OBSERVER BASED STABILIZATION OF DISCRETE-TIME NONLINEAR SYSTEMS

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Abstract

In this note we propose an observer based stabilization method for nonlinear discrete-time systems. The approach we use here is based on the stabilization method recently developed in [5] coupled with the EKO.
From the Lyapunov approach, sufficient conditions for stability are deduced and expressed in terms of LMI that depend on arbitrary matrices fixed by the user. This has the advantage to enlarge the class of systems to be considered as it can be shown through numerical examples.

1. Introduction

Over the past four decades, stabilization of nonlinear dynamical systems has received a great attention in the literature as it can be shown through basic works in this field [3], [15] and [22]. Several design methodologies have been developed for local and global stabilization problems of continuous and discrete-time nonlinear systems, see for instance [1], [6], [18], [19], [21], [23] and the references inside.
When the control laws are designed, the state variables are assumed to be available. But in general, this is not true in practice and the current state must be estimated by another dynamical system, that is a state observer.
Thus, observer based stabilization of nonlinear systems has been studied in the past few years. The main contributions, however, concern continuous time systems ; this problem has been investigated by several authors, among them [2], [9], [12], [14] and [20].
For discrete-time nonlinear systems only few designing methods have been established [6], [8] and [17]. Relevant ones have been developed by Byrnes and Lin [7] and Lin [17]. In particular the work in [17], where a global stabilization is achieved via state and output feedback, the proposed technique is judicious but only systems with stable state unforced dynamics are considered, this may be seen as a conservative condition.
The aim of this work is to analyse behaviour of the state feedback stabilization method recently developed in [5] with the use of the EKO. Thanks to simple Lypunov function, sufficient condition for stability are deduced and seem to work for a large clan of nonlinear systems even with unstable unforced dynamics. Two numerical examples are provided to show performances of the proposed method and easiness of the implementation.

2. Problem formulation

Consider a class of discrete-time multi-input multi-output (MIMO) nonlinear systems of the form

$$x_{k+1} = A(x_k)x_k + g(x_k)u_k = f(x_k, u_k) \quad (1)$$

$$y_k = h(x_k) \quad (2)$$

where $x_k \in IR^n$, $u_k \in IR^y$ and $y_k \in IR^p$ denote the state, input and output vectors respectively. The matrix A(.), g(.) and the vector h(.) are continuously differentiable nonlinear maps.

Problem :
The problem is to find a dynamic compensator

$$\xi_{k+1} = \eta(\xi_k, y_k)$$
$$u_k = \theta(\xi_k) \quad (3)$$

so that the closed loop system (1)-(3) is asymptotically stable at the equilibrium $(x, x-\xi) = (0,0)$.

3. Main result

Consider the following EKO based stabilizator

$$\xi_{k+1} = \xi_{k+1/k} + K_{k+1}e_{k+1} \quad (4.a)$$

$$u_k = -L_k\xi_k \quad (4.b)$$

and the error vectors

$$\tilde{x}_{k+1} = x_{k+1} - \xi_{k+1} \quad (5)$$

$$\tilde{x}_{k+1/k} = x_{k+1} - \xi_{k+1/k} \quad (6)$$

Where

$$\xi_{k+1/k} = f_{uk}(\xi_k) \tag{7}$$

$$f_{uk}(x_k) = f(x_k, u_k) \tag{8}$$

$$L_k = \left(\Omega_k + g_k^T P_k g_k\right)^{-1} g_k^T P_k A_k \tag{9}$$

$$P_{k+1} = \lambda_k \left(\Lambda_k^T P_k \Lambda_k + L_k^T \Omega_k L_k + Q_k\right) \tag{10}$$

$$K_{k+1} = \Sigma_{k+1/k} H_{k+1}^T \left(R_{k+1} + H_{k+1}\Sigma_{k+1/k} H_{k+1}^T\right)^{-1} \tag{11}$$

$$\Sigma_{k+1/k} = F_k \Sigma_k F_k^T + S_k \tag{12}$$

$$\Sigma_{k+1} = \left(I_n - K_{k+1} H_{k+1}\right)\Sigma_{k+1/k} \tag{13}$$

With

$$e_{k+1} = y_{k+1} - h(\xi_{k+1/k}) \tag{14}$$

$$F_k = F_{uk}(\xi_k) = \left.\frac{\partial f(x_k, -L_k x_k)}{\partial x_k}\right|_{x_k = \xi_k} \tag{15}$$

$$H_{k+1} = H_{k+1}(\xi_{k+1/k}) = \left.\frac{\partial h(x_{k+1})}{\partial x_{k+1}}\right|_{x_{k+1} = \xi_{k+1/k}} \tag{16}$$

$$\Lambda_k = A_k - g_k L_k \tag{17}$$

$$A_k = A(\xi_k),\ g_k = g(\xi_k) \text{ and } P_k = P(\xi_k) \tag{18}$$

The main result of this paper is summarized in the following theorem.

3.1 Theorem :

Assume that there exists an integer N such that:

H1) $\sqrt{\lambda_k \lambda_{k-1} .. \lambda_{k-N}} \left\| A_k A_{k-1} .. A_{k-N} \right\| < 1$.

H2) $P_k^{1/2} \Lambda_k^T \left(A_k^T \left(P_k^{-1} + g_k \Omega_k^{-1} g_k^T\right)^{-1} A_k + Q_k\right)^{-1} \Lambda_k P_k^{1/2} \le \lambda_k (1-\delta) I_n$.

H3) (1) and (2) is rank observable, i.e.,

$$rank \frac{\partial}{\partial x} \left.\begin{pmatrix} h_k(x) \\ h_{k+1} \circ f_{uk}(x) \\ \vdots \\ h_{k+N-1} \circ f_{uk+N-2} \circ \cdots \circ f_{uk}(x) \end{pmatrix}\right|_{x = x_k} = n \tag{19}$$

for all $x_k \in$ K and N-tuple of controls $(u_k, \cdots, u_{k+N-1}) \in$ U (K and U are two compact subsets of IR^n and $(IR^r)^N$, respectively).

H4) F_k and H_k are uniformly bounded matrices and F_k^{-1} exists.

H5) The instrumental matrices R_k, S_k are chosen so that

$$\left|\alpha_{i(k+1)} - 1\right| \le \overline{\alpha}_{k+1} = \sup_i \left|\alpha_{i(k+1)} - 1\right| \le \left(\frac{\underline{\sigma}(R_{k+1})}{\overline{\sigma}(H_{k+1}\Sigma_{k+1/k} H_{k+1}^T + R_{k+1})}\right)^{1/2} \quad \text{for } i = 1 \ldots p \tag{20}$$

$$\left|\beta_{jk}\right| \le \overline{\beta}_k = \sup_j \left|\beta_{jk}\right| \le \left(\frac{(1-\delta)\underline{\sigma}(F_k \Sigma_k F_k^T + S_k)}{\overline{\sigma}(F_k^T)\overline{\sigma}(\Sigma_k)\overline{\sigma}(F_k)}\right)^{1/2} \quad \text{for } j = 1 \ldots n \tag{21}$$

$\overline{\sigma}$ and $\underline{\sigma}$ denote the maximum and minimum singular values, respectively.

Then, the EKO based stabilization method (4) renders the equilibrium $(x_k, \tilde{x}_k) = (0,0)$ of the closed loop system (1)-(2)-(4.b) asymptotically stable.

The parameter α_k and β_k will be detailed later. The arbitrary positive real parameters $\{\lambda_k\}_{k=1,\ldots}$ and the positive definite matrices Ω_k and Q_k are fixed by the user.

4. Convergence Analysis

In this section, the convergence analysis of the EKO based stabilization law (1)-(2)-(5) will be performed by the standard Lyapunov approach.

First, we define a candidate Lyapunov function

$$V_{k+1} = V\begin{pmatrix} x_{k+1} \\ \tilde{x}_{k+1} \end{pmatrix} = \begin{pmatrix} x_{k+1} \\ \tilde{x}_{k+1} \end{pmatrix}^T \begin{pmatrix} P_{k+1} & 0 \\ 0 & \Sigma_{k+1} \end{pmatrix}^{-1} \begin{pmatrix} x_{k+1} \\ \tilde{x}_{k+1} \end{pmatrix} = x_{k+1}^T P_{k+1}^{-1} x_{k+1} + \tilde{x}_{k+1}^T \Sigma_{k+1}^{-1} \tilde{x}_{k+1} \tag{22}$$

by substracting both sides of (4.a) from x_{k+1}, we obtain

$$\tilde{x}_{k+1} = \tilde{x}_{k+1/k} - K_{k+1} e_{k+1} \tag{23}$$

Here, we introduce unknown diagonal matrices $\beta_k = diag(\beta_{1k}, \cdots, \beta_{nk})$ and $\alpha_{k+1} = diag(\alpha_{1k+1}, \cdots, \alpha_{pk+1})$,to model errors due to the first order linearization technique, so that we obtain the following exact equalities :

$$\tilde{x}_{k+1/k} = \beta_k F_k \tilde{x}_k \tag{24}$$

$$\alpha_{k+1} e_{k+1} = H_{k+1} \tilde{x}_{k+1/k} \tag{25}$$

The importance of this choice is given in [4].

Next, from (11) and (13), we have

$$K_{k+1} = \Sigma_{k+1} H_{k+1}^T R_{k+1}^{-1} \tag{26}$$

and

$$\Sigma_{k+1}^{-1} = \Sigma_{k+1/k}^{-1} + H_{k+1}^T R_{k+1}^{-1} H_{k+1} \tag{27}$$

substituting (26) into (23), then (23) into (22), the Lyapunov function V_{k+1} becomes

$$\begin{aligned} V_{k+1} = {} & x_{k+1}^T P_{k+1}^{-1} x_{k+1} + \tilde{x}_{k+1/k}^T \Sigma_{k+1}^{-1} \tilde{x}_{k+1/k} - \tilde{x}_{k+1/k}^T H_{k+1}^T R_{k+1}^{-1} e_{k+1} \\ & - e_{k+1}^T R_{k+1}^{-1} H_{k+1} \tilde{x}_{k+1/k} + e_{k+1}^T R_{k+1}^{-1} H_{k+1} \Sigma_{k+1} H_{k+1}^T R_{k+1}^{-1} e_{k+1} \end{aligned} \tag{28}$$

and (27) into (28)

$$\begin{aligned} V_{k+1} = {} & x_{k+1}^T P_{k+1}^{-1} x_{k+1} + V_{k+1/k} + \tilde{x}_{k+1/k}^T H_{k+1}^T R_{k+1}^{-1} H_{k+1} \tilde{x}_{k+1/k} - \tilde{x}_{k+1/k}^T H_{k+1}^T R_{k+1}^{-1} e_{k+1} \\ & - e_{k+1}^T R_{k+1}^{-1} H_{k+1} \tilde{x}_{k+1/k} + e_{k+1}^T R_{k+1}^{-1} H_{k+1} \Sigma_{k+1} H_{k+1}^T R_{k+1}^{-1} e_{k+1} \end{aligned} \tag{29}$$

with

$$V_{k+1/k} = \tilde{x}_{k+1/k}^T \Sigma_{k+1/k}^{-1} \tilde{x}_{k+1/k} \tag{30}$$

From (24) and (25), (29) becomes

$$V_{k+1} = x_{k+1}^T P_{k+1}^{-1} x_{k+1} + V_{k+1/k} + e_{k+1}^T(\alpha_{k+1} R_{k+1}^{-1}\alpha_{k+1} - \alpha_{k+1}R_{k+1}^{-1} - R_{k+1}^{-1}\alpha_{k+1} + R_{k+1}^{-1}H_{k+1}\Sigma_{k+1}H_{k+1}^T R_{k+1}^{-1})\, e_{k+1} \quad (31)$$

On the other hand, $V_{k+1/k}$ may be written as

$$V_{k+1/k} = \tilde{x}_k^T F_k^T \beta_k (F_k\Sigma_k F_K^T + S_k)^{-1}\beta_k F_k \tilde{x}_k \quad (32)$$

A decreasing sequence $\{V_k\}_{k=1,\ldots}$ means that there exists a positive scalar $0<\delta<1$ such that

$$V_{k+1} - V_k \le -\delta\, V_k \quad (33)$$

or equivalently

$$\begin{aligned}V_{k+1} - (1-\delta)V_k &= x_{k+1}^T P_{k+1}^{-1}x_{k+1} + \tilde{x}_k^T F_k^T\beta_k(F_k\Sigma_k F_K^T + S_k)^{-1}\beta_k F_k\tilde{x}_k \\ &+ e_{k+1}^T(\alpha_{k+1}R_{k+1}^{-1}\alpha_{k+1} - \alpha_{k+1}R_{k+1}^{-1} - R_{k+1}^{-1}\alpha_{k+1} + R_{k+1}^{-1}H_{k+1}\Sigma_{k+1}H_{k+1}^T R_{k+1}^{-1})\, e_{k+1} \\ &- (1-\delta)\left(x_k^T P_k^{-1}x_k + \tilde{x}_k^T\Sigma_k^{-1}\tilde{x}_k\right) \le 0 \end{aligned} \quad (34)$$

$$\begin{aligned}&= x_{k+1}^T P_{k+1}^{-1}x_{k+1} - (1-\delta)\, x_k^T P_k^{-1}x_k + e_{k+1}^T(\alpha_{k+1}R_{k+1}^{-1}\alpha_{k+1} - \alpha_{k+1}R_{k+1}^{-1} \\ &- R_{k+1}^{-1}\alpha_{k+1} + R_{k+1}^{-1}H_{k+1}\Sigma_{k+1}H_{k+1}^T R_{k+1}^{-1})\, e_{k+1} \\ &+ \tilde{x}_k^T(F_k^T\beta_k(F_k\Sigma_k F_K^T + S_k)^{-1}\beta_k F_k - (1-\delta)\Sigma_k^{-1})\tilde{x}_k \le 0 \end{aligned} \quad (35)$$

A sufficient condition to ensure (35) leads to the following nonlinear inequalities

$$x_{k+1}^T P_{k+1}^{-1}x_{k+1} - (1-\delta)\, x_k^T P_k^{-1}x_k \le 0 \quad (36)$$

$$\alpha_{k+1}R_{k+1}^{-1}\alpha_{k+1} - \alpha_{k+1}R_{k+1}^{-1} - R_{k+1}^{-1}\alpha_{k+1} + R_{k+1}^{-1}H_{k+1}\Sigma_{k+1}H_{k+1}^T R_{k+1}^{-1} \le 0 \quad (37)$$

$$F_k^T\beta_k(F_k\Sigma_k F_K^T + S_k)^{-1}\beta_k F_k - (1-\delta)\Sigma_k^{-1} \le 0 \quad (38)$$

It is easy to deduce, by matrix manipulations, that under the hypothesis H2, we ensure that :

$$\lambda_k^{-1}\Lambda_k^T\left(A_k^T\left(P_k^{-1} + g_k\Omega_k^{-1}g_k^T\right)^{-1}A_k + Q_k\right)^{-1}\Lambda_k - P_k^{-1} \le -\delta\, \mathrm{P}_k^{-1} \quad (39)$$

or

$$\Lambda_k^T P_{k+1}^{-1}\Lambda_k - P_k^{-1} \le -\delta\, \mathrm{P}_k^{-1} \quad (40)$$

Which implies

$$x_k^T\left[\Lambda_k^T P_{k+1}^{-1}\Lambda_k - P_k^{-1}\right]x_k \le -\delta\, \mathrm{x}_k^T P_k^{-1}x_k \quad (41)$$

and therefore, (36) is satisfied.

We notice that (37) may be written into an equivalent form. Indeed, by the use of (11), (26) and a simple factorisation technique, we obtain

$$(\alpha_{k+1} - I_p)R_{k+1}^{-1}(\alpha_{k+1} - I_p) - R_{k+1}^{-1} + R_{k+1}^{-1}H_{k+1}\Sigma_{k+1/k}H_{k+1}^T \times \left(H_{k+1}\Sigma_{k+1/k}H_{k+1}^T + R_{k+1}\right)^{-1} \le 0 \quad (42)$$

$$\Rightarrow (\alpha_{k+1} - I_p)R_{k+1}^{-1}(\alpha_{k+1} - I_p) - R_{k+1}^{-1} \times\left(I_p - H_{k+1}\Sigma_{k+1/k}H_{k+1}^T\left(H_{k+1}\Sigma_{k+1/k}H_{k+1}^T + R_{k+1}\right)^{-1}\right) \le 0 \quad (43)$$

using the following identity in (43)

$$I_p = \left(H_{k+1}\Sigma_{k+1/k}H_{k+1}^T + R_{k+1}\right)\left(H_{k+1}\Sigma_{k+1/k}H_{k+1}^T + R_{k+1}\right)^{-1} \quad (44)$$

Inequalities (37) and (38) become

$$(\alpha_{k+1} - I_p)R_{k+1}^{-1}(\alpha_{k+1} - I_p) - \left(H_{k+1}\Sigma_{k+1/k}H_{k+1}^T + R_{k+1}\right)^{-1} \le 0 \quad (45)$$

and

$$F_k^T\beta_k\left(F_k\Sigma_k F_k^T + S_k\right)^{-1}\beta_k F_k - (1-\delta)\,\Sigma_k^{-1} \le 0 \quad (46)$$

Using (20) and (21), we can deduce that $\{V_k\}_{k=1,\ldots}$ is a decreasing sequence.

Indeed, under (20) and (21) and using the fact that $(\alpha_{k+1} - I_p)$ and β_k are diagonal matrices, we have

$$\left[\overline{\sigma}(\alpha_{k+1} - I_p)\right]^2 \le \frac{\underline{\sigma}(R_{k+1})}{\overline{\sigma}(H_{k+1}\Sigma_{k+1/k}H_{k+1}^T + R_{k+1})} \quad (47)$$

$$\left[\overline{\sigma}(\beta_k)\right]^2 \le \frac{(1-\delta)\underline{\sigma}(F_k\Sigma_k F_k + S_k)}{\overline{\sigma}(F_k^T)\overline{\sigma}(\Sigma_k)\overline{\sigma}(F_k)} \quad (48)$$

$$\Rightarrow$$

$$\left[\overline{\sigma}(\alpha_{k+1} - I_p)\right]^2\overline{\sigma}(R_{k+1}^{-1}) \le \underline{\sigma}\left((H_{k+1}\Sigma_{k+1/k}H_{k+1}^T + R_{k+1})^{-1}\right) \quad (49)$$

$$\left[\overline{\sigma}(\beta_k)\right]^2 \le \frac{(1-\delta)\underline{\sigma}(\Sigma_k^{-1})}{\overline{\sigma}(F_k^T)\overline{\sigma}\left((F_k\Sigma_k F_k^T + S_k)^{-1}\right)\overline{\sigma}(F_k)} \quad (50)$$

as

$$\overline{\sigma}\left((\alpha_{k+1} - I_p)R_{k+1}^{-1}(\alpha_{k+1} - I_p)\right) \le \left[\overline{\sigma}(\alpha_{k+1} - \mathrm{I_p})\right]^2\overline{\sigma}\left(R_{k+1}^{-1}\right) \quad (51)$$

and

$$\overline{\sigma}\left(F_k^T\beta_k\left(F_k\Sigma_k F_k^T + S_k\right)^{-1}\beta_k F_k\right) \le \left[\overline{\sigma}(\beta_k)\right]^2\overline{\sigma}\left(F_k^T\right) \times \overline{\sigma}\left(\left(F_k\Sigma_k F_k^T + S_k\right)^{-1}\right)\overline{\sigma}(F_k) \quad (52)$$

we have then

$$\overline{\sigma}\left((\alpha_{k+1} - I_p)R_{k+1}^{-1}(\alpha_{k+1} - I_p)\right) \le \left[\overline{\sigma}(\alpha_{k+1} - \mathrm{I_p})\right]^2\overline{\sigma}\left(R_{k+1}^{-1}\right) \le \underline{\sigma}\left(\left(H_{k+1}\Sigma_{k+1/k}H_{k+1}^T + R_{k+1}\right)^{-1}\right) \quad (53)$$

$$\overline{\sigma}\left(F_k^T\beta_k\left(F_k\Sigma_k F_k^T + S_k\right)^{-1}\beta_k F_k\right) \le \left[\overline{\sigma}(\beta_k)\right]^2\overline{\sigma}\left(F_k^T\right) \times \overline{\sigma}\left(\left(F_k\Sigma_k F_k^T + S_k\right)^{-1}\right)\overline{\sigma}(F_k) \le (1-\delta)\underline{\sigma}\left(\Sigma_k^{-1}\right) \quad (54)$$

which induce that (45) and (46) are satisfied, and consequently V_k is a strictly decreasing sequence.

Now, we will prove that, the matrices P_k, and Σ_k are bounded from above and below for all k, i.e. there exists $\underline{\gamma}$, $\overline{\gamma}$, $\underline{\eta}$ and $\overline{\eta}$ such that :

$$0 < \underline{\gamma}I_n \le P_k \le \overline{\gamma}I_n \quad (55)$$

and

$$0 < \underline{\eta}I_n \le \Sigma_k \le \overline{\eta}I_n \quad (56)$$

We can verify easily, from (10), that since $\lambda_k\, Q_k$ is positive definite, we have $0 < \underline{\gamma}I_n \le P_k$. The second inequality $P_k \le \overline{\gamma}I_n$ may be deduced from the sufficient hypothesis H1. Indeed, if we consider the following auxiliary Riccati equation

$$\overline{P}_{k+1} = \lambda_k\left(A_k^T\overline{P}_k A_k + Q_k\right) \quad (57)$$

we notice that, under hypothesis H1, and for a large parameter $\bar{\gamma}$, we have

$$\bar{P}_k \leq \bar{\gamma}\ I_n \qquad \text{for all k} \qquad (58)$$

Thus, when we choose the initial condition as follow

$$P_0 \leq \bar{P}_0 \quad (< \bar{\gamma} I_n) \qquad (59)$$

we obtain, by the use of (10) and (17):

$$P_{k+1} = \lambda_k \left(A_k^T \left(P_k - P_k g_k \left(g_k P_k g_k^T + R \right)^{-1} g_k^T P_k \right) A_k + Q_k \right)$$
$$\leq \lambda_k \left(A_k^T P_k A_k + Q_k \right) \leq \bar{P}_{k+1} = \lambda_k \left(A_k^T \bar{P}_k A_k + Q_k \right) \leq \bar{\gamma} I_n \quad (60)$$

So the boundness of P_k is proved.

The proof of (56) is obtained from the local observability hypothesis H3, which ensure the boundness of Σ_k (see [10] and [24]).

Since V_k is a strictly decreasing sequence and the couple (P_k, Σ_k) is bounded, it follows that

$$0 \leq \mu \begin{pmatrix} x_k \\ \tilde{x}_k \end{pmatrix}^T \begin{pmatrix} x_k \\ \tilde{x}_k \end{pmatrix} \leq V_k \leq (1-\delta)^k V_0 \qquad (61)$$

$$\Rightarrow 0 \leq \mu \lim_{k\to\infty} \left(\begin{pmatrix} x_k \\ \tilde{x}_k \end{pmatrix}^T \begin{pmatrix} x_k \\ \tilde{x}_k \end{pmatrix} \right) \leq \lim_{k\to\infty} (V_k) \leq V_0 \lim_{k\to\infty} (1-\delta)^k = 0 \quad (62)$$

with

$$0 < \mu\, I_n \leq \begin{pmatrix} P_k & 0 \\ 0 & \Sigma_k \end{pmatrix}^{-1}$$

Therefore the convergence of both the state space and the error dynamics to zero is ensured.

4.1 Remarks :

1. we introduce the weighting factor λ_k to control boundedness of P_k and, by the way, to relax the Lyapunov stability condition of the unforced dynamic system without preliminary coordinate transformations of the initial system. A simple method to design λ_k consists to set $\lambda_k = 1$ as long as $P_k \leq \bar{\gamma} I_n$ and $\lambda_k < 1$ otherwise.

2. the two inequalities (20) and (21), connot be always checked, since the parameter α_{ik} and β_{jk} are unknown ; however, they give us the domains to which α_{ik}, β_{jk} should belong such that V_k is a decreasing sequence.

5. Simulation results

In order to show the high performances of the theory developed so far, we consider two numerical examples chosen from the literature.
The EKO control law given in (4), makes the origin asymptotically stable as we can see in the different figures.

5.1 Example 1 :

The following nonlinear discrete-time system has been considered in [7]

$$x_{k+1} = Ax_k + g(x_k)u_k \qquad (63)$$
$$y_k = Cx_k \qquad (64)$$

With

$$A = \begin{pmatrix} 0 & 2 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad g(x_k) = \begin{pmatrix} 0 & 2 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\text{and} \qquad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

where A is unstable. Fig. 1 and Fig. 2 show the convergence behaviour of x and $x-\xi$ to the equilibrium $(0,0)$, where the initialization vectors are $x_0 = [-10\ \ \text{-}7.5\ \ 10]$ and $\xi_0 = [4\ \ 3\ \ -4]$.

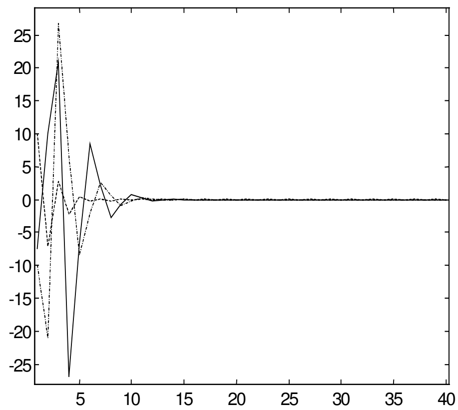

Fig. 1. The state x_k with respect to sampling time k.

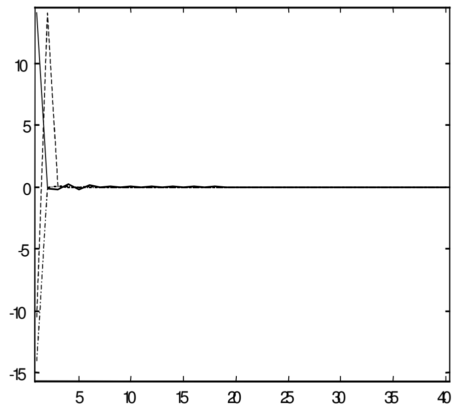

Fig. 2 : State estimation error $(x_k - \xi_k)$ with respect to sampling time k.

5.2 Example 2 :

Here, we study the example of the planar vertical take-off and Landing (PVTOL) aircraft. This example is treated in several papers in the literature [11] and [13] and [16]. The equations of motion are given by [11] :

$$\begin{aligned} \ddot{x} &= -\sin(\theta)u_1 + \varepsilon\cos(\theta)u_2 \\ \ddot{y} &= \cos(\theta)u_1 + \varepsilon\sin(\theta)u_2 - 1 \\ \ddot{\theta} &= u_2 \end{aligned} \tag{65}$$

Where x, y denote the horizontal and the vertical position of the aircraft center of mass and θ is the roll angle that the aircraft makes with the horizon. The control inputs u_1 and u_2 are the thrust (direct out of the bottom of the aircraft) and the angular acceleration (rolling moment). The parameter ε is a small coupling coefficient between the rolling moment and the lateral acceleration of the aircraft. The coefficient "-1" is the normalized gravitational acceleration. Let us use (65) with $u_1=1+v_1$, after Euler discretization of step T, we obtain the next state space model

$$x_{k+1} = A(x_k)x_k + g(x_k)u_k \tag{66}$$

with

$$A(x_k) = \begin{pmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -T\sin(\theta_k)/\theta_k & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & T(\cos(\theta_k)-1)/\theta_k & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$g(x_k) = T\begin{pmatrix} 0 & 0 \\ -\sin(\theta_k) & \varepsilon\cos(\theta_k) \\ 0 & 0 \\ \cos(\theta_k) & \varepsilon\sin(\theta_k) \\ 0 & 0 \\ 0 & 1 \end{pmatrix},$$

$$x_k = (x_{1k} \quad x_{2k} \quad y_{1k} \quad y_{2k} \quad \theta_k \quad \omega_k)^T \quad \text{and} \quad u_k = \begin{pmatrix} v_{1k} \\ u_{2k} \end{pmatrix}$$

We assume that only, the vertical and horizontal positions and the roll angle are available

$$y_k = C\,x_k \qquad \text{with} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{67}$$

We consider the discrete-time model, with T=0.007, ε =0.02 and the initializations x_0= [200 0 10 0 0.5 0], $\xi_0 = [75\ 3\ 30\ 15\ 3\ 3]$. The simulations confirm the high quality of our approach.

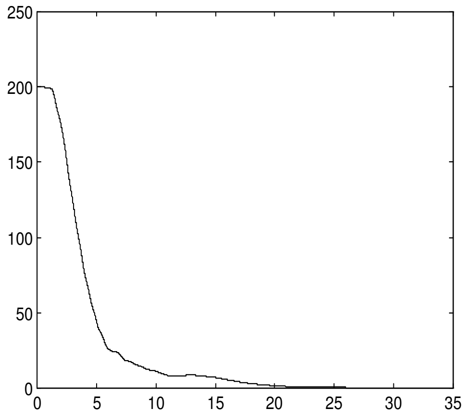

Fig. 3. EKO based control law applied to example 2, $\|x_k\|$ with respect to time t(Sec).

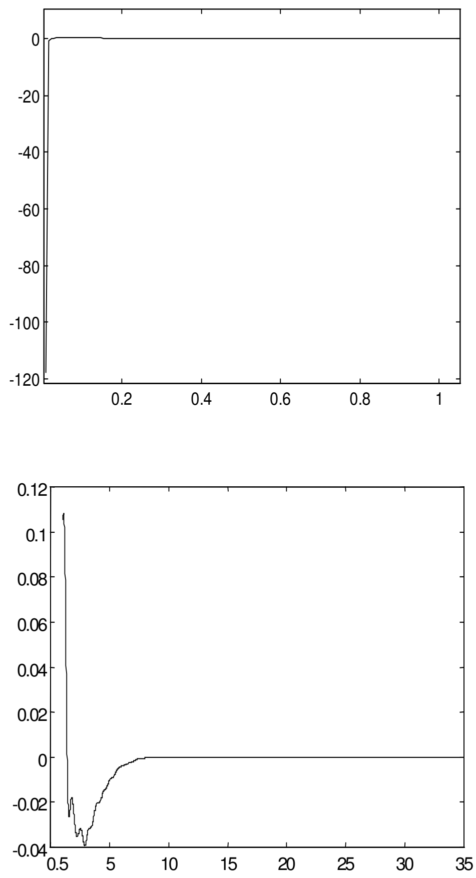

Fig. 4. State estimation error $\|x_k - \xi_k\|$ with respect to time t(Sec).(a : 0-1sec) and (b : 0.5-35sec).

6. Conclusion

In this paper, we have presented an observer based control law which asymptotically stabilize a class of discrete-time affine nonlinear systems. A simple and useful approach, using the EKO and a modified Riccati equation is given. We establish a separation principle, and the EKO based stabilization method we use gives a good results, even when the free dynamics are not Lyapunov stable. Finally, the proposed control law was successfully applied to a large numerical examples treated in the literature , and two of them are detailed in this paper.

References

[1] A. Aeyels, "Stabilization by smooth feedback of the angular velocity of a rigid body," Syst. Control Lett., vol. 5, pp. 59-63, 1985.

[2] A. N. Atassi and H. K. Khalil, "A separation principle for the control of a class of nonlinear systems," IEEE Trans. Automat. Contr., vol. 46, pp. 742-746, 2001.

[3] R. W. Brockett, "Asymptotic stability and feedback stabilization," in Differential Geometric Control Theory, R. W. Brockett, R. S. Millmann and H. J. Sussmann, Eds. Boston, MA : Birkhäuser, 1983, pp. 181-191.

[4] M. Boutayeb and D. Aubry, "A strong tracking extended Kalman observer for nonlinear discrete-time systems," IEEE Trans. Automat. Contr., vol. 44, pp. 1550-1556, 1999.

[5] M. Boutayeb, K. E. Bouazza and M. Darouach, " State feedback stabilization of discrete-time nonlinear systems," in Proc. 41th IEEE Conf. Decision Control, Las Vegas, Nevada,2002.

[6] C. Byrnes and W. Lin, "On discrete-time nonlinear control," in Proc. 32th IEEE Conf. Decision Control, San Antonio, Texas, 1993,pp. 2990-2996.

[7] C. I. Byrnes, W. Lin and B. K. Ghosh, "Stabilization of discrete-time nonlinear systems by smooth state feedback," Syst. Control Lett., vol. 21, pp. 255-263, 1993.

[8] C. Cruz, J. Alvarez and R. Castro, "Stability Robustness of linearizing controllers with state estimation for discrete-time nonlinear systems," in Proc. Of the ACC, San Diego, June1999.

[9] A. M. Dabroom and H. K. Khalil, "Discrete-time implementation of high-gain observers for numerical differentiation," Int. J. Contr., vol.72, pp. 1523-1537, 1999.

[10] J. J. Deyst and C. F. Price, "Conditions for asymptotic stability of the discrete minimum-variance linear estimator," IEEE Trans. Automat. Contr., pp. 702-705, 1968.

[11] I. Fantoni and R. Lozano, Nonlinear Control for Underactuated Mechanical Systems, London: Springer-Verlag, 2002.

[12] J. P. Gauthier and I. Kupka, "A separation principle for bilinear systems with dissipative drift," IEEE Trans. Automat. Contr., vol. 37, pp. 1970-1974, 1992.

[13] J. Hauser, S. Sastry and G. Meyer, "Nonlinear control design for slightly minimum phase systems: Application to V/STOL aircraft," Automatica, vol. 28, pp. 665-679, 1992.

[14] H. K. Khalil and F. Esfandiari, "Semiglobal stabilization of a class of nonlinear systems using output feedback," IEEE Trans. Automat. Contr., vol. 40, pp. 1412-1415, 1993.

[15] P.Kokotovic and M. Arcak, "Constructive nonlinear control: a historical perspective," Automatica, vol. 37, pp. 637-662, 1999.

[16] F. Lin, W. Zhang and R. D. Brandt, "Robust hovering control of a PVTOL aircraft," IEEE Trans. Control systems Technology, vol. 7, pp. 343-351, 1999.

[17] W. Lin, "Input saturation and global stabilization of nonlinear systems via state and output feedback," IEEE Trans. Automat. Contr., vol. 40, pp. 776-782, 1995.

[18] W. Lin, "Further results in global stabilization of discrete nonlinear systems," Syst. Control Lett., vol. 29, pp. 51-59, 1996.

[19] J. Mohseni, E. Yaz, and K. Olejniczak, "State-dependent LMI control of discrete-time nonlinear systems," in Proc. 37th IEEE Conf. Decision Control, San Diego, CA, 1998,pp. 4626-4627.

[20] H. Jo. Nam and J. H. Seo, "Local separation principle for nonlinear systems," Int. J. Contr., vol.73, pp. 292-302, 2000.

[21] H. Nijmeijer, "Local (dynamic) input-output decoupling of discrete-time nonlinear systems," IMA J. Math. Control & Info, vol. 4, pp. 237-250, 1987.

[22] H. Nijmeijer and A. J. Van der Schaft, Nonlinear dynamical control systems, Springer-Verlag, 1990.

[23] C. Qian and W. Lin, "A continuous feedback approach to global strong stabilization of nonlinear systems," IEEE Trans. Automat. Contr., vol. 46, pp. 1061-1079, 2001.

[24] Y. Song and J. Grizzle," The extended Kalman Filter as a local asymptotic observer for nonlinear discrete-time systems," J. Math., Estimation, Contr., vol.5, no.1,pp.59-78, 1995.