OBSERVER DESIGN FOR A CLASS OF DESCRIPTOR SYSTEMS

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Abstract

Descriptor systems represent dynamics of processes subject to constraint. The information about the constraint can be used in the observability analysis and observer design. A class of nonlinear descriptor systems to which a wide range of constrained mechanical systems belong are considered. The design method is illustrated by a robot manipulator example.

1 Introduction

A direct and natural outcome of modeling a dynamic process is often a set of ordinary differential equations and algebraic equations. Differential equations are obtained by applying physical laws of dynamic behaviours, while algebraic equations are determined following principles of geometric and static relations among variables representing systems' status. Dynamic systems represented by a set of differential-algebraic equations (DAEs) are called descriptor systems.

To describe complex technical processes, descriptor systems normally appear as nonlinear DAEs. As a matter of fact, in the control literature, descriptor systems were firstly introduced as nonlinear DAEs in the 70s [4, 12]. In the past, a vast of attention has been devoted to numerical analysis and algorithm development for descriptor systems [3, 7]. This has been driven by applications arising in simulations of chemical and mechanical processes described by descriptor systems. Another interesting application of descriptor systems is studied in [16] where reconstructing states of conventional systems is reformulated as numerically solving DAEs.

In the control field, most investigation had focused on linear descriptor systems in the past, see e.g. [5, 14, 18]. For certain classes of nonlinear descriptor systems, controller designs have been considered, see e.g. [2, 6, 13, 15]. As far as observer designs are concerned, it seems that no approaches are available to the class of nonlinear descriptor systems under consideration.

An observer asymptotically reconstructs all the variables of a descriptor system based on a limited number of measurements. As in the case of conventional systems, observers can be used for implementation of feedback controls, system supervision and fault diagnosis in descriptor systems. Mathematical descriptions of observers for descriptor systems may be either in the conventional differential equation form or in the DAE form. This study is interested in conventional observers because they are easily implementable.

2 Problem specification

Consider a class of descriptor systems described by

$$\begin{aligned} \dot{x} &= f_1(x) + f_2(x,u) + f_3(x,\lambda,u)\,, & (1)\\ 0 &= p(x)\,, & (2)\\ y &= h(x) & (3) \end{aligned}$$

where x and λ represent the descriptor vector, u the control vector, y the measurement vector, $p(x)$ the algebraic constraint, and f_i, p and h are vector-valued smooth functions of their arguments, and $f_1(0) \equiv 0$, $f_2(x,0) \equiv 0$ and $f_3(x,0,u) \equiv 0$. Note that an arbitrary function $f(x,\lambda,u)$ with $f(0,0,0) = 0$ can always be expressed by the three terms in (1) through defining

$$f_1(x) = f(x,0,0)\,, \quad f_2(x,u) = f(x,0,u) - f(x,0,0)\,,$$
$$f_3(x,\lambda,u) = f(x,\lambda,u) - f(x,0,u)\,.$$

It will be assumed that λ can be determined as a function of x and u. In most cases, like conventional nonlinear systems, descriptor systems may not be globally well defined. It is assumed that the system (1-3) is defined on a specified set $\mathcal{X} \times \mathcal{U}$, where $\mathcal{X} \subset R^{n_x}$ and $\mathcal{U} \subset R^{n_u}$ with $n_x = \dim x$ and $n_u = \dim u$ are open sets with respect to x and u, respectively. The input $u : [0, t_f] \to \mathcal{U}$ is analytic on the time interval $[0, t_f]$. Moreover, u is assumed to be known and the initial value $x(0)$ unknown since observer issues are being considered.

An observer for the system (1-3) is described by

$$\dot{z} = \phi_z(z,u,y)\,, \quad \hat{x} = \phi_x(z,u,y)\,, \quad \hat{\lambda} = \phi_\lambda(z,u,y)\,, \quad (4)$$

where $\hat{x} \to x$ and $\hat{\lambda} \to \lambda$ as $t \to \infty$ for arbitrary $z(0) \in R^{n_z}$ with $n_z = \dim z$ and $(x(0), u) \in \mathcal{X} \times \mathcal{U}$. In a special case, the $\hat{x}$-equation may be replaced by $\hat{x} = z$. The above observer is in the conventional form of differential equations since no constraint on the observer state z is imposed.

The notation $L_f^i g = L_f(L_f^{i-1} g)$ with $L_f g = \frac{\partial g}{\partial x} f$ and $L_f^0 g = g$ will indicate the ith-order Lie derivative of g with respect to f. f_{123} stands for $f_1 + f_2 + f_3$.

Assumption A: There exists an integer $l \geq 1$ which is called the index of the system (1-2), such that for $i = 0, \cdots, l-1$,

$L^i_{f_{123}}p = L^i_{f_1}p$, and λ can be uniquely determined from $L_{f_{123}}L^{l-1}_{f_1}p = 0$ as a vector-valued function of x and u, denoted by $\lambda = \lambda(x,u)$.

Assumption A is sufficient (but not necessary) for regularity of the descriptor system and properness of of the descriptor vector with respect to the input. These two notions are defined in the following.

Definition 1 *The system (1-2) is said to be* regular *if there exist unique x and λ satisfying (1-2) for each pair $(x(0),u) \in \mathcal{X} \times \mathcal{U}$.*

Definition 2 *The descriptor vector of x and λ is said to be* proper *with respect to u if there exists continuous solution x and λ to (1-2) for $x(0) = 0$ and arbitrary piecewise continuous u.*

It is easy to see that the descriptor vector of x and λ is proper with respect to u if and only if λ is proper with respect to u. In the discrete-time case, properness of $\lambda(k)$ means that $\lambda(k)$ is not influenced by the future input $u(j)$ for $j > k$.

It remains further to clarify the open sets $\mathcal{X}$ and $\mathcal{U}$. The set $\mathcal{X} \times \mathcal{U}$ is not only for f_i, p and h in (1-3) being well-defined as smooth functions but also for consistency of (1-2) with respect to each pair $(x(0),u)$.

Definition 3 *The system (1-2) is said to be* consistent *with pair $(x(0),u)$ if there exist x and λ satisfying (1-2) associated with this pair $(x(0),u)$.*

The consistency concerns with restrictions imposing on the choice of $x(0)$ and u.

Proposition 1 *Under Assumption A, the system (1-2) is consistent with*

(a) *$x(0)$ if and only if $L^i_{f_1}p(x(0)) = 0$ for $i = 0,\cdots,l-1$;*

(b) *each u.*

Proof: All the constraint on x and u can be deduced from (2) associated with (1). Since $\lambda = \lambda(x,u)$ is deduced from $L^l_{f_{123}}p = 0$, $L^i_{f_{123}}p = 0$ for $i > l$ no longer contains any constraint on x and u. This means that all the constraint which may impose on x and u are represented solely by $L^i_{f_{123}}p = L^i_{f_1}p = 0$ for $i = 0,\cdots,l-1$. This set of equations are purely in terms of x. The statement regarding (a) and (b) then follows. △

In the following the system descriptions and notions introduced so far are illustrated by analysing a class of mechanical descriptor systems. Consider a wide class of constrained mechanical systems described by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B(q)u + F'(q,\dot{q})\lambda\,, \quad (5)$$
$$0 = p(q,\dot{q})\,, \quad (6)$$
$$y = h(q,\dot{q})\,, \quad (7)$$

where q is the generalized displacement vector, λ the Lagrangian multiplier vector, u the control vector, y the measurement vector. $M(q)$ is the symmetric positive-definite inertial matrix, $C(q,\dot{q})\dot{q}$ the centrifugal and Coriolis vector, $G(q)$ the gravitational vector, $F(q,\dot{q})$ and $B(q)$ are matrices of appropriate dimensions, F' is the transpose of F. The term $F'\lambda$ represents the constraint force.

To make a correspondence between the descriptor systems (1-3) and (5-7), consider the special case of the mechanical system (5-7) with holonomic constraint. That means in (6) $p(q,\dot{q}) = p(q)$ and in (5) $F(q,\dot{q}) = F(q) = \frac{\partial p}{\partial q}$.

Let $x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$. Then, f_1, f_2 and f_3 in (1) read

$$f_1 = \begin{bmatrix} \dot{q} \\ -M^{-1}(C\dot{q}+G) \end{bmatrix}, \quad f_2 = \begin{bmatrix} 0 \\ M^{-1}Bu \end{bmatrix},$$
$$f_3 = \begin{bmatrix} 0 \\ M^{-1}F'\lambda \end{bmatrix}, \quad (8)$$

where arguments of the matrices are omitted.

For the holonomic constraint $p(q) = 0$, since $\frac{\partial p}{\partial x} = \begin{bmatrix} F & 0 \end{bmatrix}$, it is easy to verify that $L_{f_2}p = L_{f_3}p \equiv 0$ and $L_{f_1}p = F\dot{q}$. Hence, $L_{f_{123}}p = 0$ becomes $F\dot{q} = 0$. Furthermore, $L_{f_{123}}L_{f_1}p = 0$ leads to

$$\frac{\partial(F\dot{q})}{\partial q}\dot{q} - FM^{-1}(C\dot{q}+G-Bu) + FM^{-1}F'\lambda = 0\,. \quad (9)$$

The regularity assumption ensures $\operatorname{rank} F = \dim\lambda$ for all q, which means λ is uniquely determined by

$$\lambda = \lambda(q,\dot{q},u) = (FM^{-1}F')^{-1}\cdot \cdot\left(FM^{-1}(C\dot{q}+G-Bu) - \frac{\partial(F\dot{q})}{\partial q}\dot{q}\right). \quad (10)$$

The integer l for the holonomic system (5-6) is 2 which is one less than the widely adopted index. This is because in this study it is sufficient to obtain an expression of λ, while in other problem formulations such as in numerical analysis $\dot{\lambda}$ is required. As can be seen, an expression for $\dot{\lambda}$ generally involves $\dot{u}$ which may not be available in real applications.

3 Observability

Consider descriptor systems in a special form as

$$\dot{x} = f(x) + g(x,u)\,, \quad (11)$$
$$0 = p(x)\,, \quad (12)$$
$$y = h(x) \quad (13)$$

with $\lambda = \lambda(x,u)$ on $\mathcal{X} \times \mathcal{U}$, where f, g, p, h and r are vector-valued smooth functions of their arguments. Assume, without loss of generality, that $g(x,0) \equiv 0$. A structural restriction on this system, which corresponds to Assumption A on the system (1-3), is that the constraint of the higher-order Lie derivatives

$L_{f+g}^i p = 0$ for $i > l$ is automatically satisfied by the trajectory x governed by (11).

Under Assumption A, the descriptor system (1-3) is then equivalent to (11-13) with the specifications

$$f(x) = f_1(x) + f_3(x, \lambda(x,0), 0)\,, \tag{14}$$
$$g(x,u) = f_2(x,u) + f_3(x, \lambda(x,u), u) - f_3(x, \lambda(x,0), 0) \tag{15}$$

and $\lambda = \lambda(x,u)$ being the unique solution to $L_{f_{123}} L_{f_1}^{l-1} p = 0$. This means that both descriptions (1-3) and (11-13) with $\lambda = \lambda(x,u)$ defined on $\mathcal{X} \times \mathcal{U}$ possess the same trajectories of x, λ and y for the same initial condition and control $(x(0),\, u) \in \mathcal{X} \times \mathcal{U}$.

Definition 4 *The descriptor system (1-3) is said to be* uniformly observable *if every* $(x,\,\lambda)$ *can be uniquely determined on the basis of* y *with arbitrarily specified* $u \in \mathcal{U}$.

Because of equivalence between the descriptions (1-3) and (11-13), the following proposition is evident.

Proposition 2 *Under Assumption A, the system (1-3) is uniformly observable for* $(x,\,\lambda)$ *if and only if the system (11-13) is uniformly observable for* x.

Owing to this proposition, in the remaining part of the paper, observability and observer design will be discussed for the descriptor systems described by (11-13) with the index l.

Proposition 3 *The descriptor system (11-13) is uniformly observable on* $\mathcal{X} \times \mathcal{U}$ *if and only if for some positive integer* k *the set of equations*

$$\begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y^{(k-1)} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \psi_0(x,u) \\ \psi_1(x,u,\dot{u}) \\ \vdots \\ \psi_{k-1}(x,u,\cdots,u^{(k-1)}) \\ p(x) \\ L_f p(x) \\ \vdots \\ L_f^{l-1} p(x) \end{bmatrix} \tag{16}$$

denoted by $Y = \tilde{\psi}(x,v)$ *with* $v = \{u, \cdots, u^{(k-1)}\}$ *define an injective map on* $\mathcal{X}$, $x \mapsto Y$, *parametrised by* v.

Proof: Without the constraint (12), the system (11) with (13) becomes the conventional system. In such a case, without the part of $L_f^i p$, (16) has been proved to be necessary and sufficient for uniform observability in [9]. Now treat the constraint (12) as artificial measurements. By Assumption A, $\frac{d^i p}{dt^i} = L_{f+g}^i p = L_f^i p = L_{f_1}^i p$ for $i < l$, which forms the part of $L_f^i p$ in (16). Furthermore, $\frac{d^l p}{dt^l} = L_{f+g}^l p = 0$ is ensured by the explicit relation $\lambda = \lambda(x,u)$, and for $i > l$, $\frac{d^i p}{dt^i} = L_{f+g}^i p = 0$ is implied in (11). △

Obviously, for an unforced descriptor system $\dot{x} = f(x)$, $0 = p(x)$ $y = h(x)$ with the index l, the condition (16) is reduced to

$$\psi(x) = \begin{bmatrix} \psi_h(x) \\ \psi_p(x) \end{bmatrix}, \quad \psi_h(x) = \begin{bmatrix} h \\ L_f h \\ \vdots \\ L_f^{k-1} h \end{bmatrix},$$
$$\psi_p(x) = \begin{bmatrix} p \\ L_f p \\ \vdots \\ L_f^{l-1} p \end{bmatrix} \tag{17}$$

defining an injective map on $\mathcal{X}$. Note that the integer k can be less than n_x (the dimension of x). In fact, for the single-output case, only $k + l \geq n_x$ is required.

4 Observer design

On the strength of Proposition 2, this section deals with the problem of designing observers for descriptor systems described by (11-13). The observer design discussed in this section is rooted in a recent result of designing observers for conventional systems [9].

Under uniform observability, the map $\psi(x)$ defined through (17) is invertible. Define a map as $\xi = \psi_h(x)$. It is then easy th see that

$$\dot{\xi} = L_{f+g}\psi_h = f_\xi(\xi) + f_u(\xi,u) \tag{18}$$

with

$$f_\xi(\xi) = L_f \psi_h \circ \psi^{-1}(\xi)\,, \quad f_u(\xi,u) = L_g \psi_h \circ \psi^{-1}(\xi) \tag{19}$$

where $\psi^{-1}(\xi)$ is an inverse of $\bar{\psi}$.

Now regard $\xi = \psi_h(x)$ as a pseudo coordinate change. Note that either $n_x \geq n_\xi$ or $n_x \leq n_\xi$ is possible, and $\xi = \psi_h(x)$ may not be an injective map, where $n_\xi = \dim \xi$. Because of the structure of $\psi_h(x)$,

$$\begin{bmatrix} \dot{\xi}_1 \\ \vdots \\ \dot{\xi}_{k-1} \\ \dot{\xi}_k \end{bmatrix} = \begin{bmatrix} \xi_2 + \phi_1(\xi,u) \\ \vdots \\ \xi_k + \phi_{k-1}(\xi,u) \\ \phi_k(\xi,u) \end{bmatrix}, \quad y = \xi_1\,, \tag{20}$$

or denoted in short as

$$\dot{\xi} = A\xi + \phi(\xi,u)\,, \quad y = C\xi \tag{21}$$

with

$$A = \begin{bmatrix} 0 & I & & \\ & \ddots & \ddots & \\ & & \ddots & I \\ & & & 0 \end{bmatrix}, \quad C = \begin{bmatrix} I & 0 & \cdots & 0 \end{bmatrix},$$

$$\phi = \begin{bmatrix} \phi_1(\xi,u) \\ \vdots \\ \phi_k(\xi,u) \end{bmatrix}, \tag{22}$$

where I is the identity matrix with the dimension equal to $\dim \xi_1 = \cdots = \dim \xi_k$. According to [9], h in $\psi_h(x)$ may need to be replaced by $\bar{h}$, where $\bar{h}$ contains part of the elements of h.

The problem of designing an observer for x based on (11-13) has been converted to that for ξ based on (21). In this way, observer design for the class of descriptor systems (1-3) is equivalent to designing observers for the conventional system (21).

It is possible that sometimes in (21) $\phi(\xi, u) = \phi(y, u)$. It is then trivial to see that

$$\dot{\hat{\xi}} = A\hat{\xi} + \phi(y, u) + L(y - C\hat{\xi}), \quad \hat{x} = \psi^{-1}(\hat{\xi}) \tag{23}$$

with $A - LC$ being stable is an x-observer with linear error dynamics. More general discussion about the problem of observer error linearisation can be found in [1, 11, 19, 8].

If the nonlinear term $\phi(\xi, u)$ in (21) has a functionally triangular structure, namely $\phi_i = \phi_i(\xi_1, \cdots, \xi_i, u)$, then instead of (23), the following observer

$$\dot{\hat{\xi}} = A\hat{\xi} + \phi(\hat{\xi}, u) + L(\hat{\xi}, u)(y - C\hat{\xi}), \quad \hat{x} = \psi^{-1}(\hat{\xi}) \tag{24}$$

can be designed, where $L(\hat{\xi}, u)$ is the gain matrix having elements as nonlinear functions of $\hat{\xi}$ and u. More details about this kind of design are given in [9], for instance.

If $\phi(\xi, u)$ in (21) does not possess any particular structure but satisfies the Lipschitz condition $||\phi(\xi, u) - \phi(\bar{\xi}, u)|| \leq \alpha||\xi - \bar{\xi}||$ with α being a positive constant, an observer in form (23) can also be designed (see, e.g. [17]).

5 Illustrative example

Fig. 1 shows a typical two-link planar manipulator free of motion in a certain region of the x-y plane. The motion of the

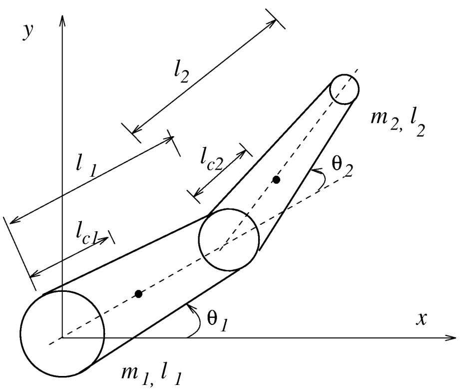

Figure 1: Two-link manipulator

manipulator becomes constrained when its end-effector moves along a surface S. The two cases are considered:

(a) The surface is a portion of a circle with radius r_0 and centered at the origin.

(b) the surface is a portion of a straight line parallel to the y-axis in the distance l_0.

Practical implication of the constrained manipulator can be revealed in the tasks like grinding or polishing by using robot manipulators.

The system parameters are as follows. m_i is the mass, l_i the length, I_i the moment of inertia, and l_{ci} the distance of the center of mass of link i.

The motion of the constrained manipulator with the measurement θ_1 is described by

$$\begin{aligned}
&\begin{bmatrix} a_1 + a_2 + a_{12} + 2a_3c_2 & a_2 + a_3c_2 \\ a_2 + a_3c_2 & a_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \\
&\begin{bmatrix} -a_3\dot{\theta}_2 s_2 & -a_3(\dot{\theta}_1 + \dot{\theta}_2)s_2 \\ a_3\dot{\theta}_1 s_2 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} b_1c_1 + b_2c_{1,2} \\ b_2c_{1,2} \end{bmatrix} \\
&= \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + F'(\theta)\lambda, \\
&0 = p(\theta), \\
&y = \theta_1,
\end{aligned}$$

where $p(\theta)$ and the associated $F(\theta)$ are to be determined for the two cases, τ_1 and τ_2 are the control torques acting respectively on the two joints,

$$\begin{aligned}
&a_1 = m_1l_{c1}^2 + I_1, \quad a_2 = m_2l_{c2}^2 + I_2, \\
&a_{12} = m_2l_1^2, \quad a_3 = m_2l_1l_{c2}, \quad c_1 = \cos\theta_1, \\
&c_2 = \cos\theta_2, \quad s_2 = \sin\theta_2, \\
&c_{1,2} = \cos(\theta_1 + \theta_2), \quad b_1 = (m_1l_{c,1} + m_2l_1)g, \\
&b_2 = m_2l_{c2}g.
\end{aligned}$$

For simplicity, let $a_1 = a_2 = b_1 = 1$ and $a_3 = b_2 = \frac{1}{2}$.

5.1 Case (a)

For the end-effector being in contact with the portion of a circle, simple geometry shows

$$r_0^2 = l_1^2 + l_2^2 + 2l_1l_2\cos\theta_2.$$

This means that to follow the surface S, θ_2 must equal the constant

$$\theta_0 = \arccos\frac{r_0^2 - l_1^2 - l_2^2}{2l_1l_2},$$

while θ_1 is free to vary within a certain range. Hence, the constraint function $p(\theta)$ and its Jacobian F are given by

$$p = \theta_2 - \theta_0, \quad F = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

Since $\frac{\partial F\dot{\theta}}{\partial\theta} \equiv 0$, from (10)

$$\lambda = \frac{1}{4}(\dot{\theta}_1 + \dot{\theta}_2)^2 s_2 + \frac{1}{4}\dot{\theta}_1^2 s_2 - \frac{1}{2}c_1 + \frac{1}{4}c_{1,2} + \frac{1}{2}\tau_1 - \tau_2.$$

For this example, $k = l = 2$, $\psi_h = \begin{bmatrix} h \\ L_f h \end{bmatrix}$,

$$\psi = \begin{bmatrix} h \\ L_f h \\ p \\ L_f p \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 - \theta_0 \\ \dot{\theta}_2 \end{bmatrix}.$$

Hence, no coordinate change is needed and thus (21) reads

$$\begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \phi_2 \end{bmatrix}, \quad y = \theta_1 \qquad (25)$$

with

$$\phi_2 = L_{f+g} L_f h = -\frac{1}{2 + c_2}(c_1 + \frac{1}{2} c_{1,2} - \tau_1).$$

This means that $\phi_2 = \phi_2(\theta_1, \theta_2, \tau_1) = \phi_2(y, \theta_0, \tau_1)$ and hence

$$\begin{bmatrix} \dot{\hat{\theta}}_1 \\ \ddot{\hat{\theta}}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\theta}_1 \\ \dot{\hat{\theta}}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \phi_2(y, \theta_0, \tau_1) \end{bmatrix} + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} (y - \hat{\theta}_1)$$

with $l_1, l_2 > 0$, is the desired observer with linear error dynamics. The estimation of λ is then

$$\hat{\lambda} = \frac{1}{2} \dot{\hat{\theta}}_1 \sin\theta_0 - \frac{1}{2} \cos\hat{\theta}_1 + \frac{1}{4} \cos(\hat{\theta}_1 + \theta_0) + \frac{1}{2} \tau_1 - \tau_2.$$

5.2 Case (b)

The constraint equation $p(\theta) = 0$ is with $p = l_1 c_1 + l_2 c_{1,2} - l_0$ and the associated Jacobian is $F = \begin{bmatrix} -l_1 s_1 - l_2 s_{1,2}, & -l_2 s_{1,2} \end{bmatrix}$. The map (25) becomes

$$\psi = \begin{bmatrix} h \\ L_f h \\ p \\ L_f p \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ l_1 c_1 + l_2 c_{1,2} - l_0 \\ -(l_1 c_1 + l_2 c_{1,2}) \dot{\theta}_1 - l_2 c_{1,2} \dot{\theta}_2 \end{bmatrix}$$

which is injective because from $\begin{bmatrix} p \\ L_f p \end{bmatrix} = 0$, θ_2 and $\dot{\theta}_2$ are determined as

$$\theta_2 = \arccos \frac{l_0 - l_1 c_1}{l_2} - \theta_1, \qquad \dot{\theta}_2 = -\frac{(l_1 s_1 + l_2 s_{1,2}) \dot{\theta}_1}{l_2 s_{1,2}}$$

namely $\theta_2 = \gamma_1(y)$, $\dot{\theta}_2 = \gamma_2(y, \dot{\theta}_1)$. Thus, if $\dot{\hat{\theta}}_1$ is an asymptotic estimation of $\dot{\theta}_1$, so is $\gamma_2(y, \dot{\hat{\theta}}_1)$ for $\dot{\theta}_2$.

As in case (a), no coordinate change is needed and (25) remains valid but ϕ_2 becomes $\phi_2 = \phi_2(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2, \tau_1, \tau_2)$ which has a long expression and hence omitted (the same is true for $\lambda = \lambda(\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2, \tau_1, \tau_2)$). An observer for reconstructing $\dot{\theta}_1$ is given by

$$\begin{bmatrix} \dot{\hat{\theta}}_1 \\ \ddot{\hat{\theta}}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\theta}_1 \\ \dot{\hat{\theta}}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \phi_2(y, \gamma_1(y), \gamma_2(y, \dot{\hat{\theta}}_1), \tau_1, \tau_2) \end{bmatrix} + \begin{bmatrix} l_1 \\ \alpha l_2 \end{bmatrix} (y - \hat{\theta}_1)$$

with $l_1, l_2 > 0$ and $\alpha > 1$. Clearly, an asymptotic estimation of λ is $\hat{\lambda} = \lambda(y, \gamma_1(y), \gamma_2(y, \dot{\hat{\theta}}_1))$.

6 Concluding remark

This paper has considered observability and observer design for a class of dynamic systems described by a set of differential-algebraic equations. The fundamental assumption imposed on the systems is closely related to regularity of descriptor systems and properness of the descriptor vector with respect to the input. According to the established analysis of observability and observer designs for linear descriptor systems, at least theoretically, the class of nonlinear descriptor systems covered in this paper is very limited. Nevertheless, a broad range of constrained mechanical systems belong to this class of nonlinear systems. The treatment in this paper can be considered as an extension of the method adopted in [10] dealing with linear mechanical systems with constraint.

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