A PERFORMANCE COMPARISON BETWEEN BACKSTEPPING AND HIGH-GAIN OBSERVER CONTROL DESIGNS

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Abstract

A non-singular performance measurement for output feedback designs is introduced. The observer backstepping design is compared to the high-gain observer design through a nonlinear output feedback system. If the initial error between the initial condition of the state and the initial condition of the observer is large, the high-gain observer design has better performance than the observer backstepping design.

1 Introduction

In this paper we will be concerned with two major classes of control designs using output feedback. The first class of controllers are based on high gain observers with saturated controls, see eg. [3, 6, 1]. We refer to this class of control designs as *Khalil* designs. The second class of controllers are based on backstepping techniques [8], and we refer to this class of controllers as *KKK* designs.

The *Khalil* designs are applicable to affine systems of full relative degree, whilst the *KKK* designs are applicable to an alternative class of systems, namely those which possess an output feedback normal form. By considering systems which are both full relative degree and have a output feedback normal form, we can compare the behaviour of the controllers on common systems[1], as initiated in [7].

The results in [7] are purely numerical, and give rise to many interesting questions, such as: When do the *KKK* designs require greater control effort than the *Khalil* designs, and vice versa? When do the *KKK* designs have superior output transients to the *Khalil* designs, and vice versa? In particular, by introducing suitable measures of performance and sensitivity we would like to be able to characterize situations in which one design is preferable to another. Such characterizations have obvious consequences for design choices, and also should lead to insight into the dynamics and trade-offs inherent in these controllers. The second problem has been studied in [10]. Here we will consider the former point, by considering a non-singular cost functional penalizing both the output transient and the control effort.

It should be observed that whilst there are many results concerning the transient performance of the output, see eg. [8], there is little work in the literature on non-singular costs for non-optimal designs, see however [5], [4], [2] for related results and techniques.

For an output feedback system Σ with input u and output y, and a controller Ξ mapping $y \mapsto u$, we consider the following cost which penalizes both the control and the output signal.

$$P(\Sigma, \Xi) = \|y\|_{L^2(T_\eta)} + \|u\|_{L^\infty(\mathbb{R}_+)}$$

where the time set T_η is defined by

$$T_\eta = \left\{ t \geq 0 \;\middle|\; |y(t)| > \eta \right\}$$

and η is a small positive number. Such a cost penalizes the input and output response of the system whilst $y(t) \notin [-\eta, \eta]$, hence for a closed loop whose goal is to regulate y to zero, keeping y, u bounded, this cost is finite and is a reasonable penalty on the transient behavior. Note that whilst that a direct L^2 penalty on the output could be considered for the designs given, the relaxation of the output penalty is physically meaningful, and considerably simplifies the technical treatment.

In Section 2 we show that a *Khalil* design out-performs with a *KKK* design when the information on initial state is poor and leads to a large initial observer error.

2 Performance of output feedback system

In this section we study the performances of a *KKK* design and a *Khalil* design for a system $\Sigma(x_0)$ which can be expressed in the output feedback form

$$\Sigma(x_0): \quad \dot{x} = Ax + \varphi(y) + Bu, \quad x(0) = x_0 \tag{1a}$$
$$y = Cx \tag{1b}$$

where

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \quad x_0 = \begin{pmatrix} x_{01} \\ x_{02} \\ \vdots \\ x_{0n} \end{pmatrix}, \quad \varphi(y) = \begin{pmatrix} \varphi_1(y) \\ \varphi_2(y) \\ \vdots \\ \varphi_n(y) \end{pmatrix}$$

[1] Note also that such systems are characterized in a coordinate free manner, [8].

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1, 0, \cdots, 0 \end{pmatrix}$$

and u is the control input, y is the measured output, x_0 is the initial condition of the state, and the functions φ_i are sufficiently smooth and Lipschitz continuous.

Let us first consider a generic observer based controller $\Xi(\hat{x}_0)$, where $\hat{x}_0$ is the initial condition for the observer. The performance of the closed loop $(\Sigma(x_0), \Xi(\hat{x}_0))$ is dependent on both the initial state x_0 and the initial condition for the observer $\hat{x}_0$. Whilst the initial state x_0 is the property of a system, the control designer has the freedom to chose the initial condition $\hat{x}_0$ for the observer.

It is intuitive that good performance results from initializing the observer state $\hat{x}_0$ to be close to the actual initial state x_0. Of course, in practice, the initial state is often unknown, so it can be hard to initialize in this manner. Nevertheless standard practice is to try to minimize

$$\|\tilde{x}_0\| = \|x_0 - \hat{x}_0\|$$

according to the best information available. However, we may well not possess complete information concerning the value of the initial condition of the state, that is we do not exactly know x_0, and hence we have to take $\hat{x}_0$ to be the best estimate to x_0. Then we are interested in studying the situation in which our estimate of x_0 is not accurate and $\|\tilde{x}_0\|$ is large, in particular how does poor information on x_0, (which causes 'bad' choices of $\hat{x}_0$), affect the performance of the controllers?

2.1 KKK design

We first consider a *KKK* design [8] which achieves global regulation of the output. Although the *KKK* design has a global region of attraction (in $(x_0, \hat{x}_0)$), we will prove that the performance of the controller can degrade arbitrarily as the initial error $\|\tilde{x}_0\|$ becomes large for any fixed initial state condition x_0.

The *KKK* design [8] for system $\Sigma(x_0)$ is as follows.

Firstly, an observer is defined by

$$\dot{\hat{x}} = A\hat{x} + k(y - \hat{y}) + \varphi(y) + Bu, \quad \hat{x}(0) = \hat{x}_0 \tag{2a}$$

$$\hat{y} = C\hat{x} \tag{2b}$$

where

$$k = (k_1, k_2, \cdots, k_n)^T, \; k_i > 0, \; 1 \le i \le n$$

is chosen such that $A - kC$ is Hurwitz.

Then define

$$\begin{aligned} \xi_1(y) =& y \\ \alpha_1(y) =& -c_1\xi_1 - d_1\xi_1 - \varphi_1(y) \\ \xi_i(y, \hat{x}_1, \cdots, \hat{x}_i) =& \hat{x}_i - \alpha_{i-1}(y, \hat{x}_1, \ldots, \hat{x}_{i-1}) \\ \alpha_i(y, \hat{x}_1, \cdots, \hat{x}_i) =& -c_i\xi_i - \xi_{i-1} - d_i\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 \xi_i \\ & - k_i(y - \hat{x}_1) - \varphi_i(y) \\ & + \frac{\partial \alpha_{i-1}}{\partial y}(\hat{x}_2 + \varphi_1(y)) \\ & + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{x}_j}(\hat{x}_{j+1} + k_j(y - \hat{x}_1) + \varphi_j(y)) \\ & i = 2, 3, \cdots, n \end{aligned}$$

where c_i, d_i, $1 \le i \le n$ are positive constants. The controller is then defined as

$$\begin{aligned} \Xi_O(\hat{x}_0): \; & u = \alpha_n(y, \hat{x}_1, \cdots, \hat{x}_n) \\ & \dot{\hat{x}} = A\hat{x} + k(y - \hat{y}) + \varphi(y) + Bu, \quad \hat{x}(0) = \hat{x}_0 \\ & \hat{y} = C\hat{x}. \end{aligned}$$

The following result summarizes the standard properties of this closed loop.

Proposition 1. *Consider the closed loop system* $(\Sigma(x_0), \Xi_O(\hat{x}_0))$. *For any initial data* $x_0 \in \mathbb{R}^n$ *and* $\hat{x}_0 \in \mathbb{R}^n$, *the following hold:*
1. The signals x, $\hat{x}$, u *and* y *and bounded;*
2. The output is regulated to zero:

$$\lim_{t \to \infty} y(t) = 0;$$

3. The performance is finite:

$$P\big(\Sigma(x_0), \Xi_O(\hat{x}_0)\big) < \infty.$$

Proof. The proof of 1, 2 can be found in [8]. Let $m(T_\eta)$ denote the Lebesgue measure of the set T_η. Note that $m(T_\eta) < \infty$ since $y(t) \to 0$ as $t \to \infty$ hence,

$$\|y\|_{L^2(T_\eta)} \le m(T_\eta)^{\frac{1}{2}} \|y\|_{L^\infty(\mathbb{R}_+)} < \infty$$

by 1. The boundedness of the performance follows directly. □

We now establish the critical performance property for the *KKK* design, which states that the performance gets arbitrarily large as the initial observer error increases.

Theorem 1. *For any choice of the controller gains* k_i, $1 \le i \le n$, *and for any fixed initial state* x_0 *of the system* $\Sigma(x_0)$, *the performance of the controller* $\Xi_O(\hat{x}_0)$ *has following property:*

$$\limsup_{\|\tilde{x}_0\| \to \infty} P\big(\Sigma(x_0), \Xi_O(\hat{x}_0)\big) = \infty. \tag{3}$$

Proof. For the convenience of notation, the following definitions are introduced:

$$\xi_i(0) = \xi_i(y, \hat{x}_1, \cdots, \hat{x}_i)|_{t=0}$$
$$\alpha_i(0) = \alpha_i(y, \hat{x}_1, \cdots, \hat{x}_i)|_{t=0}$$
$$j = 1, 2, \cdots, n.$$

To prove this theorem, it suffices to show

$$\limsup_{\|\tilde{x}_0\|\to\infty} \|u\|_{L^\infty(\mathbb{R}_+)} = \infty$$

Since $u(t)$ is continuous, to establish the above equation, we only need to show

$$\limsup_{\|\tilde{x}_0\|\to\infty} u(0) = \limsup_{\|\tilde{x}_0\|\to\infty} \alpha_n(0) = \infty. \tag{4}$$

Let $C \subset \mathbb{R}^{n-1}$ be a compact set, define

$$C_r = \left\{\hat{x}_0 \in \mathbb{R}^n \middle| (\hat{x}_{01}, \cdots, \hat{x}_{0,n-1}) \in C; \hat{x}_{0n} = r\right\}.$$

Consider the initial data of the observer $\hat{x}_0 \in C_r$. Because x_0 is fixed, if we can prove that

$$\lim_{r\to\infty} \sup_{\hat{x}_0 \in C_r} \alpha_n(0) = \infty \tag{5}$$

then (4) will hold.

By a lengthy calculation (omitted for brevity), we can show that $\alpha_n(0)$ is of the form:

$$\alpha_n(0) = r \sum_{j=2}^{n} \left(-c_j - d_j \left(\frac{\partial \alpha_{j-1}}{\partial y}\bigg|_{t=0} \right)^2 \right) + F(x_1(0), \hat{x}_1(0), \cdots, \hat{x}_{n-1}(0))$$

where F is a constant independent of r.

Because c_j and d_j are all positive numbers, and F is independent of r, we have established (5) as required. □

2.2 Khalil design

It is well-known that by a suitable coordinate transformation the system $\Sigma(x_0)$ can also be written as integrator chain with a matched nonlinearity. Concretely, we define a coordinate transformation

$$T\colon \mathbb{R}^n \to \mathbb{R}^n, \qquad z = T(x)$$

by

$$T: z_1 = x_1,\ z_2 = x_2 + \psi_1(x_1),\ \cdots,$$
$$z_n = x_n + \psi_{n-1}(x_1, x_2, \cdots, x_{n-1})$$

where

$$\psi_i(x_1, \cdots, x_i) = \varphi_i(x_1) + \sum_{j=1}^{i-1} \frac{\partial \psi_{i-1}}{\partial x_j} \left(x_{j+1} + \varphi_j(x_1) \right), 1 \le i \le n. \tag{6}$$

Then in the z coordinates, $\Sigma(x_0)$ is of the form

$$\Sigma(z_0): \quad \dot{z} = Az + B(\psi(z) + u), \quad z(0) = z_0 \tag{7a}$$
$$y = Cz \tag{7b}$$

where

$$z_0 = T(x_0)$$
$$\psi(z) = \psi_n\left(T^{-1}(z)\right) \tag{8}$$
$$\psi_n(x) = \psi_n(x_1, \cdots, x_n).$$

Remark 1. $\Sigma(z_0)$ and $\Sigma(x_0)$ actually present the same system in different coordinates, but, for convenience, we will use $\Sigma(x_0)$ and $\Sigma(z_0)$ to denote (1) and (7) respectively.

Remark 2. It can be seen from the definition of transform that T is invertible. Further more, both T and T^{-1} are smooth since $\varphi_i, 1 \le i \le n$ are smooth. Hence, the mapping T is a global diffeomorphism in $\mathbb{R}^n$.

Remark 3. Since the output y is unchanged by the transformation T, and the control input u is independent of the change of variables, the performance P independent of T.

Hence the *Khalil* designs considered in [3, 6, 1] can be applied to the system $\Sigma(z_0)$. Typical results establish semiglobal regulation of the output. The *Khalil* designs utilize a high gain observer and a nonlinear separation principle [1] which allow the observer and a globally bounded state feedback controller to be designed separately, and then combined using certainty equivalence, to ensure semiglobal results and closeness of the output feedback controllers trajectory to the underlying state feedback controller's trajectory. For the system $\Sigma(x_0)$, if φ_i and its higher derivatives are globally bounded, it is straightforward to design a globally bounded state feedback controller for $\Sigma(z_0)$, achieving bounded performance. Hence through the high gain observer we can design an output feedback controller, which, for fixed initial condition of the state $z_0 = T(x_0)$ and any initial condition of the observer $\hat{z}_0$ also has bounded performance. Furthermore, if the initial error

$$\|\tilde{z}_0\| = \|z_0 - \hat{z}_0\|$$

becomes large, this design still achieves a bounded performance independent of the initial condition of the observer.

To design an output feedback controller, we first give a state feedback controller for $\Sigma(z_0)$. The controller

$$u = -\psi(z) + v \tag{9}$$

feedback linearizes the system $\Sigma(z_0)$, yielding

$$\dot{z} = Az + Bv, \quad z(0) = z_0 \tag{10a}$$
$$y = Cz. \tag{10b}$$

We first design a bounded state feedback controller for the linear system (10). From [9] we have following lemma.

Lemma 1. *The system (10) is null controllable with bounded control (ANCBC) if and only if*
1. A has no eigenvalues with positive real part;
2. The pair (A, B) is stabilizable in the ordinary sense.

Now since all the eigenvalues of A are zero, namely, without positive real parts, and the pair (A, B) is stabilizable, the system (10) is null controllable with bounded control, and, furthermore, there exists bounded state feedback controllers for the system (10). An explicit example [9] of such a bounded state feedback controller is given by

$$v = -\sum_{i=1}^{n} \delta^i \mathrm{sat}(h_i(z)) \tag{11}$$

where $0 < \delta \leq \frac{1}{4}$, each $h_i : \mathbb{R}^n \to \mathbb{R},\ 1 \leq i \leq n$, is a linear function, and $\mathrm{sat}(\cdot)$ is the saturation function defined by

$$\mathrm{sat}(w) = \begin{cases} -1, & w < -1 \\ w, & -1 \leq w \leq 1 \\ 1, & w > 1. \end{cases}$$

This controller achieves global asymptotic stability for the resulting closed-loop system, see [9].

Consequently, the state feedback controller

$$\Xi_s : \quad u = -\psi(z) - \sum_{i=1}^{n} \delta^i \mathrm{sat}(h_i(z)) \tag{12}$$

globally asymptotically stabilizes the origin of system $\Sigma(z_0)$.

Now we design a output feedback controller for $\Sigma(z_0)$. Following [3, 1], we define the high gain observer as

$$\dot{\hat{z}} = A\hat{z} + H(y - \hat{z}_1) \qquad \hat{z}(0) = \hat{z}_0 \tag{13}$$

where

$$H = H(\epsilon) = \left(\frac{\alpha_1}{\epsilon}, \frac{\alpha_2}{\epsilon^2}, \cdots, \frac{\alpha_n}{\epsilon^n}\right)^T \tag{14}$$

and ϵ is a positive constant to be specified. The positive constants $\alpha_i\ ,\ 1 \leq i \leq n$, are chosen such that the roots of the equation

$$s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1} s + \alpha_n = 0$$

are in the open left-half plane.

To apply the nonlinear separation principle, the state feedback controller is required to be globally bounded. Generally, this property can be achieved by saturating the controller outside some set. But in our case we are interested in the initial condition of the observer becoming large. Instead, we introduce further assumptions on φ_i to ensure that ψ is globally bounded.

Lemma 2. *For system $\Sigma(x_0)$, suppose $\varphi_i \in C^{n-i}(\mathbb{R})$, $\varphi_i^{(k)} \in L^\infty(\mathbb{R}),\ 1 \leq i \leq n;\ \ 1 \leq k \leq n$, then ψ defined by equation (8) lies in $L^\infty(\mathbb{R}^n)$.*

Proof. Since $\varphi_i \in C^{n-i}(\mathbb{R})$, $\varphi_i^{(k)} \in L^\infty(\mathbb{R})$, from (6) we have that $\psi_n(x)$ is continuous and in $L^\infty(\mathbb{R}^n)$. Note that the mapping T is a global diffeomorphism, we know that $\psi(z)$ also is continuous and in $L^\infty(\mathbb{R}^n)$. □

Suppose that the conditions of Lemma 2 are satisfied, then the state feedback controller (12) is globally bounded, so an output feedback controller for system $\Sigma(z_0)$ can be taken as

$$\Xi_{H(\epsilon)}(\hat{z}_0) :\ u = -\psi(\hat{z}) - \sum_{i=1}^{n} \delta^i \mathrm{sat}(h_i(\hat{z})) \tag{15a}$$

$$\dot{\hat{z}} = A\hat{z} + H(y - \hat{z}_1) \qquad \hat{z}(0) = \hat{z}_0. \tag{15b}$$

For the system $\Sigma(z_0)$ and the output feedback controller $\Xi_{H(\epsilon)}(\hat{z}_0)$, relevant properties of the closed loop are summarized below.

Proposition 2. *For system $\Sigma(z_0)$, suppose that $z_0 = T(x_0)$, x_0 is fixed, $\varphi(0) = 0$, and the assumption of Lemma 2 is satisfied. Then for any $\tilde{z}_0 = z_0 - \hat{z}_0$ there exists ϵ^* such that for all $\epsilon : 0 < \epsilon < \epsilon^*$ the output feedback controller $\Xi_{H(\epsilon)}(\hat{z}_0)$ guarantees:*
1. The signals $z,\ \hat{z},\ u$ and y are bounded;
2. The output is regulated to zero:

$$\lim_{t\to\infty} y(t) = 0;$$

3. The following limit

$$\lim_{\epsilon\to 0} z(t, \epsilon) = \bar{z}(t)$$

holds uniformly in t for all $t \geq 0$, where $z(t, \epsilon)$ is the solution of the closed system $(\Sigma(z_0), \Xi_{H(\epsilon)}(\hat{z}_0))$; and $\bar{z}(t)$ is the solution of the state feedback control closed system $(\Sigma(z_0), \Xi_s)$.
4. The performance is finite:

$$P\big(\Sigma(z_0), \Xi_{H(\epsilon)}(\hat{z}_0)\big) < \infty.$$

Proof. Take any compact set $C \in \mathbb{R}^n$ and $\hat{C} \in \mathbb{R}^n$ such that $z_0 \in C$ and $\hat{z}_0 \in C$, then 1, 2, 3 follow directly from Theorem 1 and Theorem 2 in [1]. As to 4, the finiteness of $\|y\|_{L^2(T_\eta)}$ is obtained from 2. Note that ψ is continuous and $\hat{z}$ is bounded by 1. Hence, $\|u\|_{L^\infty(\mathbb{R}_+)}$ is also finite. So, $P\big(\Sigma(z_0), \Xi_{H(\epsilon)}(\hat{z}_0)\big)$ is finite. □

Now it is straightforward to uniformly bound the performance of system $\Sigma(z_0)$ for the *Khalil* design.

Theorem 2. *Let x_0 be fixed, $z_0 = T(x_0)$, and consider the system $\Sigma(z_0)$. Assume that $\varphi_i \in C^{n-i}(\mathbb{R})$, $\varphi_i^{(k)} \in L^\infty(\mathbb{R}),\ 1 \leq i \leq n;\ \ 1 \leq k \leq n$. Then there is a positive constant M, such that for any $\tilde{z}_0$ there exists $\epsilon > 0$ for which the controller $\Xi_{H(\epsilon)}(\hat{z}_0)$ achieves a uniformly bounded performance:*

$$P\big(\Sigma(z_0), \Xi_{H(\epsilon)}(\hat{z}_0)\big) < M. \tag{16}$$

Proof. First note that

$$\begin{aligned} &P\big(\Sigma(z_0), \Xi_{H(\epsilon)}(\hat{z}_0)\big) \\ &= \int_{T_\eta} |y|^2 dt + \|u\|_{L^\infty(\mathbb{R}_+)} \\ &= \int_{T_\eta} |z_1(t,\epsilon)|^2 dt + \|u\|_{L^\infty(\mathbb{R}_+)}. \end{aligned} \tag{17}$$

From Lemma 2, we know that $\psi(\hat{z})$ is bounded. So, the control input u has a bound which is independent of $\hat{z}_0$. By Proposition 2, if ϵ is small enough, then $z_1(t,\epsilon)$ tends uniformly in t to $\bar{z}_1(t)$, which is independent of $\hat{z}_0$ and uniformly bounded. Hence, $\bar{z}_1(t)$ has a bound that is independent of $\hat{z}_0$. Similarly the the measure of the time set T_η is also independent of $\hat{z}_0$ and finite. Hence the integral in (17) is finite and the bound is independent of $\hat{z}_0$. Therefore, we can find a constant M such that (16) holds. □

2.3 Comparison

Theorem 1 shows that for fixed initial state x_0, when the initial error $\|\tilde{x}_0\|$ becomes large, the performance of the *KKK* design is not uniformly bounded even if φ_i and its higher derivatives are globally bounded. On the other hand, Theorem 2 shows for the *Khalil* design, if φ_i and its higher derivatives are globally bounded, then for any initial error $\tilde{z}_0$, through the high gain factor, we can design a globally bounded controller, achieving a uniformly bounded performance.

Hence we obtain the following comparative result:

Corollary 1. *For the system $\Sigma(x_0)$ or $\Sigma(z_0)$, let $\varphi_i \in C^{n-i}(\mathbb{R}), \varphi_i^{(k)} \in L^\infty(\mathbb{R}), 1 \le i \le n$ and consider the controllers $\Xi_O(\hat{x}_0)$ and $\Xi_{H(\epsilon)}(\hat{z}_0)$. Then there exist $\epsilon > 0$ and $\hat{x}_0$ such that for any $\hat{z}_0$ we have:*

$$P\big(\Sigma(z_0), \Xi_{H(\epsilon)}(\hat{z}_0)\big) < P\big(\Sigma(x_0), \Xi_O(\hat{x}_0)\big).$$

Proof. The result follows directly from Theorem 1 and Theorem 2. □

3 Conclusion

Through the comparison of performances for *KKK* and *Khalil* designs, we have established the following result:

For output feedback system, the performance of *KKK* design is sensitive to the initial datum of the observer. The performance of the *KKK* design is not uniformly bounded in the initial error between the initial datum of the state and the initial datum of the observer. When the initial error becomes large, the performance becomes large. Whereas, for the *Khalil* design, for any initial error, by choosing small high-gain factor, we can design a globally bounded controller, achieving an uniformly bounded performance. Therefore, if the initial error is large or in the case that we have poor information for the initial datum of the state, the *Khalil* design has better performance than the *KKK* design.

The primary contribution of the paper is to provide rigorous statements and proofs of the intuitively reasonable trade-offs in performance between the differing classes of designs. The results have been expressed in qualitative terms only, the purpose of the paper is to illustrate the asymptotic differences between the designs. A more quantitative approach is challenging, as achieving tight bounds on non-singular performance is difficult. This is an interesting avenue for future research.

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