# STRUCTURAL SELECTION FOR DECENTRALISED CONTROL SCHEMES

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**Abstract:** This paper considers a structural approach for selection of the control structure aiming at facilitating the design of decentralised control schemes. This requires the selection of inputs, outputs, as well as their coupling (selection of decentralisation structure) that will allow the solvability of a variety of decentralised control problems, such as pole assignment by decentralised output feedback, decentralised dynamic controllers etc. The overall approach is based on the use of necessary and sufficient conditions for generic and exact solvability of decentralised control problems, which are expressed in terms of properties of Plücker invariants and Markov type matrices. The approach is based on generic classification of desirable input and output partitions and parametric design of invariants.

## 1. INTRODUCTION

The problem of control structure selection is prerequisite to Control Systems design, it is part of the Overall (Global) Instrumentation [2] and involves three major steps: (a) Classification of variables of the model into inputs, outputs and internal variables [3], (b) Definition of effective sets of inputs, outputs [6] and (c) Structuring of the feedback coupling of the Control Scheme [11]. These three classes of problems may be considered within the framework of structural methodologies for linear systems and each one of them involves concrete subproblems, which may be ordered according to the level of generality of the issues and the nature of the corresponding model (generic model with given dimensions, detailed dynamic model). The design of decentralisation schemes is a problem belonging to the third class. Such a problem may be studied using criteria based on the nature of the process, geographical location of subsystems, graph based structural analysis[12], interaction indicators and general coupling diagnostics based on generic properties and invariant based criteria [9]. The aim of this paper is to address the problem of design of decentralisation schemes in order to guarantee, or well condition the solvability of families of control problems. As such the overall approach is based on the philosophy of defining schemes, which exclude undesirable characteristics, such as fixed modes.

The current approach aims at developing the generic solvability conditions and the parametric invariant conditions linked to solvability of decentralised control problems and devise methods for design, or redesign the system in order to facilitate the solvability of such problems. Amongst the specific problems considered are: (i) Define the desirable cardinality of input, output structures to permit satisfaction of generic solvability conditions, (ii) Design the structure of input, output maps (matrices $B, C$) to eliminate the existence of fixed modes and guarantee full rank properties to the decentralised Plücker matrices [9]. The paper presents aspects of structural methodologies for design of decentralisation schemes, which are part of an overall integrated philosophy for design of such schemes. The results of the exterior algebra framework provide the means for simple tests for avoiding fixed modes, whereas the link of Plücker matrices to decentralised Markov parameters allow the effective linking of the algebraic framework to state space design.

## 2. PROBLEM STATEMENT

Consider a $k$-channel linear system $S(A,B,C)$ described by

$$\dot{x} = Ax + \sum_{i=1}^{k} B_i u_i \,,\; y_i = C_i x \qquad (1)$$

where $x, u_i, y_i$ are $n, m_i, p_i$ vectors, respectively, and $u_i$ and $y_i$ are the input and output of the ith channel in a decentralisation scheme. We use a right coprime MFD for the transfer function $G(s)$ ie

$$G(s) = N(s)D(s)^{-1} \qquad (2)$$

If local feedback laws of the type

$$u_i = K_i y_i + u_i' \,,\; K_i \in \Re^{p_i \times m_i} \,,\; i = 1,2,\cdots k \qquad (3)$$

are applied to each of the k-channels, the closed-loop pole polynomial is expressed as

$$p(s) = \det(sI - A - \sum_{i=1}^{r} B_i K_i C_i) \qquad (4)$$

$$p(s) = \det\left( \left[ I_p, K_{dec} \right] \begin{bmatrix} D(s) \\ N(s) \end{bmatrix} \right) \qquad (5)$$

where $K_{dec} = bl - diag\{K_1, \cdots K_k\}$. The above problem belongs to a more general category of pole placement problems where the multivariable feedback controller is structured. More specifically if $\Omega = \{(i,j) : 1 \le i \le p \;\; and \;\; 1 \le j \le m\}$ is the set of all possible pairs $i, j$ corresponding to closing the loop from the output $j$ to the input $i$, then any subset $\Omega_s$ of $\Omega$ defines a structure feedback problem where the permissible loop closures are described by the pairs in $\Omega_s$. In the special case of static decentralised control we have that:

$$\Omega_d = \lfloor 1, p_1 \rfloor \times \lfloor 1, m_1 \rfloor \cup \lfloor p_1 + 1, p_1 + p_2 \rfloor \times \lfloor m_1 + 1, m_1 + m_2 \rfloor .. \\ ....... \cup \left[ \sum_{i=1}^{k-1} p_i + 1, p \right] \times \left[ \sum_{i=1}^{k-1} m_i + 1, m \right] \quad (6)$$

The structured pole placement map $X^s$ is the function that maps every structured feedback $K_{dec}$ to the $n$ closed loop poles: $X^s : \Im^{|\Omega_s|} \to \Im^n$ where $|\Omega_s|$ denotes the number of free parameters in $\Omega_s$. The decentralised pole placement map, $X^d$, is defined this way to be: $X^d : \Im^{\sum m_i p_i} \to \Im^n$. The pole placement map [7] carries all the information as far as the pole placement and for the decentralised case can be considered as a restriction of the general pole placement map of the centralised output feedback case and contains a subset of the Markov parameters [9]. The latter property allows the linking of decentralised Plücker invariants to the state space parameters and thus to system design issues.

In the following, we examine issues linked to the selection of decentralisation schemes: **(a)** Generic conditions for solvability of decentralisation problems and selection of decentralisation partitioning. **(b)** Well conditioning of Plücker invariants by selection / redesign of decentralised Markov parameters. These problems are linked to necessary conditions and thus introduce possible solutions for design of decentralisation using as a vehicle the results of the decentralised pole assignment by constant output feedback. The framework can be extended by considering similar decentralised control problems, such as PI, or fixed McMillan degree solutions.

## 3. DECENTRALISED POLE ASSIGNMENT: BACKGROUND RESULTS

The study of the pole placement map is central in the investigation of solvability conditions of the decentralised pole assignment (constant or dynamic) and provides the required necessary conditions for addressing the design of the decentralisation schemes. In the following, we review two approaches. The first is based on the decentralised Plücker matrix and the second on the differential of the map $X^d$, which leads to the definition of the decentralised Markov parameters.

**Decentralised Plücker Matrices**: Using the Binet Cauchy Theorem on equation (5), we have:

$$p(s) = C_p([I_p, K_{dec}])C_p(\begin{bmatrix} D(s) \\ N(s) \end{bmatrix}) = \underline{k}_{dec}^t \underline{g}(s) \quad (7)$$

where $C_p()$ denotes p-th compound, $\underline{k}_{dec}^t$ the exterior product of the rows of $\lfloor I_p, K_{dec} \rfloor$ and $\underline{g}(s)$ the exterior product of the columns of $\left[ D(s)^t, N(s)^t \right]^t$. We may write $\underline{g}(s) = P\underline{e}_n(s)$, $\underline{e}_n(s) = \left[ s^n, \cdots, s, 1 \right]$, $\underline{g}(s)$ is the *right Grassmann representative*, $P$ the *right Plücker matrix* and they are complete invariants for the $S(A,B,C)$ system [4]. The vector $\underline{k}_{dec}^t$ has 0's at certain positions due to the block diagonal structure. If now we cut from $\underline{g}(s)$ those entries corresponding to the fixed zero locations of $\underline{k}_{dec}^t$ we get a new vector $g'(s) = P'\underline{e}_n(s)$ called the *decentralised Grassmann representative* and $P'$ the *decentralised Plücker matrix* [5]. $g'(s)$ and $P'$ are invariants under the given decentralisation scheme and their significance in the characterisation of fixed modes [1], [13]and solvability of decentralised assignment is summarised by the following result [5]:

**Theorem (1):** For a given decentralisation scheme defined on $S(A,B,C)$ by $K_{dec}$, the following properties hold true:

(i) A necessary and sufficient condition for $\lambda \in \mathbb{C}$ to be a decentralised fixed mode is that $\lambda$ is a zero of $g'(s)$, i.e. $g'(\lambda) = 0$.

(ii) A necessary condition for decentralised assignment is that $rank(P') = n + 1$.

(iii) If $rank(P') = n + 1$, then the system $S(A,B,C)$ has no decentralised fixed modes under $K_{dec}$ scheme.

□

Such results may be extended to dynamic compensation schemes [10] and can be used to test the properties of the given decentralisation scheme. However, they provide no insight in how we can modify a decentralisation scheme, since the links to the state space parameters are not explicit. Such links are established by examining the differential of the pole assignment map, which leads to the decentralised Markov parameters [9].

**Decentralised Markov Parameters**: The decentralised pole placement map $X^d$ can be factorised as:

$$X^d : \Im^{\sum m_i p_i} \xrightarrow{E} \Im^{m \times p} \xrightarrow{X^c} \Im^n \qquad (8)$$

where $X^c$ is the centralised pole placement map and

$$E(row(K_1), \dots row(K_k)) = bl.diag(K_1, \dots, K_k) \qquad (9)$$

The calculation of the differential of $X^d$ involves the decomposition $X^d = X^c \circ E$ and this implies

$$D(X^d)_k = D(X^c)_{E(k)} \circ D(E)_k \qquad (10)$$

The sets $\Omega$ and $\Omega_d$, specify the lower indices of the entries of centralised and decentralised feedback matrices respectively. We consider a basis for $T(\Im^{mp})_k$ the set of all $(\vartheta / \vartheta k_a)$, $a \in \Omega$ and for $T(\Im^{\sum m_i p_i})_k$ the set of all $(\vartheta / \vartheta k_\beta)$, $\beta \in \Omega_d$, where all the indices are lexicographically ordered. Using these bases we have a representation for the above differentials as described below [9]:

**Theorem (2):** For a given decentralised feedback gain $K$ and a system $S(A,B,C)$, a matrix representation of the differential of the decentralised pole placement map $D(X^d)_k$, with respect to the bases previously defined is denoted by $R(X^d)_k$ and it is given by

$$R(X^d)_k = R(X^c)_{E(k)} \circ R(E)_k \qquad (11)$$

$$R(x^c)_k = Q^t \left[colCB, colCHB, \dots, colCH^{n-1}B\right]^t \qquad (12a)$$

is a $n \times mp$ matrix, col maps an $m \times p$ matrix to $mp \times 1$ matrix formed by superimposing its columns, $H = A + BKC$ and $Q$ is given by

$$Q = \begin{bmatrix} 1 & p_n & \cdots & p_2 \\ 0 & 1 & \cdots & p_3 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 \end{bmatrix} \qquad (12b)$$

where the $p_i$'s are the coefficients of the closed loop polynomial and $R(E)$ is an $mp \times \sum m_i p_i$ matrix such that for $\forall \alpha, \beta \in \Omega$

$$R(E)_{\alpha\beta} = \begin{cases} 1 & if\ \alpha = \beta \\ 0 & otherwise \end{cases} \qquad (13)$$

□

Note that $R(X^d)_k$ is obtained by $R(X^c)_{E(k)}$ by keeping only those rows of $R(X^c)_{E(k)}$ which correspond to $\Omega_d$ set of indices. The column representation of the Markov parameters of $S(A,B,C)$ is obtained by computing the differential of $X^c$ at $k = 0$. Similarly, we may define the decentralised Markov parameters by using the differential of $X^d$ at $K_{dec} = 0$. In fact, if $H_i = CA^i B$, then for $K_{dec} = 0$ we have

$$R(X^d)_0 = Q^t \left[col\hat{H}_0, col\hat{H}_1, \dots col\hat{H}_{n-1}\right]^t \qquad (14)$$

where $col\hat{H}_i$ denotes the reduced column obtained from $colH_i$ after eliminating all the entries that do not correspond to the indices $\Omega_d$. The matrix

$$M_d = \left[col\hat{H}_0, col\hat{H}_1, \dots, col\hat{H}_{n-1}\right] \qquad (15)$$

is known as *decentralised Markov matrix* [9] and its properties are summarised below:

**Theorem (3):** For a system $S(A,B,C)$ and a decentralisation scheme defined by $\Omega_d$ set the following properties hold true:

(i) $rank(M_d) \le \dim(\operatorname{Im} X^d) \le n$

(ii) For a generic system $S(A,B,C)$ such that $\sum m_i p_i \ge n$, then $rank(M_d) = \mathbf{dim(Im}\ X^d\mathbf{)} = n$. Furthermore, under these conditions the arbitrary pole assignment via complex output feedback can be solved.

□

The decentralised Plücker and decentralised Markov matrices are crucial in characterising the solvability of decentralised control problems. For the centralised case the link between them is expressed as [7]:

**Proposition (1):** For the system described by $S(A,B,C)$, let $P_S = [\underline{e}_1, P_S']$ be its Plücker matrix. We may compute $M = \left[colCB, \dots, colCA^{n-1}B\right]$ from $P_S$' (the reduced Plücker matrix) as follows:

**(a)** Select the rows of $P_S$' that correspond to the set of indices $(1,2,\ldots,i-1,i+1,\ldots,p,p+j)$ for $i:1 \le i \le p$ and $j:1 \le j \le m$. This set of rows is multiplied by $(-1)^{i-1}$.

**(b)** Repeat step (a) for all $i$ and $j$ and form the $pm \times n$ matrix, which is the post-multiplied by $Q^{-1}$, where $Q$ is the $n \times n$ matrix defined in (12b) and which corresponds to the open loop pole polynomial.

□

The above result indicates that the Markov matrix $M$ is modulo $Q \in \Re^{n \times n}$, $\mathbf{det}(Q) \neq 0$, a sub-matrix of and thus the full rank of $M$ implies full rank of $P_S$, but not vice versa. This explicit relationship can thus be used as an indicator for designing systems (by selection of $C$, or $B$) such that the corresponding Plücker matrix has full rank.

**Remark (1):** If $M = \left[colCB, \ldots, colCA^{n-1}B\right]$ has full rank, then the Plücker matrix has also full matrix.

□

For the decentralised case we have the result:

**Proposition (2):** The matrix $M_dQ$ is a full row submatrix of the decentralised Plücker matrix $P_S$'.

□

**Corollary (1):** If $\sum m_i p_i \le n$ and for $M_d$, $\rho(M_d)$=$n$, then the system has no fixed modes. The condition $\rho(M_d) = n$ is necessary for solvability of the Static Decentralised Output feedback problem.

□

**Remark (2):** The presence of fixed modes is the result of the structure of the decentralisation scheme and it is independent of dynamics of decentralisation scheme. The above test stated for the decentralised constant compensation case is thus valid for all decentralised dynamic compensations having decentralisation characteristic.

□

## 4. GENERIC SOLVABILITY OF DECENTRALISED CONTROL PROBLEMS AND PARTITIONING OF INPUTS AND OUTPUTS

For the case of pole placement by decentralised control, there exist a number of results dealing with generic systems, which provide conditions for solvability based on the partitioning $\{m_i\}$, $\{p_i\}$ of $m, p$ and the number $n$. Such results are useful as a first screening of desirable partitionings and lead to parameterisations, which contain possible solutions of the decentralisation problem; these candidate solutions have to be tested with the parameter dependent invariant properties, such as those described in the previous section. Some of these results and the way they can be used are considered here. By combining Theorems (1) and (3) we have:

**Theorem (4):** A necessary condition for arbitrary pole placement via a $k$ channel static decentralised output feedback are: $\sum_{i=1}^{k} m_i p_i \ge n$, $rank(\overline{P}) = n+1$

□

Furthermore, we have [13]:

**Theorem (5):** The condition $\sum_{i=1}^{k} m_i p_i > n$ implies generic pole assignability, when either the number of all inputs, or the number of all outputs are equal.

□

More general sufficient conditions avoiding the equality of dimensions of input, or output channels have been derived and are summarised below [8]:

**Theorem (6):** Sufficient condition for generic pole assignability are: $\mathbf{min}(m_i) \cdot p > n, \quad rank(\overline{P}) = n+1$

□

A similar family of results that may lead to a parameterisation of possible partitions is based on the notion of partition of integers and on the evaluation of a function of $(m_i, p_i)$ pairs, known as the height $h(p_i, m_i)$ [8] is described below:

**Definition (1):** A binary partition $t$ of the number $n$ of length $k$ is a sequence of integers $\{t(1), t(2), \ldots, t(k)\}$ such that $n = t(1) + \ldots + t(k)$ and for every $j$, there is most one 1 in all j-th digits of the binary representations of $t(1), t(2), \ldots, t(k)$.

**Theorem (7):** A sufficient condition for arbitrary pole placement by a real static decentralised output feedback for a system with $n$ states and $k$ $(m_i, p_i)$ channels, is that $k$ length binary partition of $n$, say $\{t(1), t(2), \ldots, t(k)\}$, exists such that $t(i) \le h(p_i, m_i)$ for every $i = 1, \ldots, k$.

□

Computing $h(p_i, m_i)$ can be achieved as [8]:

**Lemma (1):** If $1 < p \le m$ and $v$ is such that $2^v \le m + p - 1 < 2^{v-1}$, then the height function $h(p, m)$ is given as

$$h(p,m) = \begin{cases} 2^{v+1} - 2 & if\ p = 2, or\ if\ p = 3\ and\ m + p = 2^v + 1 \\ 2^{v+1} - 1 & otherwise \end{cases}$$

□

Theorem (7) gives a simpler test when compared to testing the results in [13] and it is directly related to the decentralisation parameters $p_i, m_i$. Alternative parameterisations may be derived using other sufficient conditions. The existing sufficient conditions can be used as long as the necessary conditions are satisfied and this leads to a strategy for design of the decentralisation based on the conditions of Theorem (4) which is considered next.

## 5. A STRUCTURAL FRAMEWORK FOR THE SELECTION OF POSSIBLE DECENTRALISATION STRUCTURES

The design of the appropriate decentralisation scheme is based on the first instance on the specifics of the application and issues related to geographical location of process units and operational requirements are those which define the options relating to centralised versus decentralised and if decentralised the first partitioning of the decentralisation scheme. In the following we will assume that a first partitioning is given, defined by physical considerations; this initial partitioning will be referred to as the *physical decentralisation set*. This set corresponds to pairs of indices $(i, j)$ $i \in \{1,2,\dots,p\}$, $j \in \{1,2,\dots,m\}$ expressing permissible loop closures from the j-th output to the i-th input. The feasible sets are subsets of the physical decentralisation set with cardinality greater or equal to $n$. A list of desirable partitions from those coming from the feasible sets may be produced using the $\sum m_i p_i > n$ necessary conditions and then we may deploy the decentralised Markov parameter tests. The main steps of the procedure are:

### *5.1 Procedure for selection of decentralisation based on constant output feedback*

**Step (1):** Define the physical decentralisation set and from this all feasible sets corresponding to all possible cardinalities of partition $k$.

**Step (2):** Test whether the necessary conditions for centralised output feedback $mp > n$ and $rank(P) = n+1$ are satisfied. If yes, then proceed to next step; otherwise, alternative schemes based on dynamic compensation have to be used.

**Step (3):** For every element of the feasible set produced in (1) and for all possible cardinality partitions $\{(m_i, p_i), i \in \underline{K}\}$ select those partitions for which the condition $\sum_{i=1}^{k} m_i p_i > n$ is satisfied.

**Step (4):** Compute the Markov parameters set $\{H_0, H_1, \dots, H_{n-1}\}$ and the corresponding matrix $M = [colH_0, colH_1, \dots, colH_{n-1}]$. If $rank(M) = n$ then proceed to next step; otherwise, alternative schemes based on dynamic compensation have to be used, or work with the corresponding Plücker matrices.

**Step (5):** For every partition produced by (3) define the corresponding decentralised Markov matrix $M_d = [col\hat{H}_0, col\hat{H}_1, \dots, col\hat{H}_{n-1}]$ and test $rank(M_d) = n$. If the condition is not satisfied, then test the condition on the decentralised Plücker matrix; otherwise, use dynamic schemes.

**Step (6):** For every element of the feasible set coming from the previous step, use sufficient conditions for generic assignability. If such conditions are not valid, then use alternative means based on dynamics.

□

The above procedure produces schemes with no fixed modes and which satisfy the necessary conditions. If the generic sufficient conditions are not satisfied, then tests based on non-generic cases, or dynamic schemes have to be used.

### *5.2 Selection of Input, Output Matrices and the Markov Matrices Framework*

The selection of the decentralisation involves as an integral part the formulation of the selection of full rank Markov matrices for both centralised and decentralised case (with a fixed partitioning). We consider next a typical problem of this framework that is the explicit representation of the Markov matrix and the formulation of some standard design problem on it. For the system $S(A, B, C)$ the Markov matrix $M$ may be expressed in the following form, if $B = [\underline{b}_1, \underline{b}_2, \dots, \underline{b}_p]$

$$M = \left[\begin{array}{c:c:c:c} C\underline{b}_1 & CA\underline{b}_1 & \cdots & CA^{n-1}\underline{b}_1 \\ C\underline{b}_2 & CA\underline{b}_2 & \cdots & CA^{n-1}\underline{b}_2 \\ \vdots & \vdots & \ddots & \vdots \\ C\underline{b}_p & CA\underline{b}_p & \cdots & CA^{n-1}\underline{b}_p \end{array}\right] \qquad (16)$$

Assuming that the input matrix $B'$ is obtained from a model with larger dimensions $B \in \Re^{n\times q}, q \ge p$ by some appropriate reduction of number of inputs i.e. $B' = B\,R, R = [\underline{r}_1, \underline{r}_2, \dots, \underline{r}_p] \in \Re^{q\times p}$, then the resulting $M$ may be expressed as shown in condition (17) below:

$$\begin{bmatrix} [CB,\ldots CA^{n-1}B] & 0 \\ 0 & [CB,\ldots CA^{n-1}B] \end{bmatrix} \begin{bmatrix} \underline{r}_1 & & 0 \\ & \ddots & \\ 0 & & \underline{r}_1 \\ & \vdots & \\ \underline{r}_p & & 0 \\ & \ddots & \\ 0 & & \underline{r}_p \end{bmatrix} =$$

$$= M = T_p(A,B,C,)\cdot T_p(R) \tag{17}$$

The selection of $R$ that guarantees full rank $M$ is thus reduced to a design problem, where $R$ appears as a design parameter and can be expressed as: Find $R$ such that:

$$C_n\{T_p(A,B,C)\}\cdot C_n\{T_p(R)\} \neq 0 \tag{18}$$

where $T_p(A,B,C)$ is given and $T_p(R)$ is unknown. The above, as well as the corresponding problem on decentralised Markov parameters may be addressed within the exterior algebra framework [5].

## 6. CONCLUSIONS

The problem of designing the decentralisation scheme for a system $S(A,B,C)$ has been addressed using mainly as criteria the conditions for solvability of the constant output feedback problem. The overall philosophy has been based on the well conditioning of the problem by satisfying the necessary conditions which guarantee absence of fixed modes and existence of complex solutions for the generic case ($\sum m_i p_i \geq n$). Sufficient conditions guaranteeing solvability of the generic problem may then be deployed to screen further the resulting candidate solutions. Whenever such tests fail to provide solutions, dynamic problems may be considered and in particular, criteria guaranteeing low dynamics output feedback schemes [10].The advantage of the latter schemes is that they also provide means for computing the decentralised solutions, using the notion of global linearisation. An advantage of the approach based on the decentralised Markov matrices is that explicit links are established between the necessary conditions and the state space parameters and this provides the required framework for system redesign by modification of $B,C$ matrices.

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