SELF-OPTIMIZING CONTROL STRUCTURE SELECTION VIA DIFFERENTIATION

Yi Cao

School of Engineering, Cranfield University, UK
Email: y.cao@cranfield.ac.uk. Tel: 01234750111. Fax: 01234750728

Keywords: Plantwide Control, Self-Optimizing Control, Control Schemes,Decentralized Control, Cascade Control.

Abstract

A new approach using differentiation to select controlled variables for self-optimizing control [8] is proposed and applied to the evaporation process described in [7]. Based on the gradient function derived in the reduced space [1], new selection criteria, which indicate the sensitivities of the gradient function to disturbances and to implementation errors, have been derived for different controlled variables. The particular case study shows that the new selection criterion is more effective and reliable than the minimum singular value criterion proposed in [8].

1 Introduction

Chemical process plants are always controlled in different layers. For example, several local control layers are designed to maintain local controlled variables at the desired operating point whilst a plantwide optimization layer is responsible to adjust the setpoint to the local layers according to different situations (disturbances). Traditionally, these two layers are designed separately for different (economic and dynamic) objectives although they need working together. An important concept of "self-optimizing control", which can date back to the work of Morari *et. al* [6] and has been revisited recently by Skogestad [8], provides a link between these two layers. Self-optimization is a control strategy where by controlling certain specially selected variables at their nominal setpoints, it automatically achieves the optimal (or acceptable) operating conditions without re-optimization even in the presence of disturbances.

The optimality of a self-optimizing control system is strongly related to the control structure selected. In his seminal work, [8] proposed a criterion, the minimum singular value index, to select controlled variables for self-optimizing control. This criterion has been applied to several chemical processes such as the Tennessee Eastman process [5] and the evaporation process [2]. The criterion of minimum singular value is scaling dependent. The cost function affects implicitly on the criterion through the scaling of the variables in which the controlled variables are scaled with respect to the optimal range (which is dependent on the objective function used) and the implementation error. Therefore, some cautions have to be taken to properly apply this criterion for controlled variable selection.

In the previous work [1], the gradient of the constrained cost function has been derived and been directly used as controlled variables to achieve self-optimizing control. In this work, the usefulness of the gradient function is scrutinized further. It is shown that the sensitivity of the gradient function to disturbances for different controlled variable configuration is an effective and reliable criterion for controlled variable selection. By applying this measure to the evaporation process [7], a new controlled variable is identified to be the best and simplest one for self-optimizing control. The effectiveness of this new controlled variable is demonstrated through simulation.

The paper is organized as follows: The gradient of the constrained cost function as a combination of the first-order derivatives of the cost function and nonlinear model functions is represented in section 2. Then the sensitivities of the gradient to disturbances and to implementation errors for different controlled variable configurations are derived in section 3. To cope with conditionally active constraints, a cascade control structure to satisfy both optimality and constraint requirements is proposed in section 4. The sensitivity measures as criteria for controlled variable selection are applied to the evaporation process in section 5, where several controlled variables are identified as the best and simplest solutions. A comparison based on static and dynamic simulation performed for different controlled variable configurations is presented. Finally, the paper is concluded in section 6.

2 Gradient of constrained cost function

Consider the following optimization problem:

$$\begin{aligned} \min_{x,u} J \quad &= \phi(x,u,d) \\ \text{s.t.} \quad f(x,u,d) &= 0 \\ g(x,u,d) &\le 0 \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$ and $d \in \mathbb{R}^{n_d}$ are state, input and disturbance variables respectively. For a given disturbance, d, the solution of the above optimization problem is denoted as, x^* and u^*. Assume that at the optimal point, the following equalities hold:

$$F(x^*,u^*,d) = \begin{bmatrix} f(x^*,u^*,d) \\ g_1(x^*,u^*,d) \end{bmatrix} = 0 \tag{2}$$

where $f(\cdot)$ and $g_1(\cdot)$ are vector-valued functions with dimensions of n_f and n_1 respectively. If $m = (n_x + n_u) - (n_f +$

$n_1) \neq 0$, then according to the Kuhn-Tucker conditions, there are m first-order optimal conditions. Denote $u{=}[u_1^T\ u_2^T]^T$ with $u_2 \in \mathbb{R}^m$, $z = [x^T\ u_1^T]^T$ and $v = u_2$. Then the optimization problem (1) can be re-stated as:

$$\begin{aligned} \min_{z,v} J &= \phi(z,v,d) \qquad (3) \\ \text{s.t.} \quad & F(z,v,d) = 0 \end{aligned}$$

The first-order optimal conditions of the above optimization problem are:

$$J_v = \phi_v + \frac{\partial z}{\partial v}\phi_z = 0 \qquad (4)$$

$$F_v + \frac{\partial z}{\partial v}F_z = 0 \qquad (5)$$

If the Jacobian matrix, F_z is not singular, then the second condition (5) gives:

$$\frac{\partial z}{\partial v} = -F_v F_z^{-1} \qquad (6)$$

Inserting (6) into the first condition (4) leads to the following m-dimension optimal condition:

$$G(z,v,d) := \phi_v - F_v F_z^{-1}\phi_z = 0 \qquad (7)$$

Normally, the left-hand-side of the above condition is a function of x, u (u_1, and u_2) and d. For a given disturbance, d, equation (7) corresponds to an unique solution of $v^* = u_2^*$, from which all rest system variables, x^* and u_1^* can be determined.

If $F(x^*, u^*, d) = 0$ is the only active constraints for all possible disturbances, then it is clear that $G(z,v,d) = 0$ is the only condition which must be maintained to ensure the process operation is optimal. In other words, if condition $G(z,v,d) = 0$ is retained by the control system, then optimal operation can be achieved without re-optimization for different disturbances, i.e. the plant is self-optimizing controlled.

3 Sensitivity measures

When other variables rather than the gradient function itself are retained by a control system, the gradient in (7) is a function of disturbances and will not always be zero. The magnitude of the gradient indicates the optimality of the operation. Therefore, it is desirable to select controlled variables, which make the gradient as insensitive to disturbances as possible. The sensitivity of the gradient to disturbances depends on which m controlled variables selected. Assume m controlled variables selected correspond to m equations denoted as, $H(z,v,d) = 0$, then the sensitivity can be derived from the following equation set:

$$\delta = G(z,v,d) \qquad (8)$$

$$0 = F(z,v,d) \qquad (9)$$

$$0 = H(z,v,d) \qquad (10)$$

Sensitivities of (9) and (10) to disturbances are zero, i.e.

$$\frac{\partial z}{\partial d}F_z + \frac{\partial v}{\partial d}F_v + F_d = 0 \qquad (11)$$

$$\frac{\partial z}{\partial d}H_z + \frac{\partial v}{\partial d}H_v + H_d = 0 \qquad (12)$$

Since F_z is not singular, equation (11) leads to

$$\frac{\partial z}{\partial d} = -\left(\frac{\partial v}{\partial d}F_v + F_d\right)F_z^{-1} \qquad (13)$$

Inserting (13) into the second equation (12) leads to:

$$\frac{\partial v}{\partial d} = -\left(H_d - F_d F_z^{-1}H_z\right)\left(H_v - F_v F_z^{-1}H_z\right)^{-1} \qquad (14)$$

Replacing $\frac{\partial v}{\partial d}$ in (13) with (14) gives:

$$\begin{aligned} \frac{\partial z}{\partial d} &= -F_d F_z^{-1} + \qquad (15) \\ & \left(H_d - F_d F_z^{-1}H_z\right)\left(H_v - F_v F_z^{-1}H_z\right)^{-1} F_v F_z^{-1} \end{aligned}$$

Using the results in (14) and (15), the sensitivity of δ to disturbances can be yielded as follows:

$$\begin{aligned} \delta_d &= \frac{\partial z}{\partial d}G_z + \frac{\partial v}{\partial d}G_v + G_d \qquad (16) \\ &= \left(G_d - F_d F_z^{-1}G_z\right) - \left(H_d - F_d F_z^{-1}H_z\right) \\ & \left(H_v - F_v F_z^{-1}H_z\right)^{-1}\left(G_v - F_v F_z^{-1}G_z\right) \end{aligned}$$

where $\delta_d \in \mathbb{R}^{n_d \times m}$. When $H = G$, $\delta_d = 0$. This corresponds to the perfect self-optimizing control. For other controlled variables, $H \neq G$, normally $\delta_d \neq 0$. The row norms of δ_d matrix indicate how sensitive of the gradient function to the corresponding disturbances for the specified controlled variables. Therefore, row norms of δ_d can be used as a selection criterion to rank different controlled variable combinations.

The sensitivity measure, δ_d is a second-order derivative, J_{vd} of the constrained cost function. At the nominally optimal point, as explained in [8], the first-order derivative of cost function is zero. Second-order derivatives must be used to compare different controlled variable combinations. However, the minimum singular value measure, proposed as a selection criterion in [8] is only part of a second-order derivative. Therefore, it can only give a biased prediction. In contradiction to the minimum singular value measure, the sensitivity measure introduced here is a complete second-order derivative and can provide unbiased comparison for alternatives. Another important feature of the sensitivity function, δ_d is that it is independent of the scaling of controlled variables. Therefore, a comparison based on δ_d is more objective than that based on the minimum singular value measure, which is scaling dependent. The sensitivity measure can also be used to evaluate the sensitivity to measurement noise, to model uncertainties and to implementation errors if these are included in the disturbance vector.

Consider implementation errors, $\epsilon \in \mathbb{R}^m$, which associate with m controlled equations as:

$$0 = \tilde{H}(z,v,d,\epsilon) = H(z,v,d) - \epsilon \qquad (17)$$

It leads to $\tilde{H}_z = H_z$, $\tilde{H}_v = H_v$, $\tilde{H}_d = H_d$ and $\tilde{H}_\epsilon = -I$. In equation (16) replace d with ϵ and H with $\tilde{H}$ respectively and consider $F_\epsilon = 0$ and $G_\epsilon = 0$ (process equilibrium, active constraints and theoretic gradient are independent of ϵ). Then, the gradient sensitivity with respect to the implementation errors is derived as follows:

$$\delta_\epsilon = \left(H_v - F_v F_z^{-1} H_z\right)^{-1} \left(G_v - F_v F_z^{-1} G_z\right) \qquad (18)$$

where $\delta_\epsilon \in \mathbb{R}^{m\times m}$. Particularly, when, $H = G$, $\delta_\epsilon = I$, *i.e.* $\delta = \epsilon$.

For a small system, the gradient function, $G(z, v, d)$ can be derived analytically. Therefore, the sensitive measures, δ and δ_ϵ can be calculated by linearization of the plant model. Assume the nonlinear model equations and the gradient function are linearized around the nominally optimal point as follows:

$$\begin{aligned} \dot{x} &= Ax + B_1 u_1 + B_2 u_2 + B_3 d && (19)\\ y_1 &= C_1 x + D_{11} u_1 + D_{12} u_2 + D_{13} d && (20)\\ y_2 &= C_2 x + D_{21} u_1 + D_{22} u_2 + D_{23} d && (21)\\ \delta &= C_3 x + D_{31} u_1 + D_{32} u_2 + D_{33} d && (22) \end{aligned}$$

where y_1 corresponds to active constraints of $g_1(x, u, d) = 0$ and y_2 is controlled variables selected for self-optimizing control. Then the Jacobian matrices required to calculate δ_d can be obtained from the above system matrices:

$$F_z = \begin{bmatrix} A & B_1 \\ C_1 & D_{11} \end{bmatrix}^T \quad F_v = \begin{bmatrix} B_2 \\ D_{12} \end{bmatrix}^T \quad F_d = \begin{bmatrix} B_3 \\ D_{13} \end{bmatrix}^T$$

$$H_z = \begin{bmatrix} C_2 & D_{21} \end{bmatrix}^T \quad H_v = D_{22}^T \quad H_d = D_{23}^T$$

$$G_z = \begin{bmatrix} C_3 & D_{31} \end{bmatrix}^T \quad G_v = D_{32}^T \quad G_d = D_{33}^T$$

Particularly, for systems without active constraints, *i.e.* $n_1 = 0$, matrices B_1, C_1, D_{11}, D_{12}, D_{13} and D_{21} are empty. Denote steady-state gain matrices between different signals at the nominally optimal point as, $L_{yv} = D_{22} - C_2 A^{-1} B_2$, $L_{yd} = D_{23} - C_2 A^{-1} B_3$, $L_{Gv} = D_{32} - C_3 A^{-1} B_2$ and $L_{Gd} = D_{33} - C_2 A^{-1} B_3$. Then the sensitivity measures can be simplified as:

$$\begin{aligned} \delta_\epsilon^T &= L_{Gv} L_{yv}^{-1} && (23)\\ \delta_d^T &= L_{Gd} - L_{Gv} L_{yv}^{-1} L_{yd} && (24) \end{aligned}$$

The above equations clearly show how sensitivity measures are associated with the minimum singular value measure, $\sigma(L_{yv})$. If the system has no active constraints, or all active constraints have been implicitly included in equilibrium equations, and manipulated variables have been properly scaled such that $L_{Gv} = I$, then δ_ϵ is equivalent to the minimum singular value measure. Further more, only when $L_{Gd} = 0$ (no explicit dependence of gradient on disturbances) and disturbances are also properly scaled such that $L_{yd} = I$, then equivalency between δ_d and the minimal singular value measure is true. Otherwise, if $L_{Gd} \neq 0$, the minimal singular value measure can only partially predict self-optimizing properties.

For a large or complicated process, it may not be possible to get analytical expression of the gradient function. In that case, the sensitivity measures, δ_d and δ_ϵ can still be numerically calculated as the second-order derivatives, J_{vd} and $J_{v\epsilon}$ of the constrained cost function. For this purpose, the recently developed automatic differentiation techniques [3] can play an important role.

4 Conditionally active constraints

Controlled variables in a self-optimizing plant should include: stabilizing variables related to plant unstable modes, active constraint variables included in $g_1 = 0$ in (2), self-optimizing variables, G or those with small δ_d and δ_ϵ. However, active constraints of a process plant may not always be the same. Some output constraints, such as temperature and pressure limits may becomes active under certain circumstances. Traditionally, these variables are always selected as controlled variables. However, by controlling these variables at their nominal setpoints, the plant operation will not be optimal at most times.

To satisfy both requirements of self-optimization and operating constraints, a cascade control structure is proposed as shown in Figure 1. In Figure 1, an inner loop is closed for constraint

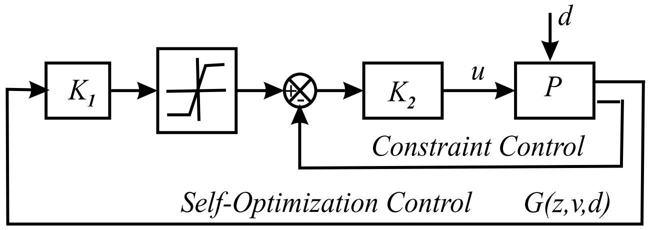

Figure 1: Cascade structure for self-optimizing and constraint control

control. The setpoint of the inner loop is determined by the outer loop, which is designated for self-optimizing control by maintaining the self-optimizing variable at constant. Within the feasible range of the process constraint, the setpoint of the inner loop is floating as a manipulated variable to perform self-optimizing control. However, when disturbances cause the process towards outside of the constraints, the saturation block will limit the setpoint within the constraint so that the controlled variable of the inner loop will be kept within feasible range. In this way, the self-optimizing control and constraint control loops alternatively become active and inactive to achieve constrained self-optimization.

5 Evaporator case study

5.1 Gradient function

The new controlled variable selection approach is applied to an evaporation process [7], shown in Figure 2.

This is a "forced-circulation" evaporator, where the concentration of dilute liquor is increased by evaporating solvent from the feed stream through a vertical heat exchanger with circulated liquor. The process variables are listed in Table 1 and model equations are given in Appendix A.

The economic objective is to minimize the operational cost [$/h] related to steam, cooling water and pump work [4, 9]:

$$J = 600F_{100} + 0.6F_{200} + 1.009(F_2 + F_3) \qquad (25)$$

The process has the following constraints related to product

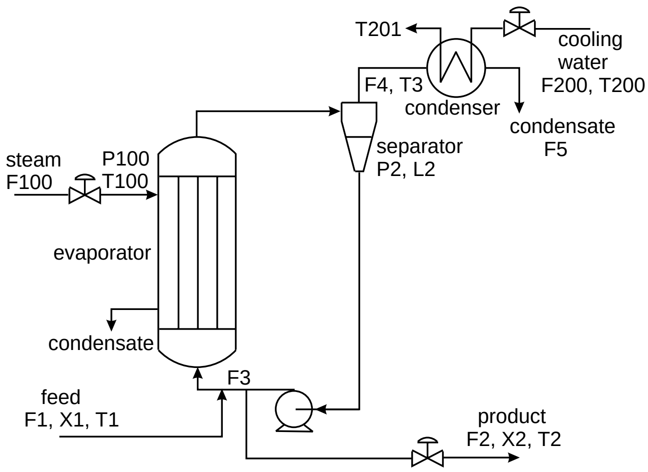

Figure 2: Evaporator System

specification, safety and design limits:

$$X_2 \geq 35 + 0.5\% \quad (26)$$

$$40 \text{ kPa} \leq P_2 \leq 80 \text{ kPa} \quad (27)$$

$$P_{100} \leq 400 \text{ kPa} \quad (28)$$

$$F_{200} \leq 400 \text{ kg/min} \quad (29)$$

$$0 \text{ kg/min} \leq F_3 \leq 100 \text{ kg/min} \quad (30)$$

Note a 0.5% back-off has been enforced on X_2 to ensure the variable remaining feasible for all possible disturbances. The process model has three state variables, L_2, X_2 and P_2 with eight degrees of freedom. Four of them are disturbances, F_1, X_1, T_1 and T_{200}. The rest four degrees of freedom are manipulable variables, F_2, P_{100}, F_3 and F_{200}. The optimization problem of (25) with process constraints, (26) to (30) has been solved under nominal disturbances:

$$d = \begin{pmatrix} F_1 & X_1 & T_1 & T_{200} \end{pmatrix}^T = \begin{pmatrix} 10 & 5 & 40 & 25 \end{pmatrix}^T \quad (31)$$

The minimum cost obtained is 6178.2 \$/h and corresponding values of process variables are shown in Table 1.

At the optimal point, there are two active process constraints, $X_2 = 35.5\%$ and $P_{100} = 400$ [kPa]. These two constraints will keep active within whole disturbance region, which is defined as $\pm$20% of the nominal disturbances. Physically, the first active constraint is because a higher outlet composition requires more solvent to be evaporated, therefore needs more steam, cooling water and pump cost. For the second constraint, since heater duty, Q_{100} is determined by both steam pressure, P_{100} and circulating flowrate, F_3, reducing P_{100} will increase F_3 due to energy balance. However, the sensitivity to steam cost of P_{100} is much lower than that of F_3. Hence, an optimal operation should keep X_2 at its lower bound and P_{100} at its higher bound.

These two active constraints plus the separator level, which has no steady-state effect on the plant operation, but must be stabilized at its nominal setpoint, consume three degrees of freedom. Therefore, the optimal condition has one degree of freedom.

Var.	Description	Value	Units
F_1	Feed flowrate	10	kg/min
F_2	Product flowrate	1.41	kg/min
F_3	Circulating flowrate	23.05	kg/min
F_4	Vapor flowrate	8.59	kg/min
F_5	Condensate flowrate	8.59	kg/min
X_1	Feed composition	5	%
X_2	Product composition	35.5	%
T_1	Feed temperature	40	^{o}C
T_2	Product temperature	91.22	^{o}C
T_3	Vapor temperature	83.61	^{o}C
L_2	Separator level	1	meter
P_2	Operating pressure	56.42	kPa
F_{100}	Steam flowrate	10.02	kg/min
T_{100}	Steam temperature	151.52	^{o}C
P_{100}	Steam pressure	400	kPa
Q_{100}	Heat duty	366.63	kW
F_{200}	Cooling water flowrate	230.54	kg/min
T_{200}	Inlet C.W. temperature	25	^{o}C
T_{201}	Outlet C.W. temperature	45.5	^{o}C
Q_{200}	Condenser duty	330.77	kW

Table 1: Variables and Optimal Values

Choose cooling water flowrate, F_{200} as v and rest manipulated variables and state variables as z, i.e.

$$z = \begin{pmatrix} L_2 & X_2 & P_2 & F_2 & P_{100} & F_3 \end{pmatrix}^T$$

By using (7), the following gradient function is obtained:

$$G = 0.6 - 0.5538 \frac{T_{201} - T_{200}}{F_{200}} \times \left(6.306 \frac{0.16(F_1 + F_3) + 0.07F_1}{T_{100} - T_2} + \frac{42F_1}{36.6} \right) \quad (32)$$

5.2 Self-optimizing variable selection

If the nonlinear gradient function, (32) is not able to be implemented in a practical system as a controlled variable, then an alternative measurement need to be selected to achieve self-optimization. It can be selected from the set of all measurable and manipulable variables. The process has twelve measurements and four manipulated variables. Three of them, L_2, X_2 and P_{100} has already been selected for stabilizing and constraint control. Amount the rest variables, F_2, F_4, F_5 have to be determined by the equilibrium of the system and T_2, T_3, P_{100}, Q_{100} and Q_{200} are dependent on some other variables. Therefore, only five variables represent independent alternatives: P_2, F_{100}, T_{201}, F_{200} and F_3. The authors of [2] have considered another controlled variable, F_{200}/F_1. A new controlled variable, $T_{201} - T_{200}$ is also considered in this work. The gradient sensitivity measures to four disturbances and to implementation errors are calculated (see Table 2) using (16) and (18) with disturbances and controlled variables both scaled by 20% of their nominal values.

Table 2 shows that for single measurement, T_{201} and F_{200} are two most promising choices. T_{201} is better when F_1 and X_1

C.V.	δ_{F_1}	δ_{X_1}	δ_{T_1}	$\delta_{T_{200}}$	δ_ϵ
$T_{201} - T_{200}$	0.0124	0.0167	0.0005	0.0064	0.2426
F_{200}/F_1	0.0124	0.0231	0.0005	0.0064	0.2426
T_{201}	0.0124	0.0167	0.0005	0.2895	0.5385
F_{200}	0.2550	0.0231	0.0005	0.0064	0.2426
P_2	1.1324	0.2044	0.0005	0.5854	0.6772
F_3	1.9840	0.3753	0.0878	0.5854	0.8516
F_{100}	12.326	1.8600	0.8544	0.5854	11.1936

Table 2: Sensitivity measures of alternative controlled variables against disturbances and implementation errors

are main disturbances, but F_{200} becomes better when T_{200} is the dominant disturbance. In all situations, $T_{201} - T_{200}$ and F_{200}/F_1 are the best controlled variables with minor difference. Implementation error is the dominant factor affecting optimality for these two choices. It is also expected that choosing either $T_{201} - T_{200}$ or F_{200}/F_1 will be as good as controlling the gradient.

5.3 Simulation results and comparison

Top four most promising controlled variables listed in Table 2 plus the gradient function in (32) are compared with constant P_2 control by static and dynamic simulation. For static simulation, 1000 sets of disturbances are randomly generated within feasible range. Static responses to these disturbances for the six control schemes are obtained. The mean value of the corresponding costs are calculated and shown in the first column of Table 5.

For dynamic simulation, all six control schemes are implemented in a decentralized cascade structure: L_2 controlled by F_3, X_2 controlled by F_2, one of the five controlled variables controlled by the setpoint of P_2, which is in turn controlled by F_{200} to satisfy both the self-optimizing and conditionally active constraint control as shown in Figure 1.

Three loops are controlled by PI controllers with parameters shown in in Table 3, whilst self-optimizing variables in all schemes are controlled with constant static gains: 1000 for G and F_{200}/F_1 loops, 20 for $T_{201} - T_{200}$ and T_{201} loops, and 10 for F_{200} loop.

Loop	Gain	Integral time [min]
(L_2,F_3)	200	5
(X_2,F_2)	36.74	4.6619
(P_2,F_{200})	200	6.667

Table 3: PI controller parameters

In the simulation all disturbances are modelled as a step signal passing through a first-order delay. The amplitudes of step changes are randomly produced within the $\pm 20\%$ range of the nominal values. The changing intervals and time constants of the first-order delays are different for different disturbance variables shown in Table 4.

With the above configuration, simulation for a 20-hour operation is performed. The total operation costs of the six schemes are shown in the second column of Table 5.

Disturbance	Interval [min]	Time constant [min]
F_1	120	20
X_1	6	2
T_1	15	5
T_{200}	15	5

Table 4: Disturbance model parameters

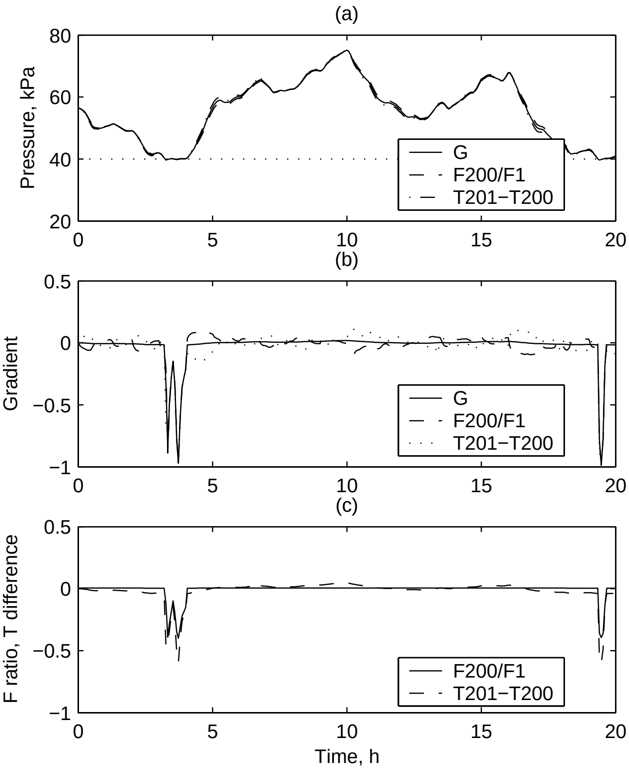

Figure 3: Simulation result comparison. (a)P_2 response. (b)G response. (c)scaled deviation of F_{200}/F_1 and $-(T_{201} - T_{200})$ from nominal steady-state.

It is shown in Table 5 that costs of all four most promising schemes are very close to the cost using G in both static and dynamic simulation. This demonstrates the concept of self-optimizing control, *i.e.* optimal or near optimal plant operation can be achieved by selecting certain controlled variables to be controlled at constant setpoints. The relative ranking of alternative controlled variables is almost coincident with the prediction of the sensitivity measure (Table 2) except that in Table 5 the schemes using F_{200}/F_1 is slightly better than using $T_{201} - T_{200}$. To explain the difference between these two configurations, the dynamic simulation results of three best schemes, using G, using F_{200}/F_1 and using $T_{201} - T_{200}$ are compared in Figure 3.

From Figure 3 it can been seen that the cascade control structure works well in all three schemes. When pressure constraint

self-optimizing c.v.	Static Mean [$]	20h Dynamic cost [$]
G	6139.80	120,823
F_{200}/F_1	6139.83	120,826
$T_{201} - T_{200}$	6139.82	120,828
F_{200}	6141.08	120,854
T_{201}	6142.42	120,857
P_2	6162.57	121,561

Table 5: Alternative controlled variables and operating costs

of P_2 is inactive, self-optimizing control is active, the gradient response has very small deviation in all three schemes. However, when P_2 reaches 40 [kPa] at 3.5 and 19.5 hour, out control loops become inactive, hence large deviations of self-optimizing variables are observed. Particularly, the scheme using $T_{201} - T_{200}$ has larger offset than the one using F_{200}/F_1 when P_2 constraint is active. The offset in scheme using $T_{201} - T_{200}$ is also more sensitive to controller gain than the one in scheme using F_{200}/F_1. Therefore, control gain of the former has to be much smaller than the one of the latter to limit the maximal deviation. However, the smaller the control gain the larger the average deviation, *i.e.* the larger the implementation error. Therefore, the loos of objective function using $T_{201} - T_{200}$ is larger than the one using F_{200}/F_1 due to different implementation error although the gradient sensitivity to implementation error is the same for both schemes.

6 Conclusions

The concept of self-optimizing control has been scrutinized. Based on the gradient of a constrained cost function described in [1], the sensitivities of the gradient function to disturbance and to implementation error have been derived and proposed as criteria for controlled variable selection in self-optimizing control system design. The sensitivity measure is a second-order derivative of the cost function and is independent of measurement scaling. Therefore, it can provide objective and unbiased comparison for controlled variable selection. The gradient sensitivity can be calculated from the linearized model when the gradient is available analytically, or numerically calculated by applying the newly developed automatic differentiation techniques. The evaporator case study demonstrates the effectiveness of this new selection measure. Two better controlled variables are able to be identified by using these sensitivity criteria. The case study also demonstrates the success of using cascade control to cope with conditionally active constraints.

Acknowledgements

The work is supported by the EPSRC UK under grant GR/R57324.

References

[1] Y. Cao. Constrained self-optimizing control via differentiation. Submitted to ADCHEM 2003, 2002.

[2] M.S. Govatsmark and S. Skogestad. Control structure selection for an evaporation process. In *Proceedings of ESCAPE'11*, pages 657–662, Kolding, Denmark, May 2001.

[3] A. Griewank. *Evaluating Derivatives*. SIAM, Philadelphia, PA, 2000.

[4] J.A. Heath, I.K. Kookos, and J.D. Perkins. Process control structure selection based on economics. *AICHE J.*, 46(10):1998–2016, 2000.

[5] T. Larsson, K. Hesterun, E. Hovland, and S. Skogestad. Self-optimization control of a large-scale plant: The tennessee eastman process. *Ind. Eng. Chem. Res.*, 40:4889–4901, 2001.

[6] M. Morari, Y. Arkun, and G. Stephanopoulos. Studies in the synthesis of control structures for chemical process, Part I: Formulation of the problem. Process decomposition and the classification of the control tasks. Analysis of the optimizing control structures. *AIChE Journal*, 26(2):220–232, 1980.

[7] R.B. Newell and P.L. Lee. *Applied Process Control – A Case Study*. Prentice Hall, Englewood Cliffs, NJ, 1989.

[8] S. Skogestad. Plantwide control: the search for the self-optimizing control structure. *J. Process Control*, 10(5):487–507, 2000.

[9] F.Y. Wang and I.T. Cameron. Control studies on a model evaporation process – constrainted state driving with conventional and higher relative degree systems. *J. Process Control*, 4(2):59–75, 1994.

A Model equations

$$\frac{\mathrm{d}L_2}{\mathrm{d}t} = \frac{F_1 - F_4 - F_2}{20} \tag{33}$$

$$\frac{\mathrm{d}X_2}{\mathrm{d}t} = \frac{F_1 X_1 - F_2 X_2}{20} \tag{34}$$

$$\frac{\mathrm{d}P_2}{\mathrm{d}t} = \frac{F_4 - F_5}{4} \tag{35}$$

$$T_2 = 0.5616P_2 + 0.3126X_2 + 48.43 \tag{36}$$

$$T_3 = 0.507P_2 + 55.0 \tag{37}$$

$$F_4 = \frac{Q_{100} - 0.07F_1(T_2 - T_1)}{38.5} \tag{38}$$

$$T_{100} = 0.1538P_{100} + 90.0 \tag{39}$$

$$Q_{100} = 0.16(F_1 + F_3)(T_{100} - T_2) \tag{40}$$

$$F_{100} = Q_{100}/36.6 \tag{41}$$

$$Q_{200} = \frac{0.9576F_{200}(T_3 - T_{200})}{0.14F_{200} + 6.84} \tag{42}$$

$$T_{201} = T_{200} + \frac{13.68(T_3 - T_{200})}{0.14F_{200} + 6.84} \tag{43}$$

$$F_5 = \frac{Q_{200}}{38.5} \tag{44}$$