SENSORLESS CONTROL OF INDUCTION MOTORS WITH EXPONENTIAL STABILITY PROPERTY

Marcello Montanari*, Sergei Peresada†, Andrea Tilli*

* CASY-Center for Research on Complex Automated Systems “G. Evangelisti”
DEIS, University of Bologna
Viale Risorgimento 2, 40136 Bologna, ITALY
Tel.: +39-051-2093024, Fax:+39-051-2093073, E-mail: {mmontanari, atilli}@deis.unibo.it.
† Department of Electrical Engineering,
National Technical University of Ukraine “Kiev Politechnical Institute”
Prospect Pobedy 37, Kiev 252056, UKRAINE
E-mail: peresada@i.com.ua

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Abstract

In this paper a sensorless controller for induction motor is presented. It provides local exponential stability properties with an explicit estimation of the convergence domain since Lyapunov direct method is adopted in the design/investigation process. Full-order model of the induction motor is considered and no simplifying assumptions on the speed dynamics (as negligibility of the speed time-derivative) are introduced. The load torque is assumed constant but unknown. Simulation results are provided.

1 Introduction

High performance drives based on induction motor (IM) can be implemented by means of a speed/flux controller which relies on field orientation concepts [1, 7]. This control algorithm is an output feedback controller based on the measured currents and rotor speed/position, normally obtained with a shaft encoder. The position sensor reduces the robustness and reliability of the IM drive and increases its cost. Hence, in recent years speed sensorless controllers for IM (i.e. without the speed/position measure) have become an attractive task from the industrial perspective and as a benchmark for different nonlinear control techniques.

Excellent surveys on different control techniques applied to sensorless control of IM can be found in [10, 11] and [4]. In [3] a sensorless control algorithm which guarantees semi-global exponential rotor velocity/rotor flux tracking for the full-order nonlinear model of the IM is presented. In particular, an observer, designed assuming known mechanical dynamics and external load torque, provides the speed estimation with assignable convergence rate. The initial conditions for rotor fluxes are assumed to be known and the rotor flux is estimated by means of an on-line pure integration algorithm. In [8] a sensorless controller for the current-fed IM, which guarantees local exponential stability and global asymptotic stability of the closed-loop dynamics, is proposed. The speed is estimated using an adaptive observer based on the magnetic dynamics of the IM. The following assumptions are made: the estimation of the rotor flux is performed by means of pure integration of the stator winding electric model, the load torque is assumed known. In [12] sliding-mode technique has been exploited to develop a speed-flux observer and a torque controller, which are used in the speed control assuring local asymptotic stability properties.

In this work the sensorless control scheme proposed in [9] is deeply revised in order to achieve stronger theoretical and practical results relaxing some hypothesis on the speed error dynamics. A flux controller based on indirect field oriented control and a speed controller designed assuming *unknown* constant load torque are realized. Since the latter controllers depend on the estimated speed, an adaptive speed/flux observer, based on the current regulation error dynamics, is designed, providing an exponential speed estimation convergence. It is worth noting that the speed estimator is realized in a closed-loop way, since it is integrated in the current regulator. In this way, reduction of the dynamic order of the speed estimator is obtained. A model-based flux observer, similar to those in [3] and [8], is added to speed-up the speed estimation dynamics, but it is *not* used in flux control or to obtain direct field orientation of the reference frame. In the controller design, motor parameters are assumed constant and exactly known and the initial conditions of IM state-variables are assumed to be known.

The proposed solution provides local exponential speed tracking and local exponential flux amplitude regulation and field orientation, with an explicit estimation of the domain of attraction of the zero equilibrium point of the error model. The overall system stability is proved using Lyapunov-like technique applied to two feedback interconnected subsystems: speed tracking and flux regulation dynamics on one side, and current regulation dynamics with the speed/flux observer on the other side. The stability proof is carried on proving the exponential stability of the above subsystems and taking into account features of the coupling terms in order to evaluate the properties of the whole dynamics.

The paper is organized as follows. In Section 2 the IM model, the control objectives and the regulation/estimation error definitions are reported. The proposed solution is presented in Section 3. In 4 the stability properties of the closed-loop system are analyzed. In Section 5 some simulations are reported; particular attention is devoted to the robustness with respect to mechanical parameter uncertainties.

2 Induction motor model and control objectives formulation

Assuming linear magnetic circuits and balanced operating conditions, the induction motor (IM) model expressed in an arbitrary two-phase rotating reference frame (d,q) is given by the fifth-order model [7]:

$$\begin{aligned}
\dot\omega &= \mu(\psi_d i_q - \psi_q i_d) - \frac{T_L}{J}\\
\dot i_d &= -\gamma i_d + \omega_0 i_q + \alpha\beta\psi_d + \beta\omega\psi_q + \frac{1}{\sigma}u_d\\
\dot i_q &= -\gamma i_q - \omega_0 i_d + \alpha\beta\psi_q - \beta\omega\psi_d + \frac{1}{\sigma}u_q\\
\dot\psi_d &= -\alpha\psi_d + \omega_2\psi_q + \alpha L_m i_d\\
\dot\psi_q &= -\alpha\psi_q - \omega_2\psi_d + \alpha L_m i_q
\end{aligned}\tag{1}$$

where the state variables are the rotor speed ω, the stator currents (i_d, i_q), the rotor fluxes (ψ_d, ψ_q); the control inputs are the applied stator voltages (u_d, u_q); the load torque T_L is an unknown constant external input, i.e. $\dot T_L = 0$. The slip frequency is defined as $\omega_2 = \omega_0 - \omega$. Parameters of the above model are related to the mechanical and electrical IM parameters as:

$$\sigma = L_s\Big(1-\frac{L_m^2}{L_sL_r}\Big),\ \beta = \frac{L_m}{\sigma L_r},\ \alpha = \frac{R_r}{L_r}$$
$$\gamma = \frac{R_s}{\sigma} + \alpha L_m\beta,\ \mu = \frac{3}{2}\frac{L_m}{JL_r}$$

where R_s, R_r, L_s, L_r are stator/rotor resistances and inductances, L_m is the magnetizing inductance, J is the rotor inertia.

The rotating reference frame (d,q) has angular speed ω_0 and relative angular position ε_0, with dynamics given by $\dot\varepsilon_0 = \omega_0$, with respect to the stationary reference frame (a,b). The following relations are introduced to link variables in the stationary reference frame (a,b), where physical variables are defined, with the corresponding ones expressed in the rotating reference frame (d,q):

$$\begin{bmatrix} i_d\\ i_q\end{bmatrix} = \mathbf{e}^{-\mathbf{J}\varepsilon_\mathbf{0}}\begin{bmatrix} i_a\\ i_b\end{bmatrix},\quad \begin{bmatrix} u_a\\ u_b\end{bmatrix} = \mathbf{e}^{\mathbf{J}\varepsilon_\mathbf{0}}\begin{bmatrix} i_d\\ i_q\end{bmatrix}$$

where

$$\mathbf{e}^{\mathbf{J}\varepsilon_\mathbf{0}} = \begin{bmatrix}\cos\varepsilon_0 & -\sin\varepsilon_0\\ \sin\varepsilon_0 & \cos\varepsilon_0\end{bmatrix}.$$

In sensorless control, the measured variables are the stator currents (i_d, i_q), while the variables to be controlled are the rotor velocity ω and the rotor flux amplitude $\sqrt{\psi_d^2+\psi_q^2}$. In the proposed scheme an explicit estimation of the mechanical speed and load torque are adopted. Let define the following notations for regulation errors:

$$\begin{aligned}
\tilde i_d &= i_d - i_d^* \qquad \tilde\psi_d = \psi_d - \psi^* \qquad \tilde\omega = \omega - \omega^*\\
\tilde i_q &= i_q - i_q^* \qquad \tilde\psi_q = \psi_q
\end{aligned}$$

and for estimation errors of speed and torque correspondingly:

$$\tilde{\tilde\omega} = \omega - \hat\omega = \tilde\omega - \hat{\tilde\omega} \quad \tilde T_L = \frac{T_L}{J} - \hat T_L$$

where ψ^* is the constant reference flux, ω^* is the smooth reference speed signal, with bounded time-derivatives $\dot\omega^*, \ddot\omega^*$. Reference currents i_d^*, i_q^*, estimated speed tracking error $\hat{\tilde\omega}$ and estimated (normalized) load torque $\hat T_L$ are signals to be defined later. The estimated speed is defined as $\hat\omega = \omega^* + \hat{\tilde\omega}$. The asymptotic speed tracking objective corresponds to impose the convergence to zero of the speed tracking error $\tilde\omega$ while the asymptotic flux regulation and field orientation of the (d,q) reference frame correspond to impose the convergence to zero of $\tilde\psi_d$ and $\tilde\psi_q$. Asymptotic estimation of the mechanical speed is achieved iff the speed estimation error $\tilde{\tilde\omega}$ tends to zero.

3 Controller definition

3.1 Flux and speed controller

Considering indirect field orientation as the framework, the following flux and speed controllers are designed:

$$\begin{aligned}
i_d^* &= \frac{1}{\alpha L_m}(\dot\psi^* + \alpha\psi^*)\\
\omega_0 &= \omega^* + \hat{\tilde\omega} + \frac{\alpha L_m}{\psi^*}i_q^*
\end{aligned}$$

and

$$\begin{aligned}
i_q^* &= \frac{1}{\mu\psi^*}(-k_\omega\hat{\tilde\omega} + \hat T_L + \dot\omega^*)\\
\dot{\hat T}_L &= -k_{\omega i}\hat{\tilde\omega}
\end{aligned}$$

where $k_\omega, k_{\omega i}$ are control gains and the estimated speed tracking error $\hat{\tilde\omega}$ will be defined later. The controller state $\hat T_L$ is introduced to compensate for the unknown constant load torque.

The magnetic and mechanical regulation error dynamics are

$$\begin{aligned}
\dot{\tilde\psi}_d &= -\alpha\tilde\psi_d + (\omega_0-\omega)\tilde\psi_q + \alpha L_m\tilde i_d\\
\dot{\tilde\psi}_q &= -\alpha\tilde\psi_q - (\omega_0-\omega)\tilde\psi_d + \alpha L_m\tilde i_q + \psi^*\tilde{\tilde\omega}
\end{aligned}\tag{2}$$

$$\begin{aligned}
\dot{\tilde\omega} &= -k_\omega\tilde\omega - \tilde T_L + k_\omega\tilde{\tilde\omega} + \phi_1(\tilde i_d, \tilde i_q, \tilde\psi_d, \tilde\psi_q, \tilde{\tilde\omega}, \tilde\omega, \tilde T_L)\\
\dot{\tilde T}_L &= k_{\omega i}\tilde\omega - k_{\omega i}\tilde{\tilde\omega}
\end{aligned}\tag{3}$$

with

$$\begin{aligned}
&\phi_1(\tilde i_d, \tilde i_q, \tilde\psi_d, \tilde\psi_q, \tilde{\tilde\omega}, \tilde\omega, \tilde T_L) =\\
&\qquad \mu(\tilde\psi_d\tilde i_q - \tilde\psi_q\tilde i_d) + \mu\psi^*\tilde i_q + \mu i_q^*\tilde\psi_d - \mu i_d^*\tilde\psi_q.
\end{aligned}\tag{4}$$

3.2 Current controller and speed/flux estimator

The current controller is defined as

$$u_d = \sigma\left(\dot{i}_d^* + \gamma i_d^* - k_{id}\tilde{i}_d - \omega_0 i_q - \alpha\beta\psi^* + \nu_d\right)$$
$$u_q = \sigma\left(\dot{i}_q^* + \gamma i_q^* - k_i\tilde{i}_q + \omega_0 i_d + \beta\psi^*\hat{\omega} + \nu_q\right)$$

where k_i, k_{id} are controller parameters and ν_d, ν_q are auxiliary signals to be defined later.

Hence, the current error dynamics is

$$\dot{\tilde{i}}_d = -(\gamma + k_{id})\tilde{i}_d + \alpha\beta\tilde{\psi}_d + \beta\omega\tilde{\psi}_q + \nu_d$$
$$\dot{\tilde{i}}_q = -(\gamma + k_i)\tilde{i}_q + \alpha\beta\tilde{\psi}_q - \beta\omega\tilde{\psi}_d - \beta\psi^*\tilde{\omega} + \nu_q \qquad (5)$$

In order to design an adaptive speed observer, defining the signal $\hat{\omega}$, let introduce the following variables

$$z_d = i_d + \beta\psi_d, \qquad z_q = i_q + \beta\psi_q.$$

From (1), the dynamics of the z-variables are expressed by

$$\dot{z}_d = -\frac{R_s}{\sigma}i_d + \omega_0 z_q + \frac{1}{\sigma}u_d$$
$$\dot{z}_q = -\frac{R_s}{\sigma}i_q - \omega_0 z_d + \frac{1}{\sigma}u_q.$$

Hence, current error dynamics (5) is rewritten as

$$\dot{\tilde{i}}_d = -k_d\tilde{i}_d + \alpha z_d - \alpha\beta\psi^* - \alpha i_d^* + \omega(z_q - i_q) + \nu_d$$
$$\dot{\tilde{i}}_q = -k_I\tilde{i}_q + \alpha z_q - \alpha i_q^* - \omega(z_d - i_d) + \beta\psi^*\hat{\omega} + \nu_q$$

with $k_d = \gamma + \alpha + k_{id}$, $k_I = \gamma + \alpha + k_i$. Let define the auxiliary signals:

$$\nu_d = -\alpha\hat{z}_d + \alpha\beta\psi^* + \alpha i_d^* - \hat{\omega}(\hat{z}_q - i_q)$$
$$\nu_q = -\alpha\hat{z}_q - \beta\psi^*\hat{\omega} + \alpha i_q^* + \hat{\omega}(\hat{z}_d - i_d)$$

which are based on the following (z_d, z_q) estimator:

$$\dot{\hat{z}}_d = -\frac{R_s}{\sigma}i_d + \omega_0\hat{z}_q + \frac{1}{\sigma}u_d$$
$$\dot{\hat{z}}_q = -\frac{R_s}{\sigma}i_q - \omega_0\hat{z}_d + \frac{1}{\sigma}u_q.$$

Define the estimation errors $\tilde{z}_d = z_d - \hat{z}_d$, $\tilde{z}_q = z_q - \hat{z}_q$ whose dynamics is

$$\dot{\tilde{z}}_d = \omega_0\tilde{z}_q$$
$$\dot{\tilde{z}}_q = -\omega_0\tilde{z}_d. \qquad (6)$$

Assuming known initial conditions for the IM state variables and imposing that

$$\hat{z}_d(0) = z_d(0) = i_d(0) + \beta\psi_d(0)$$
$$\hat{z}_q(0) = z_q(0) = i_q(0) + \beta\psi_q(0)$$

from (6) it follows that

$$\tilde{z}_d(t) = 0$$
$$\tilde{z}_q(t) = 0, \quad \forall t.$$

Defining the speed estimation law

$$\dot{\hat{\omega}} = -\frac{1}{\gamma_1}\beta\psi^*\tilde{i}_q - k_\omega\hat{\omega} + \hat{\phi}_1$$
$$\hat{\phi}_1 = \mu\left[\left(\frac{\hat{z}_d - i_d}{\beta} - \psi^*\right)\tilde{i}_q - \frac{\hat{z}_q - i_q}{\beta}\tilde{i}_d\right] + \mu\psi^*\tilde{i}_q +$$
$$+ \mu i_q^*\left(\frac{\hat{z}_d - i_d}{\beta} - \psi^*\right) - \frac{\mu}{\beta}i_d^*(\hat{z}_q - i_q)$$

with the positive gain γ_1, and recalling the definition of ν_d, ν_q, the current and speed estimation error $\tilde{\omega}$ dynamics are rewritten in matrix form

$$\begin{bmatrix}\dot{\tilde{i}}_d\\ \dot{\tilde{i}}_q\end{bmatrix} = \begin{bmatrix}-k_d & 0\\ 0 & -k_I\end{bmatrix}\begin{bmatrix}\tilde{i}_d\\ \tilde{i}_q\end{bmatrix} + \begin{bmatrix}0\\ -\beta\psi^*\end{bmatrix}\tilde{\omega} + \begin{bmatrix}\beta\tilde{\psi}_q\tilde{\omega}\\ -\beta\tilde{\psi}_d\tilde{\omega}\end{bmatrix} \qquad (7)$$
$$\dot{\tilde{\omega}} = \frac{1}{\gamma_1}\beta\psi^*\tilde{i}_q - \tilde{T}_L. \qquad (8)$$

It is worth noting that the z-variable estimator is a sort of flux estimator based on an on-line pure integration algorithm, which may suffer from drift problems due to measurement offset, parameter uncertainties and numerical implementation. The same solution is adopted also in other works [3, 8] and still remains an open problem for sensorless control. Different technical solutions have been proposed in order to reduce the sensitivity of the open-loop estimation algorithm[6, 5].

4 Closed-loop system stability

The full-order error dynamics is represented by the 7^{th} order system given by: a) the "outer" magnetic and mechanical regulation error dynamics, described by (2), (3); b) the "inner" dynamics given by the current regulation dynamics and the speed estimation law, represented by (7) and (8). The subsystems are interconnected by means of linear and bilinear terms, as synthetically depicted in Fig. 1.

System stability is investigated, first considering stability properties of the isolated outer and inner subsystems, then exploiting the features of the coupling, by means of a composite Lyapunov function.

Let assume the following technical hypothesis, in order to simplify the stability proof:

$$k_{\omega i} = \frac{k_\omega^2}{2} \qquad k_d = \frac{k_I}{2} \qquad \frac{k_I^2}{2} = \frac{\beta^2\psi^{*2}}{\gamma_1}$$

Define $\tilde{\psi} = (\tilde{\psi}_d, \tilde{\psi}_q)^T$ and

$$w_1 = \frac{k_\omega}{2}\tilde{\omega} \quad w_2 = \frac{k_\omega}{2}\tilde{\omega} + \tilde{T}_L \quad w = (w_1, w_2)^T$$
$$\xi_1 = \frac{k_I}{2}\tilde{i}_q \quad \xi_2 = \frac{k_I}{2}\tilde{i}_q + \beta\psi^*\tilde{\omega} \quad \xi = (\xi_1, \xi_2, \tilde{i}_d)^T.$$

Full-order system dynamics is rewritten with respect to the new state variables as

$$\dot{\tilde{\psi}}_d = -\alpha\tilde{\psi}_d + \omega_2\tilde{\psi}_q + \alpha L_m\tilde{i}_d$$

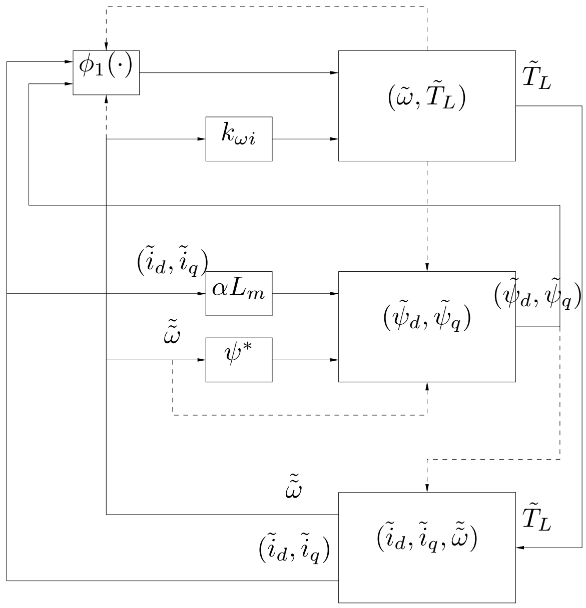

Figure 1: Closed-loop system structure. (solid lines: linear/bilinear interconnection, dashed line: bilinear interconnection)

$$
\begin{aligned}
\dot{\tilde\psi}_q &= -\alpha\tilde\psi_q - \omega_2\tilde\psi_d + \frac{2\alpha L_m}{k_I}\xi_1 + \frac{1}{\beta}(\xi_2-\xi_1)\\
\dot w_1 &= -\frac{k_\omega}{2}w_1 - \frac{k_\omega}{2}w_2 + \frac{k_\omega^2}{2\beta\psi^*}(\xi_2-\xi_1) + \frac{k_\omega}{2}\phi_1\\
\dot w_2 &= \frac{k_\omega}{2}w_1 - \frac{k_\omega}{2}w_2 + \frac{k_\omega}{2}\phi_1\\
\dot{\tilde i}_d &= -\frac{k_I}{2}\tilde i_d + \frac{\tilde\psi_q}{\psi^*}(\xi_2-\xi_1)\\
\dot\xi_1 &= -\frac{k_I}{2}\xi_1 - \frac{k_I}{2}\xi_2 - \frac{k_I}{2}\frac{\tilde\psi_d}{\psi^*}(\xi_2-\xi_1)\\
\dot\xi_2 &= \frac{k_I}{2}\xi_1 - \frac{k_I}{2}\xi_2 - \frac{k_I}{2}\frac{\tilde\psi_d}{\psi^*}(\xi_2-\xi_1) - \beta\psi^*(w_2-w_1)
\end{aligned}
$$

where ϕ_1 is defined in (4). Recall that $\frac{T_L}{J}+\dot\omega^*$ is bounded. In the following, let assume that the Euclidean norm $\|\cdot\|$ of the flux regulation error is bounded, i.e. suppose that $\|\tilde\psi(t)\| < \Psi$, where $0 < \Psi < \min\left(\frac{(1-\varepsilon_1)}{4}\psi^*, \frac{(1-\varepsilon_2)}{1+\frac{4}{k_I}}\psi^*\right)$, where $\varepsilon_1, \varepsilon_2$ are chosen such that $0<\varepsilon_1<1$, $0<\varepsilon_2<1$. At the end of the stability proof, it will be shown that this hypothesis is satisfied with proper initial conditions of the state variables.

First, let consider the two dynamics which represent the outer subsystem. For the flux subsystem, it is worth noting that the following inequality holds for ω_2:

$$|w_2| \le B_1 + \left(\frac{2}{\beta\psi^*} + \frac{2\alpha L_m k_\omega}{\mu\beta\psi^{*3}}\right)\|\xi\| + \frac{2\alpha L_m}{\mu\psi^{*2}}\|w\|$$

where

$$B_1 > \left|\frac{\alpha L_m}{\mu\psi^{*2}}\left(\frac{T_L}{J}+\dot\omega^*\right)\right|,$$

hence ω_2 is bounded if the state variables are bounded. Considering the following Lyapunov function for the flux regulation error subsystem (2)

$$V_\psi(t) = \frac{1}{2}\left(\tilde\psi_d^2 + \tilde\psi_q^2\right)$$

the following inequality holds for its time-derivative:

$$\dot V_\psi(t) \le -\alpha\|\tilde\psi\|^2 + C_1\|\tilde\psi\|\|\xi\|$$

with

$$C_1(k_I) \ge \alpha L_m + \frac{2\alpha L_m}{k_I} + \frac{2}{\beta}.$$

Hence, the isolated flux regulation error dynamics is globally exponentially stable.

Let define

$$A_1(k_\omega,k_I) = \frac{2\mu}{k_I} + \mu + \frac{2k_\omega}{\beta\psi^{*2}}, \quad A_2 > \mu i_d^* + \frac{1}{\psi^*}\left|\frac{T_L}{J}+\dot\omega^*\right|$$

such that

$$\|\phi_1\| < A_1(k_\omega,k_I)\|\tilde\psi\|\|\xi\| + A_2\|\tilde\psi\| + \frac{2\mu\psi^*}{k_I}\|\xi\| + \frac{2}{\psi^*}\|\tilde\psi\|\|w\|.$$

Let consider the following candidate Lyapunov function for the mechanical subsystem:

$$V_w(t) = \frac{1}{2}\left(w_1^2 + w_2^2\right).$$

The time derivative of $V_w(t)$ along the trajectory of (3) satisfies

$$
\begin{aligned}
\dot V_w(t) \le{}& -\varepsilon_1\frac{k_\omega}{2}\|w\|^2 + k_\omega A_2\|\tilde\psi\|\|w\| +\\
&+ k_\omega\left(\frac{k_\omega}{\beta\psi^*} + \frac{2\mu\psi^*}{k_I} + A_1(k_\omega,k_I)\Psi\right)\|\xi\|\|w\|.
\end{aligned}
$$

Hence, the isolated linear system (3) (i.e. with null inputs $(\xi,\tilde\psi)=0$, and hence $\phi_1=0$) is globally exponentially stable.

Let define the candidate Lyapunov function for the inner subsystem

$$V_\xi(t) = \frac{1}{2}(\xi_1^2 + \xi_2^2 + \tilde i_d^2)$$

with time-derivative along the trajectories of (7), (8) given by

$$\dot V_\xi \le -\varepsilon_2\frac{k_I}{2}\|\xi\|^2 + 2\beta\psi^*\|\xi\|\|w\|.$$

Hence, the inner subsystem is globally exponentially stable. Note that this property is achieved if the flux reference is positive ($\psi^*>0$), in accordance with the results presented in [9], related to a condition on persistency of excitation that has to be satisfied to obtain the estimation convergence.

Let consider now the full-order system, whose state variables are $X = (\tilde\psi^T, w^T, \xi^T)^T$. Let define

$$V(t) = V_\psi(t) + \eta_w V_w(t) + \eta_\xi V_\xi(t)$$

with $\eta_w,\ \eta_\xi > 0$ to be defined later. Exploiting the unidirectional linear interconnection between the magnetic and mechanical subsystems and the freedom in the choice of the rate of convergence of the inner subsystem (through the selection of the control parameter k_I), and applying Young's inequalities, after some lengthy computations the following relation holds for the time-derivative of $V(t)$:

$$\dot{V}(t) \leq -\alpha_\psi \|\tilde{\psi}\|^2 - \alpha_w \|w\|^2 - \alpha_\xi \|\xi\|^2$$

where, $\delta_i > 0,\ i = 1 \dots 4,\ \eta_\omega, \eta_\xi,\ \alpha_\psi, \alpha_w, \alpha_\xi$ and control gains k_ω, k_i are chosen in order to satisfy the following relations

$$\alpha - \frac{\eta_\omega k_\omega A_2}{2\delta_1} - \frac{\delta_2 C_1(k_I)}{2} \geq \alpha_\psi > 0$$

$$\eta_\omega k_\omega \left(\frac{\varepsilon_1}{2} - \frac{A_2 \delta_1}{2} - \frac{\delta_3}{2} \left(\frac{k_\omega}{\beta \psi^*} + \frac{2\mu\psi^*}{k_I} + A_1(k_\omega, k_I)\Psi \right) \right) - \\ -\delta_4 \eta_\xi \beta \psi^* \geq \alpha_w > 0$$

$$\frac{\eta_\xi \varepsilon_2 k_I}{2} - \frac{\eta_\omega k_\omega}{2\delta_3} \left(\frac{k_\omega}{\beta\psi^*} + \frac{2\mu\psi^*}{k_I} + A_1(k_\omega, k_I)\Psi \right) \\ - \frac{\eta_\xi \beta \psi^*}{\delta_4} - \frac{C_1(k_I)}{2\delta_2} \geq \alpha_\xi > 0$$

Defining $a_1 = \frac{1}{2}\min(1, \eta_\omega, \eta_\xi)$, $a_2 = \frac{1}{2}\max(1, \eta_\omega, \eta_\xi)$, $a_3 = \frac{1}{2}\min(\alpha_\psi, \alpha_\omega, \alpha_\xi)$, noting that $a_1\|X\|^2 \leq V(t) \leq a_2\|X\|^2$ and $\dot{V}(t) \leq -a_3\|X\|^2$ if $\|\tilde{\psi}(t)\| < \Psi$ and applying standard techniques from Lyapunov stability analysis, it follows that

$$\|X(t)\| \leq \sqrt{\frac{a_2}{a_1}} \exp\left(-\frac{a_3}{2a_2} t \right) \|X_0\|$$

where $X(0) = X_0$ are the initial conditions for the state variables. Hence, imposing that $\|X_0\| < \sqrt{\frac{a_1}{a_2}}\Psi = X_M$, it follows that $\|\tilde{\psi}\| < \Psi$ and hence $\dot{V}(t) < 0$. Therefore, the state variables $X(t)$ of the error model is bounded and tend exponentially to zero. The local exponential stability of the origin of the full-order system, with estimated domain of attraction $\|X(0)\| < X_M$, has been proved.

In the stability proof, due to intensive use of inequalities and implicit conservative nature of Lyapunov-like method, strict conditions on control parameters and initial condition have been obtained. Nevertheless, from a practical point of view, it is possible to use a set of control parameters which does not satisfy the given conditions but that still achieves local exponential stability.

5 Simulation results

Examples showing the performance attainable with the proposed controller are illustrated. The parameters of the 6Nm induction motor tested by means of simulation are reported in the Appendix. The tuning parameters of the controllers and the observer are $k_\omega = 40,\ k_{\omega i} = 800,\ k_i = 250, k_{id} = 3$, $\gamma_1 = 0.0025$. In the first test, the operating sequences, reported in Fig. 2, are the following:

1. the machine is excited during the initial time interval 0-0.28s using a smooth flux reference trajectory with steady-state value equal to 0.9Wb,

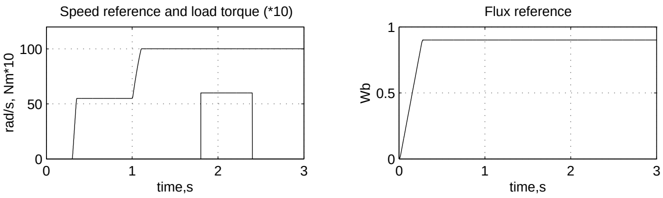

Figure 2: Reference trajectories and load torque profile (first test)

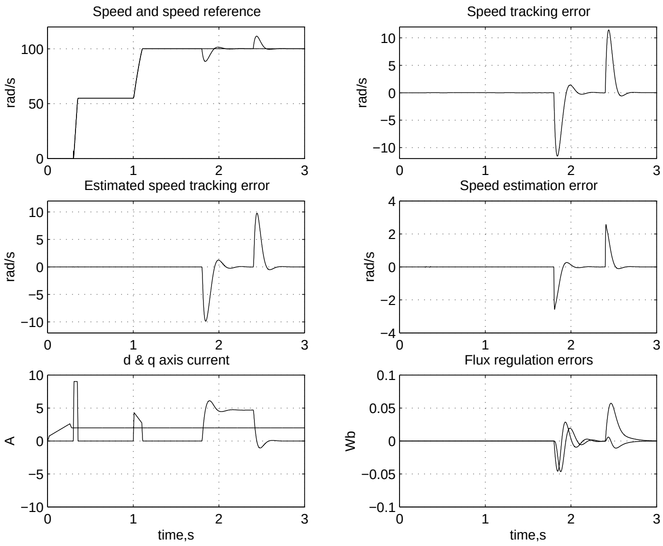

Figure 3: Transients during speed tracking, with load torque $T_L = 6Nm$.

2. the unloaded motor is required to track the smooth speed reference trajectory, starting at t = 0.3s from zero initial value, characterized by two time intervals with constant reference speed respectively equal to 55 rad/s and 100 rad/s,
3. at time t=1.8s a constant load torque, equal to the 100% of the motor rated value (6Nm), is applied; at time t = 2.4s load torque is set to zero.

Simulation results are reported in Fig. 3. The speed is correctly estimated, and speed tracking, flux amplitude regulation and field orientation are obtained with unknown load torque.

In the second test, the same flux reference trajectory is applied. The speed reference, reported in the first graph of Fig. 4, with a transient from 0 rad/s to 50 rad/s with maximum acceleration equal to $1050\mathrm{rad/s}^2$, followed by a sinusoidal profile with frequency equal to 40 rad/s, is applied to the IM, with null load torque. Since the controller is based on the mechanical model of the IM, it is interesting to evaluate the performance of the controller with respect to uncertainties on the rotor inertia J, considering a 20% error (the value of J used in the controller is $\hat{J} = 0.012\mathrm{kg}\cdot\mathrm{m}^2$). Results are depicted in Fig. 4. Due to uncertainties in the feed-forward actions in the controller, speed tracking error is non-null, but it still remains bounded. Since the mechanical dynamics bandwidth is 28 rad/s, while

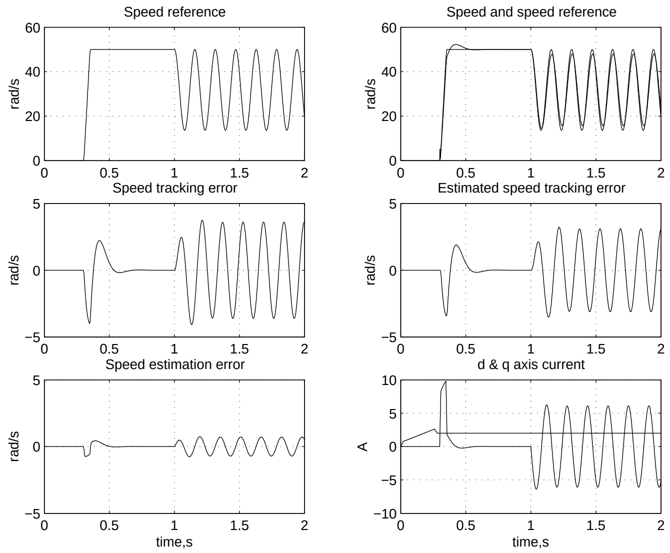

Figure 4: Transients during speed tracking, with no load torque, with 20% uncertainties on J.

the speed estimation dynamics bandwidth is 350 rad/s, during the sinusoidal speed reference the speed estimation error amplitude is small, thanks to the wide bandwidth of the speed estimator.

6 Conclusions

The proposed sensorless controller for induction motor provides local exponential speed tracking and flux regulation proven by means of Lyapunov direct method applied to the full order dynamics. Simulation results show that the controller is effective to track the reference speed and to reject the applied load torque even if relevant errors on the mechanical model are present.
In order to achieve these results the only significant requirement is to impose a non null flux reference. This fact is in accordance with the "induction motor physics": the machine must be excited to produce torque.
No restrictions on the speed reference and the load torque are present. This is a very relevant feature since it indicates that the system could work in any condition, provided sufficiently small initial error. It is worth noting that this result seems achievable only if exact knowledge of the fluxes is assumed, i.e. the fluxes are obtained by pure integration of stator equation with known initial conditions, in fact there exist conditions where different speed-flux trajectories are not distinguishable from voltage-current behavior (e.g. when $\omega_0 = 0$ with constant voltages and currents)[2]. Anyway it is well known that, from a practical perspective, obtaining reliable pure integration of the stator equations is a very involved task, in particular in condition of constant voltages and currents.
Further studies will be devoted to better analyze the above consideration in order to develop more robust solutions.

IM parameters

rated power	1.9kW	rated voltage	380V
rated speed	3000rpm	rated current	4.1A
rated torque	6Nm	magnetizing current	1.4A
pole number	1		

$$R_s = 6.6\Omega, R_r = 5.3\Omega,$$
$$L_s = L_r = 0.475H, L_m = 0.45H, J = 0.01\mathrm{kg}\cdot\mathrm{m}^2$$
$$\gamma = 233, \alpha = 11.2, \sigma = 0.0487, \beta = 19.5, \mu = 142$$

References

[1] F. Blaschke. The principle of field orientation applied to the new transvector closed-loop system for rotating field machines. *Siemens-Rev.*, vol.39, pp. 217–220, (1972).

[2] C. Canudas, A. Youssef, J. Barbot, P. Martin, F. Malrait. Observability conditions of induction motors at low frequencies. *Proc. of IEEE Conf. on Decision and Control (CDC00)*, pp. 2044–2049, (2000)

[3] M. Feemster, P. Aquino, D.M. Dawson, A. Behal. "Sensorless rotor velocity tracking control for induction motors", *IEEE Trans. Contr. Systems Techn.*, vol. 9, no.4 , pp. 645–653, (July 2000).

[4] J. Holtz, Sensorless control of induction motor drives. *Proc. of the IEEE*, vol. 90, no. 8, pp. 1359-1394, (Aug. 2002).

[5] J. Holtz, J. Quan, "Sensorless vector control of induction motors at very low speed using a nonlinear inverter model and parameter identification", *IEEE Trans. on Industry Applications*, vol. 38, no. 4, pp. 1087–1095, (July-Aug. 2002)

[6] K.D. Hurst, T.G. Habetler, G. Griva, F. Profumo. "Zero-speed tacholess IM torque control: simply a matter of stator voltage integration", *IEEE Trans. on Industry Applications*, vol. 34, no. 4, pp. 790–795 (July-Aug. 1998)

[7] W. Leonhard. *Control of Electrical Drives*. Springer-Verlag, Berlin, (1996).

[8] R. Marino, P. Tomei, C.M. Verrelli. "A new global control scheme for sensorless current-fed induction motors", *Proc. 15th Triennial World Congress IFAC 2002*, Barcelona, Spain, (July 2002).

[9] M. Montanari, S. Peresada, A. Tilli. "Sensorless Indirect Field Oriented Control of Induction Motor via Adaptive Speed Observer", *Proc. 2003 American Control Conference (ACC2003), Denver, CO, (June 2003)*.

[10] K. Rajashekara, A. Kawamura, K. Matsuse. *Sensorless Control of AC Motor Drives*. IEEE Press, Piscataway, NJ, (1996).

[11] P. Vas. *Sensorless vector and direct torque control*. Oxford University Press, 1998.

[12] Z. Yan, C. Jin, V. Utkin. "Sensorless sliding-mode control of induction motors", *IEEE Trans. Ind. Electron.*, vol. 47, no. 6 , pp. 1286–1297, (Dec. 2000).