A TWO DIMENSIONAL BOUNDED ERROR OBSERVER FOR A CLASS OF BIOREACTOR MODELS

V. Lemesle *, **J.L. Gouzé** *

* COMORE, INRIA, BP93, 06902 Sophia-Antipolis cedex, France
fax: + 33 4 92 38 78 58, phone: +33 4 92 38 {76 35, 78 75}
e-mail: Valerie.Lemesle,Jean-Luc.Gouze@sophia.inria.fr

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Abstract

In this paper, we build bounded error observers for a common class of partially known bioreactor models in two dimensions. The main idea is to build bounded observers "between" the high gain observer, which has an adjustable rate of convergence but requires a perfect knowledge of the model, and an asymptotic observer which is very robust towards uncertainty but with a fixed rate of convergence. We build a two dimensional bounded error observer which reconstructs the two states variables with an error depending on the knowledge of the model; this error is as small as we want. Thus, we obtain better convergence rate for the estimate states than the asymptotic observer.

1 Introduction

The bioreactor is a continuous device where microorganisms consume a nutrient to grow. This nutrient is provided by a constant inflow, and a blend of nutrient and of microorganisms is retrieved in the constant outflow [1]. Generally, no reliable biological sensors for each variable of a biological system exist. In this context, the idea of observers is very interesting to estimate the concentration of the main chemical or biological species in the bioreactor.

First of all we recall the general definition of observers. Consider a dynamical system such that :

$$\begin{array}{l} \dot{X} = F(X,U) \\ Y = h(X) \end{array} \qquad (1)$$

with $X \in \mathbb{R}^n, U \in \mathbb{R}^m \; m \leq n, Y \in \mathbb{R}^p \; p < n$.

An observer for (1) is a dynamical system

$$\dot{\hat{X}} = \hat{F}(\hat{X}, U, Y)$$

whose task is state estimation. It is expected to provide an estimate state $\hat{X}$ of the state X of the original system. One usually requires at least that $||\hat{X} - X||$ goes to zero as t tends to ∞ ; in some cases, exponential convergence is required [10].

Often, it happens that some functions of the state variables are partially known in the dynamical model [9]. Then, we define a bounded error observer giving $\hat{X}$ with $||\hat{X} - X||$ bounded by a "reasonable" constant; "reasonable" meaning that it is small enough to have a good approximation of the unmeasured state.

In the following of the paper, we will consider a classical class of bioreactor models, [1], describe by:

$$\left\{ \begin{array}{l} \dot{x} = \mu(s)x - dx \\ \dot{s} = -\alpha\mu(s)x + ds_{in} - ds \end{array} \right. \qquad (2)$$

where $d = \frac{q}{V}$ is the dilution rate with V the volume of the bioreactor and q the constant flow passing through the bioreactor, α the growth yield, s_{in} is the input substrate concentration, $\mu(s)$ is the specific growth rate per unit of biomass. Different models exist in the literature; for example, one often use the Monod model $\mu(s) = \frac{\mu_m s}{k+s}$ where μ_m and k are the maximum growth rate and the half saturation constant, respectively.

We propose to adapt the observer design to the available knowledge of the growth rate $\mu(s)$. We first recall the classical observers built for the bioreactor model. When the growth rate $\mu(s)$ is perfectly known, a high gain observer which have an adjustable convergence rate is built; if $\mu(s)$ is unknown, then an asymptotic observer which have a constant rate of convergence is considered. Then, we propose an intermediate approach to deal with a partial knowledge of $\mu(s)$ where we obtain a bound on the error depending on the knowledge we have on the model, and where it can be adjusted in some way as in [4]. We build some hybrid observers evolving between the two limit cases : the high gain observer and the asymptotic one as the asymptotic-Kalman observer proposed by [3]. We recall the one dimensional bounded observer already obtained in [7] then we propose a two dimensional bounded observer to improve the first one. It can be seen in some way as a switch between the high gain observer and the asymptotic one. Finally, we illustrate all the results by simulation studies.

2 Classical observers for the bioreactor model

We recall some facts concerning the high gain observer and the asymptotic one for the classical bioreactor model (2).

2.1 The high gain observer

First, we recall briefly the notion of an high gain observer for general system. Consider the differential system (1) defined on a domain $\Omega \subset \mathbb{R}^n$ where $F(X,U,Y) = f(X) + g(X)U$ that is:

$$\left\{\begin{array}{l}\dot{X}=f(X)+g(X)U\\ Y=h(X)\end{array}\right. \quad (3)$$

where f, g $\mathbb{R}^n \rightarrow \mathbb{R}^n$ and the observation function h $\mathbb{R}^n \rightarrow \mathbb{R}^p$ are smooth. Moreover if we assume that some hypotheses hold [5], we can design high gain observer. Notice that for biological system, this hypothesis are often verified [2]. Then, we recall the high gain observer definition.

Proposition 2.1 *For θ large enough the following differential system (4) is an exponential observer for (3) :*

$$\hat{X}=f(\hat{X})+Ug(\hat{X})-\left[\frac{\partial\Phi}{\partial x}\right]^{-1}_{X=\hat{X}}S_\theta^{-1}C(h(\hat{X})-Y) \quad (4)$$

with S_θ the solution of the equation $\theta S_\theta+A^tS_\theta+S_\theta A=C^tC$ where

$$A=\begin{pmatrix}0&1&0&\dots 0\\0&0&1&\dots 0\\ \vdots&\vdots&\vdots&\vdots\\0&0&0&\dots 1\\0&0&0&\dots 0\end{pmatrix} \text{ and } C=(0 \quad 0\cdots 1)$$

S_θ can be analytically computed

$$S_\theta(i,j)=\frac{(-1)^{i+j}}{\theta^{i+j-1}}\frac{(i+j-2)!}{(i-1)!(j-1)!}$$

In (4), Φ denotes the diffeomorphism, globally defined on Ω (L_f denotes the Lie derivative of h along the field f) $\Phi : x \rightarrow (h(x)L_fh(x)\dots L_f^{n-1}h(x))^t$.

In particular, assuming that the rate of growth is given by the "Monod model", we get the differential standard equations :

$$\left\{\begin{array}{l}\dot{s}=-\alpha\dfrac{\mu_m sx}{k+s}-ds+ds_{in}\\ \dot{x}=\dfrac{\mu_m sx}{k+s}-dx\\ y=s\end{array}\right. \quad (5)$$

We obtain the following high gain observer for the system (5) applying the Proposition (2.1) [5]:

$$\left\{\begin{array}{l}\dot{\hat{s}}=\dfrac{-\alpha\mu_m\hat{x}\hat{s}}{k+\hat{s}}+d(s_{in}-\hat{s})-2\theta(\hat{s}-y)\\ \dot{\hat{x}}=\dfrac{\mu_m\hat{s}\hat{x}}{\hat{s}+k}-d\hat{x}+(2\theta\dfrac{k\hat{x}}{(k+\hat{s})\hat{s}}+\theta^2\dfrac{\hat{s}+k}{\alpha\mu_m\hat{s}})(\hat{s}-y)\end{array}\right.$$

Simulations

We take for our parameters values : $s_{in}=50$, $d=0.1$, $\alpha=1$, $\mu(s)=\frac{s}{140+s}$ and $\hat{\mu}(s)=\frac{0.8s}{140+s}$. Moreover, we take $\theta=3$. In the two following simulations, θ is fixed and s is measured. In dotted line we can see the high gain observer when the growth rate is partially known, in plus line the high gain observer when the model is perfectly known, in plain line the model. Moreover, we take $\hat{s}(0)=10$, $\hat{x}(0)=20$.

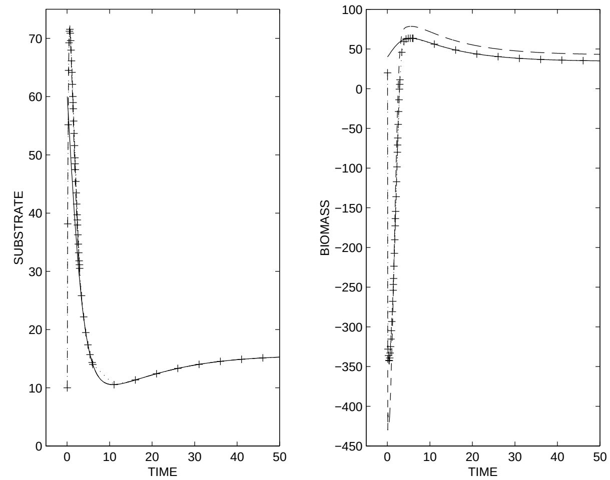

Figure 1: In dotted line the high gain observer when the growth rate is partially known, in plus line the high gain observer when the model is perfectly known, in plain line the model

A very strong peak appear at the beginning of the simulations. The value of the gain θ and the big initial output error are the causes of this phenomenon. Moreover, to obtain this exponential observer, the model must be perfectly known : we can see that the observer converges towards the model rapidly. If we don't know the model ($\mu(s)$ is replaced by $\hat{\mu}(s)$ in the observer equation), we can see that the error does not go to zero.

Therefore, in some cases, we want to obtain a better bound for this error.

2.2 The asymptotic observer

The main idea of the asymptotic observer is to eliminate the unknown function. Consider the dynamical system (2) and assume $z=\alpha x+s$. We suppose that s is exactly measured and the function $\mu(s)$ is unknown [1].

The dynamics of z is given by the following equation :

$$\dot{z}=ds_{in}-dz \quad (6)$$

An asymptotic observer for (6) is given by $\dot{\hat{z}}=ds_{in}-d\hat{z}$. If we consider the error $e=\hat{z}-z$, we can immediately conclude that $\dot{e}=-de$ that is to say the asymptotic observer converges towards z with a constant convergence rate given by d. We can, moreover reconstruct x considering $\hat{x}=\frac{\hat{z}-s}{\alpha}$. The advantage of this kind of observer is its robustness in comparison with the high gain observer but its convergence rate is fixed by the model.

3 Bounded error observer

We define a bounded error observer as an observer such that we no longer require the error between the estimate state and the original model to converge to zero but to be bounded by a "reasonable" constant, "reasonable" meaning that this constant is small with respect to the measurement errors; we impose that this bound is zero if the model is perfectly known.

Definition 3.1 *A bounded error observer of (1) will be a dynamical system $\dot{\hat{X}} = \hat{F}(\hat{X}, U, Y)$ with $\lim_{t\to\infty} ||\hat{X} - X|| \leq m$, m a positive real constant depending on the knowledge of F such that $m = 0$ if F is perfectly known.*

In this paper, m depends in particular on the difference between $\mu(s)$ and $\hat{\mu}(s)$. Indeed, we know the bounds of the growth rate such that

$|\hat{\mu}(s) - \mu(s)| < a$ with a a positive real constant. Moreover, we assume that $\hat{\mu}(0) = \mu(0) = 0$.

3.1 A one dimensional bounded error observer

First, we build a one dimensional bounded error observer. We suppose that s is measured. Then, we want to reconstruct the biomass variable x. Consider the system :

$$\begin{cases} \dot{s} = -\alpha\mu(s)x - ds + ds_{in} \\ \dot{x} = \mu(s)x - dx \\ y = s \end{cases}$$

and make the change of variable $(s, x) \to (s, z)$ with

$z = \alpha x + \theta s$ where θ is a fixed real constant.

The dynamics of z is :

$$\dot{z} = (1-\theta)\mu(s)\alpha x - dz + \theta ds_{in} \tag{7}$$

Proposition 3.1 *The system $\dot{\hat{z}} = (1-\theta)\hat{\mu}(s)\alpha\hat{x} - d\hat{z} + \theta ds_{in}$ is a bounded error observer of (7) where $\hat{\mu}(s)$ is chosen such as $|\hat{\mu}(s) - \mu(s)| < a$ with $a \in \mathbb{R}$ $a > 0$ and θ is a gain ($\theta > 1$).*

Proof See [7].

Comments

This bounded observer has a constant error depending on θ. The error is equal to zero if $\theta = 1$ and is fixed if θ is large. Then, when θ is time dependent (large at the beginning of the integration and equal to 1 at the end), this bounded observer can be seen as a switch between the high gain observer and the asymptotic one.

The major problem of this bounded observer is that we can not improve the gain to have a better convergence rate. Indeed, as we don't reconstruct the measured variable we cannot use the measured error as a control parameter. Now, we want to improve this with the following view : when the error between the measurement and the observed variable is large, a kind of high gain observer is considered; when the error is smaller enough, a kind of asymptotic observer is considered. Thus, we construct a two dimensional bounded error observer.

3.2 A two dimensional bounded error observer

To build this observer, we use the same idea that for the one dimensional bounded observer, that is to say a high gain bounded observer to go to a bounded error rapidly then an asymptotic like one to converge to an error as small as we want.

We consider the system:

$$\begin{cases} \dot{s} = -\alpha\mu(s)x - ds + ds_{in} \\ \dot{x} = \mu(s)x - dx \\ y = s \end{cases}$$

We only have a partial knowledge on $\mu(s)$ that is to say that $|\hat{\mu}(s) - \mu(s)| \leq a$.

We make the change of variables $(s, x) \to (s, z)$ with

$z = \alpha x + s$. We obtain the new dynamical system :

$$\begin{cases} \dot{s} = -\hat{\mu}(s)(z-s) - ds + ds_{in} + (\hat{\mu}(s) - \mu(s))(z-s) \\ \dot{z} = -dz + ds_{in} \\ y = s \end{cases} \tag{8}$$

Proposition 3.2 *The dynamical system*

$$\begin{cases} \dot{\hat{s}} = -\hat{\mu}(s)(\hat{z} - \hat{s}) - d\hat{s} + ds_{in} - k_1\theta(\hat{s} - s) \\ \dot{\hat{z}} = -d\hat{z} + ds_{in} - k_2\theta^2(\hat{s} - s) \end{cases} \tag{9}$$

with θ a positive constant gain, k_1 and k_2 constant gains verifying (10), k_2 depends on the error $\hat{s} - s$ such that $k_2 = 0$ when $\hat{s} - s < \epsilon$, ϵ a fixed small constant, is a bounded error observer for (8) where $\hat{\mu}(s)$ is chosen as: $|\hat{\mu}(s) - \mu(s)| \leq a$ with $a \in \mathbb{R}$, $a > 0$.

First of all, we suppose that the hypotheses in [6] hold.To prove the proposition, we need the following lemma.

Lemma 3.1 *There exists $\epsilon > 0$ a chosen constant, such that $s(0) > \epsilon$ implies $s(t) > \epsilon$ and $\mu(s(t)) > \mu(\epsilon)$ for all t.*

The proof is easy using standard technics for invariant regions [8].

Thanks to this lemma, we could always choose ϵ such that $\hat{\mu}(s) > \hat{\mu}(\epsilon) = l$. We recall also that it is well known that the variables s and x are bounded.

Proof

The ideas are the same as in [6] for the high gain observer. Consider e the error in s, and z such that $e = \begin{pmatrix} e_s = \hat{s} - s \\ e_z = \hat{z} - z \end{pmatrix}$.
It verifies the following equation :

$$\dot{e} = \begin{pmatrix} -k_1\theta & -\hat{\mu}(s) \\ -k_2\theta^2 & 0 \end{pmatrix} e + \begin{pmatrix} \hat{\mu}(s) - d & 0 \\ 0 & -d \end{pmatrix} e + (\mu(s) - \hat{\mu}(s))(z-s)\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Taking $e_1 = \Delta_\theta^{-1} e = \begin{pmatrix} \frac{1}{\theta} e_s \\ \frac{1}{\theta^2} e_z \end{pmatrix}$ with $\Delta_\theta = \begin{pmatrix} \theta & 0 \\ 0 & \theta^2 \end{pmatrix}$.

We obtain the following equation for e_1 :

$$\begin{array}{rcl} \dot{e_1} & = & \Delta_\theta^{-1} \begin{pmatrix} -k_1\theta & -\hat{\mu}(s) \\ -k_2\theta^2 & 0 \end{pmatrix} \Delta_\theta e_1 \\ & + & \Delta_\theta^{-1} \begin{pmatrix} \hat{\mu}(s) - d & 0 \\ 0 & -d \end{pmatrix} \Delta_\theta e_1 \\ & + & \Delta_\theta^{-1} (\mu(s) - \hat{\mu}(s))(z - s) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{array}$$

That is to say :

$$\dot{e_1} = \theta \begin{pmatrix} -k_1 & -\hat{\mu}(s) \\ -k_2 & 0 \end{pmatrix} e_1 + \begin{pmatrix} \hat{\mu}(s) - d & 0 \\ 0 & -d \end{pmatrix} e_1 + (\mu(s) - \hat{\mu}(s))(z - s) \begin{pmatrix} \frac{1}{\theta} \\ 0 \end{pmatrix}$$

Consider the matrix $A = \begin{pmatrix} 0 & -\hat{\mu}(s) \\ 0 & 0 \end{pmatrix}$, $C = (1\ 0)$. Then, there are a real constant $\lambda > 0$, a vector $K \in \mathbb{R}^2$, $K^t = (k_1\ k_2)$ and a symmetric, positive definite 2×2 matrix S depending the bounds of $\hat{\mu}(s)$ only such that :

$$S(A - KC) + (A - KC)^t S \leq -\lambda Id \tag{10}$$

A proof of this lemma can be see in [6]. We can notice that $A - KC$ is stable, that is to say $k_1 > 0$ and $k_2 < 0$. With matricial notation, we obtain the equation for e_1 :

$$\dot{e_1} = \theta(A - KC)e_1 + Be_1 + (\mu(s) - \hat{\mu}(s))(z - s) \begin{pmatrix} \frac{1}{\theta} \\ 0 \end{pmatrix}$$

Consider a Liapounov function $V = \frac{1}{2} e_1^t S e_1$. We want to prove that $\dot{V} < 0$. We have :

$$\begin{array}{rl} \dot{V} < & -\theta\frac{\lambda}{2}||e_1||_2^2 + N(S).N(B).||e_1||_2^2 \\ & + \frac{1}{\theta}||e_1||_2.N(S).|\mu(s) - \hat{\mu}(s)|.|z - s| \end{array}$$

where $N(S)$, $N(B)$ are the induced matrix norm corresponding to the Euclidean one that is to say

$$N(M) = max\{\sqrt{\lambda}, \lambda \in Spect(M^* M)\}$$

We can remark that in our case $N(B)$ is equal to $\hat{\mu}(s) - d$ which is between $\hat{\mu}(\epsilon) - d$ and $\hat{\mu}_{max} - d$ using lemma (3.1). Moreover, the states variables are bounded :

$$\dot{V} < (-\frac{\theta\lambda}{2} + N(S).N(B))||e_1||_2^2 + \frac{\alpha a x_{max} N(S)}{\theta}||e_1||_2$$

As all the norms are equivalent in $\mathbb{R}^n$, we have $\gamma_1||e_1||_S \leq ||e_1||_2 \leq \gamma_2||e_1||_S$. Hence:

$$\dot{V} < 2\gamma_1^2(-\frac{\theta\lambda}{2} + N(S).N(B))V + 2\gamma_2\frac{\alpha a x_{max} N(S)}{\theta}\sqrt{V}$$

Thus :

$$\frac{\dot{V}}{2\sqrt{V}} = \frac{d\sqrt{V}}{dt} < \gamma_1^2(-\frac{\theta\lambda}{2} + N(S).N(B))\sqrt{V} + \gamma_2\frac{\alpha a x_{max} N(S)}{\theta}$$

Using the Gronwall lemma :

$$\begin{array}{rl} \sqrt{V} < & (\sqrt{V(0)} - \frac{\gamma_2 \alpha a x_{max} N(S)}{\theta\gamma_1^2(-\frac{\theta\lambda}{2} + N(S).N(B))}) e^{\gamma_1^2(-\frac{\theta\lambda}{2} + N(S).N(B))t} \\ & - \frac{\gamma_2 \alpha a x_{max} N(S)}{\theta\gamma_1^2(-\frac{\theta\lambda}{2} + N(S).N(B))} \end{array}$$

Let us denote the previous equation :

$$\sqrt{V} < \varphi(t)$$

We must make another change of variable to conclude on the convergence of e.

We can prove by a simple computation that $V > \frac{M^2}{\theta^4} e^t e$ with M a positive real constant chosen such that $\Delta_\theta^{-1} S \Delta_\theta^{-1} - \frac{M^2}{\theta^4} Id$ positive. Finally we conclude:

$$\sqrt{e^t e} < \frac{\theta^2}{M}\varphi(t)$$

We can go asymptotically as fast as we want to a bounded error by chosen θ large:

$$lim_{t \to \infty}||e|| < \frac{2\gamma_2 \alpha a x_{max} N(S)}{\gamma_1^2 \lambda M} \tag{11}$$

We switch to an asymptotic like observer taking $k_2 = 0$ when $(\hat{s} - s)$ stays during some time less or equal than ϵ, ϵ a fixed small constant. We obtain the new bounded error observer for (2):

$$\begin{array}{l} \dot{\hat{s}} = -\hat{\mu}(s)(\hat{z} - \hat{s}) - d\hat{s} + ds_{in} - k_1\theta(\hat{s} - s) \\ \dot{\hat{z}} = -d\hat{z} + ds_{in} \end{array} \tag{12}$$

The equation of the error become:

$$\dot{e} = \begin{pmatrix} -k_1\theta & -\hat{\mu}(s) \\ 0 & 0 \end{pmatrix} e + \begin{pmatrix} \hat{\mu}(s) - d & 0 \\ 0 & -d \end{pmatrix} e + (\mu(s) - \hat{\mu}(s))(z - s) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Thus solving the second equation and injecting it in the first one, we obtain :

$$\begin{array}{rl} \dot{e_s} = & -k_1\theta e_s - \hat{\mu}(s) e_z(0) e^{-dt} \\ & + (\hat{\mu}(s) - d)e_s + (\mu(s) - \hat{\mu}(s))(z - s) \\ & \\ e_z = & e_z(0)e^{-dt} \end{array}$$

We call this observer an asymptotic like one because the convergence rate of e_z is fixed by the model and is equal to d;

moreover, e_z goes asymptotically to zero. Let us considered $|e_s|$. It dynamics is given by $\dot{|e_s|} = sgn(e_s)\dot{e_s}$ that is:

$$\begin{aligned} \dot{|e_s|} = \quad & sgn(e_s)(\mu(s) - \hat{\mu}(s))(z - s) \\ & + (\hat{\mu}(s) - d - k_1\theta)|e_s| - sgn(e_s)\hat{\mu}(s)e_z(0)e^{-dt} \end{aligned}$$

But $sgn(e_s) \leq 1$ and for all s, $\hat{\mu}(s) - d - k_1\theta < 0$ by chosen θ large, $k_1 > 0$ we get :

$$\dot{|e_s|} \leq a\alpha x_{max} + (\hat{\mu}_{max} - d - k_1\theta)|e_s| + \hat{\mu}_{max}|e_z(0)|e^{-dt}$$

Hence:

$$\begin{aligned} |e_s| \leq \quad & (|e_s(0)| + \frac{\hat{\mu}_{max}|e_z(0)|}{\hat{\mu}_{max} - k_1\theta})e^{(\hat{\mu}_{max} - d - k_1\theta)t} \\ & + \frac{a\alpha x_{max}}{-\hat{\mu}_{max} + d + k_1\theta}e^{(\hat{\mu}_{max} - d - k_1\theta)t} \\ & + \frac{\hat{\mu}_{max}|e_z(0)|}{-\hat{\mu}_{max} + k_1\theta}e^{-dt} + \frac{a\alpha x_{max}}{-\hat{\mu}_{max} + d + k_1\theta} \end{aligned}$$

Thus :

$$lim_{t\to\infty}|e_s| \leq \frac{a\alpha x_{max}}{-\hat{\mu}_{max} + d + k_1\theta} \qquad (13)$$

We remark that for $k_1\theta$ large , we get $lim_{t\to\infty}|e_s| = 0$ with a convergence rate as large as we want.

Comments :

Before the switch between the two different observers, one can see that the error of the first is a fixed bound (11). Then, to obtain a faster convergence rate than the asymptotic one, the initial output error between the bounded observer and the model must be bigger than this fixed bound. Under this condition, the first bounded observer which can be seen as a high gain observer goes rapidly towards the bound (11), then when the observation error is small enough we switch to the asymptotic like observer.

One can notice that the final error bound (13) depend on θ, that is to say if θ is large this bound goes to zero, and we go as fast as we want as near as we want : it is the idea of "practical observer" [4]. Thus, a static small final error can be observed.

Simulations

We take for our parameters values : $s_{in} = 50$, $d = 0.1$, and the difference a between $\mu(s)$ and $\hat{\mu}(s)$ equal to 0.2. Moreover, we take $k_2 = -1.5$ when the absolute value error between the observation and the model of the substrate is bigger than 0.1 else we take $k_2 = 0$. The other gains are $\theta = 3$ and $k_1 = 5$. We take for initial conditions $\hat{s}(0) = 10$, $\hat{z}(0) = 30$ that is to say $\hat{x}(0) = 20$.

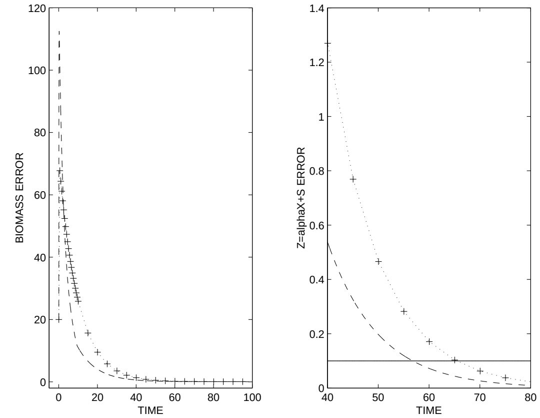

Figure 2: In dotted line the biomass error and the substrate error of the asymptotic observer in plus line and of the two dimensional bounded observer in dotted line

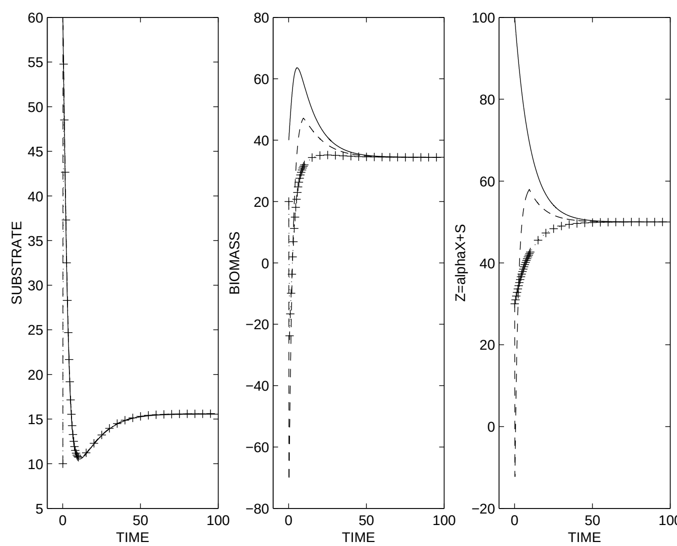

Figure 3: In dotted line the two dimensional bounded observer, in plus line the asymptotic one, in plain line the model

The peak which appears at the beginning of the simulations returns non positive observer variables; it is the same phenomenon that for the high gain observer when the gain and the output error are large.

We can see that the two dimensional observer converges faster than the asymptotic one; indeed, if we choose $z - \hat{z} = 0.1$ (see the second part of Figure(1)), we can see than the two dimensional observer reaches this bound for $t \simeq 55$ and the asymptotic observer reaches this bound for $t \simeq 65$ and after this bound the two dimensional bounded observer is always below the asymptotic one.

4 Conclusion

The purpose of bounded observers is simply to provide a tool allowing the state variable estimation when the model is poorly known, that is usually the case in biology.

Then, we build observers reconstructing variables with a reasonable error. In one dimension, the convergence rate of this observer can not be improve because we cannot consider the output error as a control parameter. Thus we build a two dimensional observer and we obtain a faster convergence rate than the asymptotic observer if the initial error is large enough.

A way to improve the convergence seems to build an adaptive version of the two dimensional observer. Some simulations studies seem to show this result; an theoretical proof is currently in study. In this paper, we only consider two dimensional system, a generalization to higher dynamical system is evidently possible.

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References

[1] G. Bastin and D. Dochain. *On-line Estimation and Adaptative Control of Bioreactors*, volume 1. Elsevier, Amsterdam, 1990.

[2] O. Bernard, G. Sallet, and A. Sciandra. Nonlinear Observers for a Class of Biological Systems Application to Validation of Phytoplanktonic Growth Model. *IEEE Transactions on Automatic Control*, 43:1056–1065, 1998.

[3] P. Bogaerts. A Hybrid Asymptotic-Kalman Observer for Bioprocesses. *Bioprocess Engineering*, 20(3):101–113, 1999.

[4] M. Farza, K. Busawon, and H. Hammouri. Simple Nonlinear Observers for On-line Estimation of Kinetics Rates in Bioreactors. *Automatica*, 34:301–318, 1998.

[5] J. Gauthier, H. Hammouri, and S. Othman. A simple Observer for Nonlinear Systems . Applications to Bioreactors. *IEEE Transactions on Automatic Control*, 37:875–880, 1992.

[6] J. Gauthier and I. Kupka. Observability and Observers for nonlinear systems. *SIAM Journal of Control and Optimization*, 32:975–994, 1994.

[7] J. L. Gouzé and V. Lemesle. A bounded error observer for a class of bioreactor models. In *Proceedings of ECC01, Porto, September 4-7 2001*, 2001.

[8] M. Z. Hadj-Sadok and J. L. Gouzé. Interval Observers for Uncertain Models of Wastewater treatment processes. In *CONTROLO'98* 3^{rd} *Portuguese Conference on Automatic Control, Coimbra, Portugal, p.673–678*, 1998.

[9] M. Z. Hadj-Sadok and J. L. Gouzé. Estimation of uncertain models of activated sludge process with interval observers. *Journal of Process Control*, 11(3):299–310, 2001.

[10] D. Luenberger. *Introduction to Dynamic Systems. Theory, Models,and Applications*. John Wiley and Sons, New York, 1979.