to find the controller transfer matrices $M(s), L(s) \in RH_\infty$ that minimizes the H_∞ norm of the transfer matrix $T_{zw}(s)$ defined as,

$$\begin{aligned} T_{zw}(s) &= T_m(s) - \left[I - T(s)L(s)\right]^{-1}T(s)M(s) = \\ &= T_m(s) - T(s)\left[I - L(s)T(s)\right]^{-1}M(s) \end{aligned}$$

such that the proper rational matrices $T_m(s)=H(sI-F)^{-1}+J$ and $T(s)=C(sI-A)^{-1}B+D$ are given as the model and the system transfer matrices respectively.□

3 The solution of the H_∞ MMP in 2DOF control structure

In this section, the LMI based solvability conditions of the H_∞ MMP in 2DOF control structure is derived. For this aim, consider a plant $P(s)$ described by,

$$\frac{d}{dt}\begin{bmatrix} x(t) \\ q(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix}\begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} w(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \tag{5}$$

$$z(t) = y_m(t) - y_s(t) = \begin{bmatrix} -C & H \end{bmatrix}\begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + J\,w(t) - D\,u(t) \tag{6}$$

$$y(t) = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} w(t) + \begin{bmatrix} D \\ 0 \end{bmatrix} u(t) \tag{7}$$

a controller $K(s)$ defined as,

$$K(s) = \begin{bmatrix} L(s) & M(s) \end{bmatrix} \tag{8a}$$

and Figure 2, so the closed loop transfer matrix $T_{zw}(s)$ is obtained as,

$$
\begin{aligned}
T_{zw}(s) &= T_m(s) - \\
&- T(s)K(s)\left(\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} - \begin{bmatrix} T(s) \\ 0 \end{bmatrix} K(s)\right)^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} \\
&= T_m(s) - \left[I - T(s)L(s)\right]^{-1} T(s)M(s)
\end{aligned}
\tag{9a}
$$

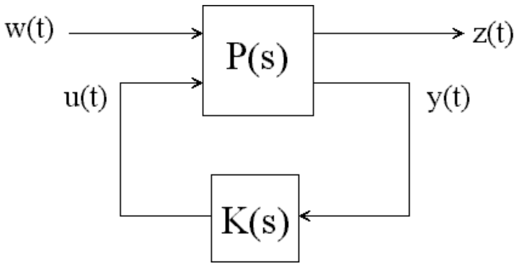

Figure 2

Note that $P(s)$ in Figure 2 is described as follows,

$$
P(s) = \begin{bmatrix} J & -D \\ 0 & D \\ I & 0 \end{bmatrix} + \begin{bmatrix} -C & H \\ C & 0 \\ 0 & 0 \end{bmatrix} \left[\begin{pmatrix} sI & 0 \\ 0 & sI \end{pmatrix} - \begin{pmatrix} A & 0 \\ 0 & F \end{pmatrix}\right]^{-1} \begin{bmatrix} 0 & B \\ G & 0 \end{bmatrix}
\tag{10a}
$$

To use the results on standard H_∞ OCP for solving H_∞ MMP in 2DOF control structure, we assume $D=0$, namely the system to be controlled must be strictly proper, thus the system will be well posed. As it is known, the solution of the standard H_∞ OCP gives all admissible controllers $K(s)$ for $P(s)$ shown in Figure 2, such that $\|T_{zw}(s)\|_\infty$ are minimized. The following Preposition provides the existence conditions of internally stabilizing controllers for the plant defined by (10a).

Preposition 1: A necessary and sufficient condition for the existence of internally stabilizing $K(s)$ for Figure 2 and the plant $P(s)$ given in (10) is that $\left(\begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix}, \begin{bmatrix} B \\ 0 \end{bmatrix}, \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}\right)$ is stabilizable, namely F is Hurwitz and (A, B, C) is stabilizable.

Proof: See *Preposition 5.6* in [3]. □

Throughout the paper, we assume that (A, B, C) is stabilizable, i.e., there exists a constant matrix K such that A-BK is Hurwitz, and (A, C) is detectable i.e., there exists a constant matrix L such that A-LC is Hurwitz. The synthesis theorem for H_∞ OCP in formulation of LMIs given in [6] can be written for the H_∞ MMP in 2DOF control structure as the following Lemma.

Lemma 1 A controller $K(s)=[L(s)\ M(s)]$ with order $n_K \geq dim\ A + dim\ F$ which holds $\|T_{zw}(s)\|_\infty < \gamma$ exists for the plant described by (5-7) and closed-loop system is internally stable for H_∞ Optimal Control Problem if and only if there exist symmetric matrices $X>0$ and $Y>0$ such that

$$
\begin{bmatrix} N_o & 0 \\ 0 & I \end{bmatrix}^*
\begin{bmatrix}
\begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} X + X \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} & X \begin{bmatrix} 0 \\ G \end{bmatrix} & \begin{bmatrix} -C^* \\ H^* \end{bmatrix} \\
\begin{bmatrix} 0 & G^* \end{bmatrix} X & -\gamma I & J^* \\
\begin{bmatrix} -C & H \end{bmatrix} & J & -\gamma I
\end{bmatrix}
\begin{bmatrix} N_0 & 0 \\ 0 & I \end{bmatrix} < 0
\tag{11}
$$

$$
\begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix}^*
\begin{bmatrix}
\begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} Y + Y \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} & Y \begin{bmatrix} -C^* \\ H^* \end{bmatrix} & \begin{bmatrix} 0 \\ G \end{bmatrix} \\
\begin{bmatrix} -C & H \end{bmatrix} Y & -\gamma I & J \\
\begin{bmatrix} 0 & G^* \end{bmatrix} & J^* & -\gamma I
\end{bmatrix}
\begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix} < 0
\tag{12}
$$

$$
\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0
\tag{13}
$$

where N_o and N_c are full rank matrices whose images satisfy

$$
Im\ N_o = Ker \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & I_{m_1} \end{bmatrix}
\tag{14}
$$

$$
Im\ N_c = Ker \begin{bmatrix} B^* & 0 & 0 \end{bmatrix}
\tag{15}
$$

Proof: The claims of the Lemma are the same as those of the synthesis theorem for H_∞ OCP in the framework of LMIs presented in [6], they are only rewritten for the system $P(s)$ given in (5-7). □

In order to obtain some specific results for H_∞ MMP, it should be studied on Lemma 1. For this aim, the following Lemmas are given.

Lemma 2 Suppose A and Q are square matrices and $Q>0$. Then A is Hurwitz if and only if there exists the unique solution,

$$X = \int_0^\infty e^{A^* t} Q e^{At} dt > 0$$

to the Lyapunov equation $A^* X + XA + Q = 0$.

Proof: See [3]. □

Lemma 3 The block matrix,

$$\begin{bmatrix} P & M \\ M^* & N \end{bmatrix} < 0$$

if and only if $N<0$ and $P-MN^{-1}M^*<0$. In the sequel, $P-MN^{-1}M^*$ will be referred to as the Schur Complement of N.

Proof: See [3].□

Lemma 4 Suppose A, C, X and Y are square matrices and $\gamma \in R$. If the matrix A is Hurwitz, then for every pair of $\gamma>0$ and $Y>0$, there always exists a matrix $X>0$ such that holds the following inequalities,

$$A^* X + XA + \frac{1}{\gamma} C^* C < 0 \tag{16}$$

$$X - Y^{-1} \geq 0 \tag{17}$$

Moreover, some matrices with satisfying (16) and (17) are generated by the following explicit relation,

$$X = \varepsilon\, P_0 + \frac{1}{\gamma} L_0 \tag{18}$$

in which, $\varepsilon \in R^+$ and holds,

$$\varepsilon \geq \lambda_{max}\left[(P^{-1})^* (Y^{-1} - \frac{1}{\gamma} L_0) P^{-1} \right] \tag{19}$$

and L_0 is Observabilty Gramian of (A,C) as

$$L_0 = \int_0^\infty e^{A^* t} \frac{1}{\gamma} C^* C\, e^{At} dt \geq 0 \tag{20}$$

and $P_0>0$ is a solution of the equation $A^* P_0 + P_0 A + Q = 0$ for $Q>0$, and P is a nonsingular matrix with satisfying $P_0 = P^* P$.

Proof: See [8].□

Lemma 5 Suppose (A, C) detectable and $Im\ N = Ker\ C$, there exist some $X>0$ such that the following inequality holds,

$$N^* \left(A^* X + XA \right) N < 0 \tag{21}$$

Furthermore, these matrices $X>0$ with satisfying (21) can be generated by the following relation,

$$X = \varepsilon\, X_0, \quad X_0 = \int_0^\infty e^{A_1^* t} e^{A_1 t} dt > 0 \tag{22}$$

where $A_1 = A - LC$ and Hurwitz and $\varepsilon \in R^+$.

Proof: Since (A, C) is detectable then there always exist the matrices L with compatible dimensions and $X>0$, such that $(A-LC)$ is Hurwitz and so the following inequality holds,

$$(A - LC)^* X + X(A - LC) < 0 \tag{23}$$

because of Lemma 2. Since $(A-LC)\,N = A\,N$ and so $N^*(A-LC)^* = N^* A^*$ and the inequality (21) is obtained by pre- and post-multiplying (23) with N^* and N respectively, then the proof is completed by using Lemma 2.□

It will be useful to give the following Conclusion as a straightforward result of last two Lemmas, to provide an easy proof of a theorem on the H_∞ MMP in 2DOF control structure, which will be given after that.

Conclusion 1: Suppose F Hurwitz and (A, C) detectable and $Im\ N = Ker\ C$, then for every pair of $\gamma>0$ and $Y>0$, there always exists a matrix $X>0$ such that hold the following inequalities,

$$\begin{bmatrix} N^* & 0 & 0 \\ 0 & I_{n_1} & I_{p_1} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} X + X \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} & \begin{bmatrix} -C^* \\ H^* \end{bmatrix} \\ \begin{bmatrix} -C & H \end{bmatrix} & -\gamma I \end{bmatrix} \begin{bmatrix} N^* & 0 \\ 0 & I_{n_1} \\ 0 & I_{p_1} \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \tag{25}$$

The matrices $X>0$ which satisfy (24) and (25) can be generated by the following relations,

$$X = \begin{bmatrix} X_1 & 0 \\ 0 & X_3 \end{bmatrix},\ X_1 = \varepsilon X_0,$$
$$X_3 = \varepsilon\ P_0 + \frac{1}{\gamma} L_0 \tag{26}$$

in which $\varepsilon \in R^+$ and holds,

$\varepsilon \geq \lambda_{max}\left[(P^{-1})^* (Y^{-1} - \frac{1}{\gamma} L_0) P^{-1} \right]$, and

$X_0 = \int_0^\infty e^{A_1^* t} e^{A_1 t} dt > 0$ such that $A_1 = A - LC$ and Hurwitz , and L_0 is Observabilty Gramian of (F,H) as,

$$L_0 = \int_0^\infty e^{F^* t} \frac{1}{\gamma} H^* H e^{Ft} dt \geq 0$$

and $P_0 > 0$ is a solution of the equation,

$$F^* P_0 + P_0 F + Q = 0$$

for $Q>0$, and P is a nonsingular matrix with satisfying $P_0 = P^* P$.

Proof: Let the matrix X be block diagonal with appropriate dimensions as $X = \begin{bmatrix} X_1 & 0 \\ 0 & X_3 \end{bmatrix}$, then the LMI (24) can be written as $\begin{bmatrix} \Psi_1 & 0 \\ 0 & \Psi_3 \end{bmatrix} < 0$ where,

$$\Psi_1 = N^* \left(A^* X_1 + X_1 A \right) N < 0,$$
$$\Psi_3 = \begin{bmatrix} I & I \end{bmatrix} \begin{bmatrix} F^* X_3 + X_3 F & H^* \\ H & -\gamma I \end{bmatrix} \begin{bmatrix} I \\ I \end{bmatrix} < 0 \tag{27}$$

so the proof is completed by applying Lemma 3 to (27) and using Lemma 4 and Lemma 5.□

The following theorem can be presented on LMI based solution of the H_∞ MMP in 2DOF control structures as a reduced version of Lemma 1.

Theorem 1 A controller ***K(s)=[L(s) M(s)]*** with order $n_K \geq \dim A + \dim F$, which the transfer matrices $T_{zw}(s)$ given in (9) hold $\|T_{zw}(s)\|_\infty < \gamma$, exists for the plant described by (5-7) and the closed-loop systems are internally stable, i.e., there exists a solution of the H_∞ MMP in 2DOF control structure for the system and model given by (2) and (3) respectively, if and only if there exists a symmetric matrix $Y>0$; such that the following inequality holds,

$$\begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix}^* \begin{bmatrix} \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} Y + Y \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} & Y \begin{bmatrix} -C^* \\ H^* \end{bmatrix} & \begin{bmatrix} 0 \\ G \end{bmatrix} \\ \begin{bmatrix} -C & H \end{bmatrix} Y & -\gamma I & J \\ \begin{bmatrix} 0 & G^* \end{bmatrix} & J^* & -\gamma I \end{bmatrix} \begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix} < 0 \tag{28}$$

where N_c is a full rank matrix with,

$$Im\ N_c = Ker \begin{bmatrix} B^* & 0 & 0 \end{bmatrix}$$

and (A, B, C) and (F, G, H, J) are the state space description of the $T(s)$ and $T_m(s) \in RH_\infty$ respectively, such that (A, B) is stabilizable and (A, C) is detectable .

Proof: It is easily seen that the claim of the Theorem is the same as the condition (12) of Lemma 1. To complete the proof, it will be sufficient to show that the conditions (11) and (13) are already satisfied. For this aim, the condition (11) in Lemma 1 can be rewritten as follows,

$$\begin{bmatrix} N^* & 0 & 0 & 0 \\ 0 & I_{n_1} & 0 & I_{p_1} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} X + X \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} & X \begin{bmatrix} 0 \\ G \end{bmatrix} & \begin{bmatrix} -C^* \\ H^* \end{bmatrix} \\ \begin{bmatrix} 0 & G^* \end{bmatrix} X & -\gamma I & J^* \\ \begin{bmatrix} -C & H \end{bmatrix} & J & -\gamma I \end{bmatrix} \begin{bmatrix} N & 0 \\ I_{n_1} & 0 \\ 0 & 0 \\ 0 & I_{p_1} \end{bmatrix} < 0 \tag{29}$$

since $Im\ N_0 = Ker \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & I_{m_1} \end{bmatrix}$ so,

$N_0 = \begin{bmatrix} N & 0 \\ 0 & I_{n_1} \\ 0 & 0 \end{bmatrix}$ such that $ImN = KerC$.

Furthermore, the inequality (29) can also be written as follows,

$$\begin{bmatrix} N^* & 0 & 0 \\ 0 & I_{n_1} & I_{p_1} \end{bmatrix}$$

$$\begin{bmatrix} \begin{bmatrix} A^* & 0 \\ 0 & F^* \end{bmatrix} X + X \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} & \begin{bmatrix} -C^* \\ H^* \end{bmatrix} \\ \begin{bmatrix} -C & H \end{bmatrix} & -\gamma I \end{bmatrix} \tag{30}$$

$$\begin{bmatrix} N & 0 \\ 0 & I_{n_1} \\ 0 & I_{p_1} \end{bmatrix} < 0$$

Since ***(A, C)*** is detectable and ***F*** is Hurwitz, it can easily be seen that there always exist some ***X>0*** with satisfying the inequalities (30) and (13) by using Conclusion 1. This means that the conditions (11) and (13) given in Lemma 1 are already satisfied, hence the proof is completed. □

In order to construct the controllers ***L(s)*** and ***M(s)***, it must be useful to give a brief procedure; suppose the matrix ***Y>0*** and the minimum value of $\gamma_{opt} \in R^+$ are found as a solution of (28) by using LMI toolbox [7]. Then a matrix ***X>0*** is found by using (26), such that the inequalities given in (30) and (13) hold. Finally, the controller transfer matrix ***K(s)***, which minimizes the $\| T_{zw}(s) \|_\infty$ given in (9a) is obtained as,

$$K(s) = D_k + C_k (sI - A_k)^{-1} B_k \tag{31}$$

by using the matrices ***X*** and ***Y***, via the controller reconstruction procedure given in [6]. Thus the transfer matrices of the feedback and the feed-forward controllers ***L(s)*** and ***M(s)*** respectively, i.e., the solution of the H∞ MMP in 2DOF control structure for the system and model given by (2) and (3) respectively, are found from the definition ***K(s)=[L(s) M(s)]***.

6 Conclusions

In this paper, we have studied on the H∞ MMP in the 2DOF control structure. The LMI-based solution of the problem by using this control structure has been presented with including some relations with the solution of OPC.

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