

INDEPENDENT DESIGN OF DECENTRALIZED CONTROLLERS FOR SPECIFIED CLOSED-LOOP PERFORMANCE

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Abstract: The paper presents an original frequency domain approach to decentralized controller design for specified performance. The novelty of the proposed approach consists in assessing the influence of interactions by means of characteristic functions/loci of the plant interaction matrix, and further using them to modify mathematical models of subsystems thus defining the so called „equivalent subsystems“. The independent design carried out for equivalent subsystems provides local controllers that guarantee fulfilment of performance requirements without any deterioration due to the effect of interactions. Theoretical conclusions are supported with results obtained from the solution of several examples, one of which is included.

Keywords: decentralized controller, generalized Nyquist stability criterion, spectral Nyquist plot, characteristic loci, independent design

1 Introduction

Industrial plants are complex systems typical by multiple inputs and multiple outputs (MIMO systems). Usually they arise as interconnection of a finite number of self-contained units – subsystems. If strong interactions within the plant are to be compensated for then multivariable controllers are used. However, there may be practical reasons that make restrictions on controller structure necessary or reasonable. In an extreme case, the controller is split into several local feedbacks and becomes a decentralized controller. Compared with centralized full-controller systems such controller structure constraints bring about certain performance deterioration. However, this drawback is weighted against important benefits such as hardware simplicity, operation simplicity and reliability improvement as well as design simplicity [2,7]. Due to them, decentralized control (DC) design techniques remain probably the most popular among control engineers, in particular the frequency domain ones which provide insightful solutions and link to the classical control theory.

Major important multivariable frequency-response Nyquist-type design techniques based on the generalized concept of the return difference were developed in the late 60' and throughout the 70's: the Inverse-Nyquist

Array (INA) and Direct Nyquist Array (DNA) methods by Rosenbrock, the Sequential design technique by Mayne. Almost simultaneously the non-interacting Characteristic locus (CL) technique was developed [3,4].

Development of decentralized control (DC) techniques dates back to the 70'; the research, though not so excessive, is still going on. The principle of the frequency-domain DC design techniques consists in achieving dominance (diagonal, block diagonal, quasi-block diagonal), reducing sensitivity or improving chosen performance measure in subsystems (principle of dominant subsystems [2]). The DC design has two main steps: first, a suitable control structure (pairing inputs with outputs) has to be selected; then, local controllers are designed for individual subsystems. Depending on the manner of coping with interaction, there are three general approaches to the local controller design: simultaneous, sequential and independent designs [2, 7].

According to the independent design, effect of interactions on the full system is assessed first, and then transformed into bounds for individual subsystem closed-loops. These bounds are to be considered in the local controller design in order to guarantee stability and required performance of the full system. Main advantages with this approach are failure tolerance and a direct design of local controllers [2, 7].

In what follows, it is assumed that the control structure selection step has already been completed. To account for plant interactions, an original approach is proposed based upon representing interactions by means of their characteristic loci (eigenvalue loci) [1,3,4,5] and further using them to modify mathematical models of individual subsystems. To these modified subsystems denoted as „equivalent subsystems“, an independent design procedure is applied then. Designed local controllers guarantee fulfilment of performance requirements imposed on the full system. Theoretical results are supported by an example.

The paper is organized as follows: theoretical preliminaries of the proposed technique are surveyed in Section 2, problem formulation is in Section 3, main results along with the proposed design procedure are presented in

Section 4 and verified on an example in Section 5. Conclusions are given at the end of the paper.

2 Preliminaries

Consider a MIMO system $G(s)$ and the controller $R(s)$ in a standard feedback configuration (Fig. 1)

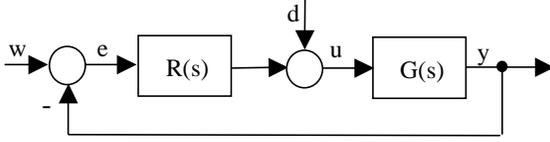


Figure 1: Standard feedback configuration

where $G(s) \in R^{m \times l}$ and $R(s) \in R^{l \times m}$ are transfer function matrices and w, u, y, e, d are respectively vectors of reference, control, output, control error and disturbance of compatible dimensions. Hereafter, only square matrices will be considered, i.e. $m=l$.

The feedback system in Fig.1 is internally stable if and only if the transfer matrix from $[d \ w]^T$ to $[u \ e]^T$ given by

$$\begin{bmatrix} (I + RG)^{-1} & (I + RG)^{-1}R \\ -(I + GR)^{-1}G & (I + GR)^{-1} \end{bmatrix} \quad (1)$$

is stable.

Another test for internal stability is the Nyquist encirclement criterion. Both the internal stability condition and the Nyquist stability theorem provide necessary and sufficient condition for the closed-loop stability. Note that if the system is internally stable then it is stable with respect to all state and output variables and in the sequel it will simply be called “stable”.

The multivariable stability theory relies on the concept of the system return difference [5, 8] defined as

$$F(s) = [I + Q(s)] \quad (2)$$

where

$F(s) \in R^{m \times m}$ is the system return-difference matrix

$Q(s) \in R^{m \times m}$ is the (open) loop transfer function matrix, for the system in Fig.1 $Q(s) = G(s)R(s)$

$H(s) \in R^{m \times m}$ is the closed-loop transfer function matrix,

$$H(s) = Q(s)[I + Q(s)]^{-1}$$

R denotes the field of rational functions in s .

The *Nyquist D-contour* is a large contour in the complex plane consisting of the imaginary axis $s = j\omega$ and an infinite semi-circle into the right-half plane. It has to avoid locations where $Q(s)$ has $j\omega$ -axis poles (e.g. if $R(s)$ includes integrators) by making small indentations around these points such as to include them to the left-half plane. Thus, unstable poles of $Q(s)$ will be considered those in the *open* right-half plane. *Nyquist plot* of a complex function $g(s)$ is the image of the

Nyquist under $g(s)$, whereby the D -contour is traversed clockwise; $N[k, g(s)]$ denotes number of anticlockwise encirclements of the point $(k, j0)$ by the Nyquist plot of $g(s)$.

Consider the characteristic polynomial of the system in Fig. 1

$$\det F(s) = \det[I + Q(s)] = \det[I + G(s)R(s)] \quad (3)$$

If $Q(s)$ has n_q unstable poles, the closed-loop stability can be determined using the generalized Nyquist stability criterion [1, 3, 4, 5, 8].

Theorem 1 (Generalized Nyquist Stability Theorem)

The feedback system in Fig. 1 is stable if and only if

- $\det F(s) \neq 0$
- $N[0, \det F(s)] = n_q$ (4)

where n_q is the number of its open-loop unstable poles. □

As rational functions in s form a field, standard arithmetical manipulations with transfer functions can be carried out. Thus, eigenvalues of a square matrix $Q(s) \in R^{m \times m}$ (whose elements are rational functions in s) are themselves rational functions in s . They are called *characteristic functions* [3, 4, 5]. The m characteristic functions $q_i(s)$, $i = 1, \dots, m$ of $Q(s)$ are given by

$$\det[q_i(s)I_m - Q(s)] = 0 \quad i = 1, \dots, m \quad (5)$$

Then

$$\det F(s) = \det[I + Q(s)] = \prod_{i=1}^m [I + q_i(s)] \quad (6)$$

Characteristic loci (CL) denoted $q_i(j\omega)$, $i = 1, 2, \dots, m$ are the set of loci in the complex plane traced out by the characteristic functions of $Q(s)$ as s traverses the Nyquist D -contour. This set is called the *spectral Nyquist plot* [1]. The *degree* of the spectral Nyquist plot is the sum of anticlockwise encirclements with respect to a specified point in the complex plane, contributed by the characteristic loci of $Q(s)$.

A stability test analogous to *Theorem 1* has been derived in terms of the CL's [1, 3, 4, 5, 8].

Theorem 2

The closed-loop system with the open-loop transfer function $Q(s)$ is stable if and only if $\det[I + Q(s)] \neq 0$ and the degree of the spectral Nyquist plot of $Q(s)$ with respect to $(-1, 0j)$ fulfils the following condition (with n_q being the number of unstable poles of $Q(s)$)

$$\sum_{i=1}^m N[-1, q_i(s)] = n_q \quad (7a)$$

or alternatively, if considering the spectral Nyquist plot of $[I + Q(s)]$

$$\sum_{i=1}^m N\{0, [I + q_i(s)]\} = n_q \quad (7b)$$

Remarks:

1. In the sequel, whenever possible, matrices and their corresponding characteristic transfer functions/loci will be denoted by like capital and small letters, respectively.
2. The CL's are not uniquely defined since whenever two particular loci cross, the identity of the correct continued path is lost. Furthermore, each locus does not in general form a closed contour. Fortunately, as the underlying characteristic functions are piecewise analytic, it is possible to concatenate the m CL's of $[I + Q(s)]$ to form one or more close contours [1].
3. Theorems 1 and 2 are equivalent, therefore

$$N\{0, \det[I + Q(s)]\} = \sum_{i=1}^m N\{0, [I + q_i(s)]\} = n_q \quad (8)$$

In general, for points other than $(0,0j)$, degrees of the respective plots are different [1].

3 Problem Formulation

Consider a complex system with m subsystems ($m=1$), described by a square transfer matrix $G(s) \in R^{m \times m}$ that can be split into the diagonal and off-diagonal parts. The transfer matrix collecting diagonal entries of $G(s)$ is the model of decoupled (isolated) subsystems; interactions between subsystems are represented by the off-diagonal entries, i.e.

$$G(s) = G_d(s) + G_m(s) \quad (9)$$

where $G_d(s) = \text{diag}\{G_i(s)\}_{i=1, \dots, m}$

whereby $\det G_d(s) \neq 0 \quad \forall s$

and $G_m(s) = G(s) - G_d(s)$

For the system (9), a decentralized controller is to be designed

$$R(s) = \text{diag}\{R_i(s)\}_{i=1, \dots, m}, \quad \det R(s) \neq 0 \quad \forall s \quad (10)$$

where $R_i(s)$ is transfer function of the i -th subsystem local controller.

In compliance with the independent design framework, the point at issue is to appropriately assess the effect of interactions and to translate it into constraints for local controller designs to guarantee stability of the full closed-loop system as well as its required performance.

4 Main Results

The proposed decentralized control design technique is based upon factorising the closed-loop characteristic polynomial of the full system (3) under the decentralized controller (10) in the following way

$$\det F(s) = \det\{I + R(s)[G_d(s) + G_m(s)]\} = \det R(s) \det[R^{-1}(s) + G_d(s) + G_m(s)] \quad (11)$$

□ Existence of $R^{-1}(s)$ is implied by the assumption that $\det R(s) \neq 0$. Denote

$$F_I(s) = R^{-1}(s) + G_d(s) + G_m(s) \quad (12)$$

Then, with respect to (9)-(11), the necessary and sufficient stability conditions of *Theorem 1* can be modified using the following corollary.

Corollary 1

A closed-loop system comprising the system (9) and the decentralized controller (10) is stable if and only if

a. $\det F_I(s) \neq 0$

$$b. N\{0, \det F_I(s)\} + N\{0, \det R(s)\} = n_q \quad (13)$$

If $R(s)$ is stable, $N\{0, \det[R(s)]\} = 0$ and the encirclement condition (13b) reduces to

$$N\{0, \det F_I(s)\} = N\{0, \det[R^{-1}(s) + G_d(s) + G_m(s)]\} = n_q \quad (14)$$

□

As the term $[R^{-1}(s) + G_d(s)]$ in (12) is diagonal and refers just to subsystems and includes all necessary information on their dynamics, it is possible to specify a required performance (including stability) of individual subsystems using an appropriately chosen diagonal matrix $P(s)$:

$$R^{-1}(s) + G_d(s) = P(s) \quad (15)$$

where $P(s) = \text{diag}\{p_i(s)\}_{i=1, \dots, m}$

From (15) results

$$I + R(s)[G_d(s) - P(s)] = 0 \quad (16)$$

or, on the subsystem level

$$I + R_i(s)G_i^{eq}(s) = 0 \quad i = 1, 2, \dots, m \quad (17)$$

where

$$G_i^{eq}(s) = G_i(s) - p_i(s) \quad i = 1, 2, \dots, m \quad (18)$$

$G_i^{eq}(s)$ denotes the i -th subsystem transfer function modified by $p_i(s)$ and will be called the transfer function of the i -th equivalent subsystem or simply the equivalent transfer function. Similarly, (17) will be denoted the i -th equivalent characteristic equation.

If we modify (12) using (15) we obtain

$$\det F_I(s) = \det[P(s) + G_m(s)] \quad (19)$$

or using another formal modification of (19)

$$\det F_I(s) = \det[I + P(s) + G_m(s) - I] = \det[I + K(s)] \quad (20)$$

where $K(s) = P(s) + G_m(s) - I$

Considering (19) and (20) we can formulate the encirclement stability conditions for the closed-loop system under a decentralized controller in terms of the spectral Nyquist plot of $F_I(s)$.

Corollary 2

A closed-loop system comprising the system (9) and a stable decentralized controller (10) is stable if

1. there exists such a diagonal matrix $P(s) = \text{diag}\{p_i(s)\}_{i=1,\dots,m}$ that each equivalent

subsystem $G_i^{eq}(s) = G_i(s) - p_i(s)$, $i = 1, \dots, m$

can be stabilized by its related local controller $R_i(s)$, i.e. each equivalent characteristic polynomial

$$CLCP_i^{eq} = 1 + R_i(s)G_i^{eq}(s) \quad i = 1, 2, \dots, m$$

has stable roots;

2. any of the following conditions are satisfied

$$N\{0, \det\{I + [P(s) + G_m(s) - I]\}\} = n_k \quad (21a)$$

$$\sum_{i=1}^m N[-1, k_i(s)] = n_k \quad (21b)$$

where $k_i(s)$, $i = 1, \dots, m$ are characteristic functions of $K(s) = P(s) + G_m(s) - I$; n_k is the number of its unstable poles; or

$$N\{0, \det\{P(s) + G_m(s)\}\} = n_m \quad (22a)$$

$$\sum_{i=1}^m N\{0, m_i(s)\} = n_m \quad (22b)$$

where $m_i(s)$, $i = 1, \dots, m$ are characteristic functions of $M(s) = P(s) + G_m(s)$; n_m is the number of its unstable poles. \square

Thus far, only stability in the DC design has been considered. Any additional performance requirements need to be included in the design by a suitable choice of $P(s) = \text{diag}\{p_i(s)\}_{i=1,\dots,m}$. Next, the choice of $p_i(s)$ is being discussed in detail.

4.1. Decentralized Controller Design for Performance

Referring to requirements of the independent design approach [2], the $p_i(s)$, $i = 1, \dots, m$ actually represent bounds for local controller designs. Therefore, to guarantee closed-loop stability of the full system they should be chosen such as to appropriately account for the interaction term $G_m(s)$.

According to (5), characteristic functions of $G_m(s)$ are defined

$$\det\{g_i(s)I - G_m(s)\} = 0 \quad i = 1, \dots, m \quad (23)$$

If we consider identical entries in the diagonal of $P(s)$, substituting (15) in (13) and equating to zero actually defines the m characteristic functions of $[-G_m(s)]$

$$\det\{p_i(s)I + G_m\} = 0 \quad i = 1, \dots, m \quad (24)$$

To be able to include the performance issue in the design, the following considerations about $P(s) = p(s)I$ motivated by (23) and (24) have been adopted:

1. If $p(s) = p_1(s) = -g_1(s)$ for fixed $\mathbf{1} \in \{1, \dots, m\}$ then

$$\det F_1(s) = \prod_{i=1}^m [p(s) + g_i(s)] = \prod_{i=1}^m [-g_1(s) + g_i(s)] = 0 \quad (25)$$

and the closed-loop system is not stable (it is on the stability boundary);

2. If $p(s) = p_1[s(a)] = -g_1[s(a)]$, fixed $\mathbf{1} \in \{1, \dots, m\}$

where we consider the generalized frequency $s = -a + jw$, $a \geq 0$; $w \in (-\infty, \infty)$ and $s(a)$ indicates the particular case when $a > 0$, then

$$\begin{aligned} \det F_1[s(a)] &= \prod_{i=1}^m [p_1[s(a)] + g_i[s(a)]] = \\ &= \prod_{i=1}^m [-g_1[s(a)] + g_i[s(a)]] = 0 \end{aligned} \quad (27)$$

Thus, if considering $s = -a + jw$ (a being the decay rate), the imaginary axis of the Nyquist complex plane is "shifted to $-a$ ". As $\det F_1(s, a) = 0$, the modified closed-loop system is exactly on the stability boundary "shifted to $(-a)$ ", hence it is stable with the decay rate a . The equivalent subsystems transfer functions are

$$G_i^{eq}[s(a)] = G_i[s(a)] - p_1[s(a)] \quad i = 1, 2, \dots, m \quad (28)$$

It is noteworthy that a closed-loop system stable with a decay rate $a \geq 0$ it is necessarily stable also for $a = 0$.

According to (22b) and considering (25) and (26), the encirclement condition for the closed-loop stability in terms of the spectral Nyquist plot of $F_1(s)$ is then

$$\sum_{i=1}^m N\{0, [p_1[s(a)] + g_i(s)]\} = \sum_{i=1}^m N\{0, m_i^{eq}[s(a)]\} = n_m \quad (29)$$

where

$$\begin{aligned} m_i^{eq}[s(a)] &= \{p_1[s(a)] + g_i(s)\} = \\ &= \{-g_1[s(a)] + g_i(s)\}, \quad i = 1, \dots, m \end{aligned}$$

will be called *equivalent characteristic functions* of $M(s) = [P(s) + G_m(s)]$ and n_m is the number of unstable poles of $M(s)$.

However, if for any $s = -a + jw$, $a \geq 0$, $w \in (-\infty, \infty)$

$$\begin{aligned} \det F_1(s) &= \prod_{i=1}^m [p_1[s(a)] + g_i(s)] = \\ &= \prod_{i=1}^m [-g_1[s(a)] + g_i(s)] = 0 \end{aligned} \quad (30)$$

i.e. if at any frequency, say w_1 , the $p_1(-a + jw_1)$ and any characteristic locus $g_i(w_1)$, $i \in \{1, \dots, m\}$ happen to cross, then the closed loop system is not stable.

The main theoretical results from the preceding section are summarized next.

Definition 1 (Set of stable characteristic functions)

$p_1[s(a)]$ will be called a stable characteristic function if $\forall g_i(s), i=1, \dots, m, s=-a+jw, a \geq 0, w \in (-\infty, \infty)$

1. $\{p_1[s(a)] + g_i(s)\} \neq 0$;
2. $\sum_{i=1}^m N\{0, m_i^{eq}[s(a)]\} = n_m$

where $m_i^{eq}[s(a)] = \{p_1[s(a)] + g_i(s)\}$ and n_m is the number of unstable poles of $M(s) = [P(s) + G_m(s)]$.

The set of all stable characteristic functions for a system will be denoted R_S . \square

Lemma 1

The closed-loop system comprising the system (9) and a stable decentralized controller (10) is stable if

1. $p_1[s(a)] = -g_1[s(a)]$, fixed $\mathbf{l} \in \{1, \dots, m\}$, $s = -a + jw, a \geq 0, w \in (-\infty, \infty)$ belongs to R_S and
2. the closed-loop characteristic polynomials of all equivalent subsystems (equivalent characteristic polynomials) $CLCP_i^{eq} = 1 + R_{ii}(s)G_i^{eq}(s), i = 1, 2, \dots, m$ are stable. \square

Proof of Lemma 1 results from previous considerations.

4.2. Decentralized Controller Design Procedure

1. Partition the controlled system into subsystems (diagonal part) and interactions (off-diagonal part)

- a. $G(s) = G_d(s) + G_m(s)$
- b. specify $a > 0$;

2. Find the characteristic functions of $G_m(s): g_i(s), i = 1, \dots, m$
3. Choose $p_1(s, a): p_1(s, a) = -g_1(s, a); p_1(s, a) \in R_S$ by examining equivalent characteristic loci $m_i^{eq}[s(a)] = \{p_1[s(a)] + g_i(s)\}, i = 1, \dots, m$

If no such $p_1[s(a)] \in R_S$ can be found, no stabilizing decentralized controller can be designed using this approach, and the procedure stops; else

4. Design local controllers $R_i(s)$ for all m equivalent subsystems

$$G_i^{eq}[s(a)] = G_i[s(a)] - p_1[s(a)] \quad i = 1, \dots, m;$$

using any suitable design technique, e.g. the Neymark D-partition method [6].

The proposed procedure is illustrated on an example in the next Section.

5 Example

Consider a MIMO system with two inputs and two outputs given by the transfer function matrix

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \quad \text{where}$$

$$G_{11}(s) = \frac{0.01675s^2 - 0.1018s + 0.438}{s^3 + 2.213s^2 + 2.073s + 0.6106}$$

$$G_{12}(s) = \frac{0.01555s^2 - 0.0375s - 0.1106}{s^3 + 2.554s^2 + 1.783s + 0.5433}$$

$$G_{21}(s) = \frac{0.01325s^2 - 0.03415s + 1.018}{s^3 + 3.927s^2 + 5.815s + 3.547}$$

$$G_{22}(s) = \frac{0.01575s^2 - 0.1252s + 0.442}{s^3 + 3.514s^2 + 2.01s + 0.3872}$$

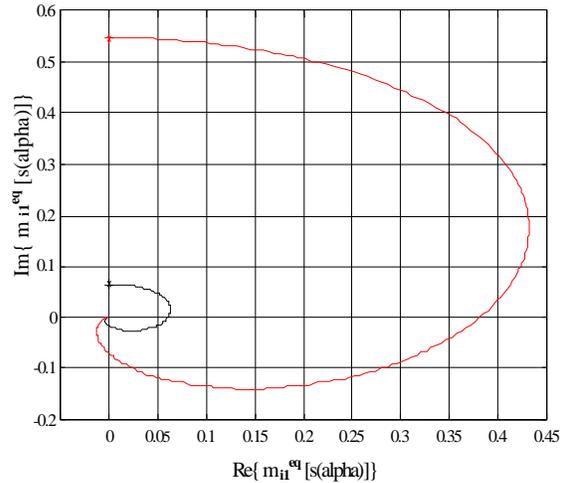


Figure 2: Equivalent characteristic loci for $\mathbf{l} = 1$ and $a = 0.1$

For the related equivalent subsystems, local PI controllers in form $R(s) = r_0 + \frac{r_1}{s}$ were designed applying the Neymark D-partition method to both equivalent characteristic equations

$$R_i^{-1}(s) + \{G_i[s(a)] - p_1[s(a)]\} = 0 \quad i = 1, 2$$

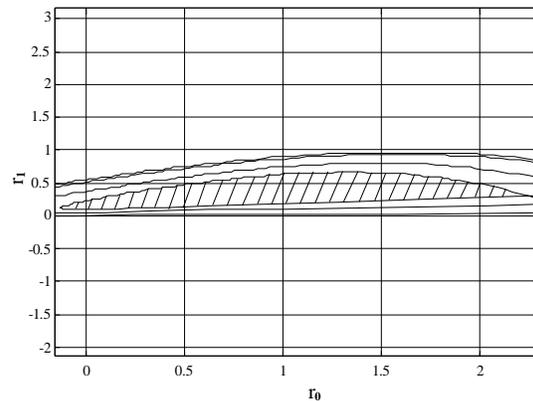


Figure 3: Neymark D-contours: 1st equivalent subsystem

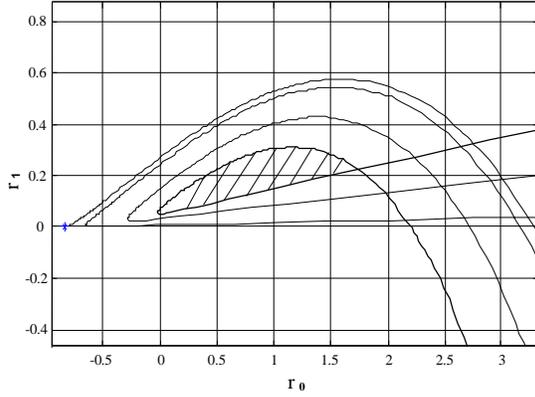


Figure 4: Neymark D-contours: 2nd equivalent subsystem

For both subsystems, parameters of local PI controllers have been chosen

- from inside of the hatched areas in Figures 3 and 4, respectively, which correspond to closed-loop values $a > 0.1$;
- from the boundary of the hatched areas in Figures 3 and 4, respectively, which correspond to closed-loop values $a = 0.1$

The other contours in Figures 3 and 4 specify areas for the individual above-chosen decay rates.

The design results are summarized in Table 1.

Sub-syst.	Controller	Choice of α	Achieved decay rate α
1	$R_1(s) = 1.2786 + \frac{0.3553}{s}$	from inside of the hatched areas in Figs.3, 4	0.1549
2	$R_2(s) = 0.9327 + \frac{0.1916}{s}$		
1	$R_1 = 1.2740 + \frac{0.2149}{s}$	from the boundary of the hatched areas in Figs.3, 4	0.1007
2	$R_2 = 0.9222 + \frac{0.1429}{s}$		

Table 1: Results of the local controller design

Pertinent closed-loop eigenvalue sets $L_i = 1,2$ are as follows

$$L_1 = \{-0.1549; -0.1825; -0.2286 \pm 0.4133j; -0.3464 \pm 0.6648j; -0.4129 \pm 0.3888j; -1.079 \pm 0.9291j; -1.2811; -1.7029; -1.8115; -2.9768\}$$

$$L_2 = \{-0.1007 \pm 0.0091j; -0.2499 \pm 0.4113j; -0.367 \pm 0.7166j; -0.4163 \pm 0.3871j; -1.0801 \pm 0.9296j; -1.3172; -1.7043; -1.8145; -2.9918\}$$

Conclusion

In this paper a novel frequency-domain approach to the decentralized controller design for performance has been proposed. Its main advantage consists in that the plant interactions are included in the design of local controllers through their characteristic functions, modified so as to achieve a required closed-loop performance (in terms of a specified decay rate) of the full system. The independent design is carried out on the subsystem level for the so called „equivalent subsystems“ which are actually mathematical models of individual decoupled subsystems modified using characteristic functions of the plant interaction matrix. Local controllers designed for equivalent subsystems guarantee fulfilment of performance requirements imposed on the full system without any performance deterioration brought about by the effect of interactions. Theoretical results are supported with results obtained by solving several examples one of which is included.

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