SIMULTANEOUS STATE AND PARAMETER ESTIMATION IN ASYNCHRONOUS MOTORS UNDER SENSORLESS SPEED CONTROL

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Abstract

This paper proposes an exact (Kalman-like) observer for the simultaneous state and parameter estimation in asynchronous motors - including speed and resistant torque estimation -, as well as a simulation study of its use under an estimate-based speed control scheme. The simulation setup includes the model of a PWM-controlled voltage source inverter, and the results are quite promising.

1 Introduction

The asynchronous motor is a multivariable, nonlinear, highly coupled process with time-varying parameters, which has motivated a lot of work in the control community during the last decade (see e.g. [5] for a recent overview). One of the challenging problems the literature has dealt with is the sensorless speed control, which implies the estimation of the mechanical speed from the only measurements of the stator currents. Another well-known problem is that electrical parameters might not be accurately known, or might significantly vary when the motor is operating, which has motivated various proposals for their identification. This results in problems of simultaneous state and parameter estimation that sometimes include *speed* estimation. Among the wide range of available contributions in this direction, one can find results for the estimation of a limited number of required electrical parameters, or a limited number of required states, or a combination of both, under more or less simplifying assumptions (see e.g. [1, 9, 12, 13, 14, 15, 17] to cite a few).

In the present paper, following previous results on simultaneous estimation of states and parameters under stator currents *and speed* measurement [2], the considered problem is that of speed control under *only stator currents measurement*, with on-line estimation of all state variables - including the speed itself (and the resistant torque) and all identifiable electrical parameters. A full description and justification of the proposed observation scheme is given, and its performances are illustrated in simulation, using a classical Park model for the motor [11], to-

gether with a commutation-based model for a PWM-controlled voltage source inverter [10] and a DTC-like control law for the speed [11, 5].

Section 2 formulates the considered problem, while section 3 presents the observer design proposed as a solution. Some corresponding simulation set-up and results are proposed in section 4, and section 5 draws the final conclusions.

2 Problem statement

Under classical simplifying assumptions (no saturation, iron losses, end-windings and slot effects), a continuous-time model of the machine can be considered in the stator reference frame (α, β) [11], with stator currents $I_{s\alpha}$, $I_{s\beta}$ and stator fluxes $\phi_{s\alpha}$, $\phi_{s\beta}$ as state variables, stator voltages $U_{s\alpha}$, $U_{s\beta}$ as input variables, and the first two state variables (stator currents) as measured outputs:

$$\dot{x} = \left[\begin{array}{cc} -\left(\frac{R_r}{\sigma L_r} + \frac{R_s}{\sigma L_s} \right) + p\Omega J & \frac{R_r}{\sigma L_s L_r} I - p\Omega \frac{1}{\sigma L_s} J \\ -R_s I & Z \end{array} \right] x$$
$$+ \left[\begin{array}{c} \frac{1}{\sigma L_s} \\ I \end{array} \right] u \qquad (1)$$

$$y = \left[\begin{array}{cc} I & Z \end{array} \right] x \qquad (2)$$

with:

$$I = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right] \quad Z = \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \end{array} \right] \quad J = \left[\begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right]$$

and where R (resp. L) stands for the resistance (resp. inductance) parameter with index s (resp. r) to refer to the stator (resp. rotor), σ is equal to $1 - \frac{M^2}{L_s L_r}$ (with M the maximum mutual inductance between a winding on the stator and one on the rotor), and p denotes the number of pairs of poles of one stator winding. The differential equation of the currents also involves the mechanical speed of the rotor Ω, whose dynamical behavior is described by:

$$\dot{\Omega} = -\frac{f_v}{J_m} \Omega + \frac{1}{J_m} p y^T J z - \frac{1}{J_m} \tau_l \qquad (3)$$

where $z = \left[\begin{array}{cc} Z & I \end{array} \right] x$ represents the stator flux. In (3), τ_l is the resistant torque, J_m is the total inertia momentum (rotor

plus load), and f_v is the viscous friction coefficient. Throughout this paper, the values of J_m and f_v will be assumed to be known, while the value of the resistant torque τ_l will be estimated as a constant parameter.

As regards the electrical parameters, notice that not all five parameters that fully characterize the electrical part of the motor (L_s, L_r, R_s, R_r, M) are identifiable from the above input-output mapping and one can only identify four parameters [3], which are in bijection with:

$$\frac{L_r}{R_r}, R_s, L_s, \sigma. \tag{4}$$

The stator currents are assumed to be the only available measurements, while the stator flux (z in the model (1)-(2)-(3)) as well as the rotor speed Ω are to be estimated. Since the model is nonlinear with respect to all these unknown variables (states and parameters), there is no systematic solution to their estimation. Usually, a general approach to this problem can be given by the so-called *extended Kalman filter*, namely a Kalman filter based on a linear approximation of the model (see e.g. [7, 8, 16] for some particular cases), but it does not guarantee the asymptotic convergence of the estimations. Instead, a global exponential observer has been recently proposed for state and parameter estimation when the speed is assumed to be available through measurement [2], based on the transformation of the considered model into a representation that is *affine with respect to all to-be-estimated variables*. Under *sufficiently exciting input signals* assumption, this representation allows the use of an *exact* Kalman-like observer.

In the present paper, it is shown how the transformation method can be extended to the additional estimation of the rotor speed Ω and the resistant torque τ_l. Moreover, in order to reproduce more realistic operating conditions, the motor is simulated together with a voltage source inverter, which in turn injects harmonics in the inputs of the motor, and can thus help in the identification process. Studying the effect of these harmonics is of particular interest when the speed is regulated close to zero, where the system is known to become unobservable if no additional excitation signal is present [6].

The employed control law is roughly a "Direct Torque Control" law (see e.g. [5]) and it will not be described here any further. Notice that it only uses estimated variables, namely it is based on the variables given by the observer, whenever such variables are required.

3 State and parameter estimation through Kalman observer

The goal in this section is to show how the nonlinear model of the asynchronous motor (1)-(2)-(3) can be transformed into a representation which is *affine with respect to the unmeasured variables*. The estimation of its states allows the retrieval of the original unmeasured states and parameters of interest (z, Ω, τ_l and the parameters in (4)). The estimation of the unmeasured states in the new representation can indeed be given by a classi-

cal Kalman observer provided the input signals are sufficiently exciting for the system.

Let us first define the following constant parameters:

$$\theta_1 := \frac{1}{\sigma T_r}; \; \theta_2 := \frac{1}{T_s}; \; \theta_3 := \frac{1}{\sigma T_r T_s}$$

$$\theta_4 := \frac{1}{\sigma L_s}; \; \theta_5 := \frac{1}{\sigma L_s L_r}; \; \theta_6 := R_s$$

$$\theta_7 := \frac{R_s}{\sigma T_s}; \; \theta_8 := \tau_l; \; \theta_9 := \tau_l \theta_2; \; \theta_{10} := \tau_l \theta_4$$

where $T_r = \frac{L_r}{R_r}$ and $T_s = \frac{L_s}{R_s}$.

Then, since the stator currents coincide with the measured outputs y, their differential equation admits the following equivalent expression:

$$\begin{aligned} \dot{y} &= -y \cdot \theta_1 - y \cdot \theta_2 + pJy \cdot \Omega + \theta_5 z \\ &\quad -pJ \cdot \Omega \theta_4 z + u \cdot \theta_4 \end{aligned} \tag{5}$$

From this, in order to get an affine representation, let us consider five new state variables: θ_1, θ_2, $\theta_5 z$, $\Omega \theta_4 z$, θ_4. Each one of them satisfies a new state equation; it is trivial that the derivative is zero if the state is constant.

As regards the speed, its state equation is:

$$\dot{\Omega} = -\frac{f_v}{J_m} \cdot \Omega + \frac{1}{J_m} py^T J \cdot z - \frac{1}{J_m} \cdot \theta_8 \tag{6}$$

which is already affine w.r.t. three states: Ω, z, and a new constant one, θ_8. The same happens for z w.r.t. θ_6 since $\dot{z}$ reads (from (1)):

$$\dot{z} = -R_s y + u = -y \cdot \theta_6 + u. \tag{7}$$

Now if we set $z_1 := \theta_5 z$ and $z_2 := \theta_4 z$, we get:

$$\dot{z_1} = \theta_5 \dot{z} = -y \cdot \theta_3 + u \cdot \theta_5 \tag{8}$$
$$\dot{z_2} = \theta_4 \dot{z} = -y \cdot \theta_2 + u \cdot \theta_4 \tag{9}$$

which are linear w.r.t. $\theta_2, \theta_3, \theta_4, \theta_5$.
We still need the differential equation of Ωz_2. We have:

$$\widehat{\dot{\Omega z_2}} = \dot{\Omega} z_2 + \Omega \dot{z_2}. \tag{10}$$

First, we compute:

$$\begin{aligned} \dot{\Omega} z_2 &= -\frac{f_v}{J_m} \Omega z_2 + \frac{1}{J_m} py^T J z z_2 - \frac{1}{J_m} \tau_l z_2 \\ &= -\frac{f_v}{J_m} \cdot \Omega z_2 \\ &\quad + \frac{p}{J_m} \left[\begin{array}{c} y(2) \cdot \theta_4 z(1)^2 - y(1) \cdot \theta_4 z(1) z(2) \\ y(2) \cdot \theta_4 z(1) z(2) - y(1) \cdot \theta_4 z(2)^2 \end{array} \right] \\ &\quad - \frac{1}{J_m} \tau_l z_2 \end{aligned} \tag{11}$$

which is linear if four new state variables are considered, namely $\theta_4 z(1)^2$, $\theta_4 z(2)^2$, $\theta_4 z(1)z(2)$ and $\tau_l z_2$. For $\theta_4 z(1)^2$ and $\theta_4 z(2)^2$ we have successively:

$$
\begin{aligned}
\widehat{\dot{\theta_4 z(1)^2}} &= \theta_4 2z(1)\dot{z}(1) \\
&= 2z(1)\theta_4[-y(1)\theta_6 + u(1)] \quad (12) \\
&= -2y(1)\cdot\theta_2 z(1) + 2u(1)\cdot\theta_4 z(1)
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\dot{\theta_4 z(2)^2}} &= \theta_4 2z(2)\dot{z}(2) \\
&= 2z(2)\theta_4[-y(2)\theta_6 + u(2)] \quad (13) \\
&= -2y(2)\cdot\theta_2 z(2) + 2u(2)\cdot\theta_4 z(2)
\end{aligned}
$$

If we define:

$$
z_3 = \theta_2 z = \begin{bmatrix} \theta_2 z(1) \\ \theta_2 z(2) \end{bmatrix}
$$

with:

$$
\dot{z}_3 = \theta_2 \dot{z} = -y\cdot\theta_7 + u\cdot\theta_2 \quad (14)
$$

then (12) and (13) are equivalent to:

$$
\begin{bmatrix} \widehat{\dot{\theta_4 z(1)^2}} \\ \widehat{\dot{\theta_4 z(2)^2}} \end{bmatrix} = 2\begin{bmatrix} u(1) & 0 \\ 0 & u(2) \end{bmatrix}\cdot z_2 \\
- 2\begin{bmatrix} y(1) & 0 \\ 0 & y(2) \end{bmatrix}\cdot z_3 \quad (15)
$$

Notice the apparition of two new state variables, z_3 and θ_7. Next, using the definitions of z_2 and z_3, the differential equations of $\theta_4 z(1)z(2)$ and $\tau_l z_2$ are respectively:

$$
\begin{aligned}
\widehat{\dot{\theta_4 z(1)z(2)}} &= \theta_4 \dot{z}(1)z(2) + \theta_4 z(1)\dot{z}(2) \quad (16) \\
&= -y(1)\cdot\theta_2 z(2) + u(1)\cdot\theta_4 z(2) \\
&\quad -y(2)\cdot\theta_2 z(1) + u(2)\cdot\theta_4 z(1) \\
&= [\; u(2) \quad u(1)\;]\cdot z_2 - [\; y(2) \quad y(1)\;]\cdot z_3
\end{aligned}
$$

and:

$$
\widehat{\dot{\tau_l z_2}} = \tau_l \dot{z}_2 = -y\cdot\theta_9 + u\cdot\theta_{10} \quad (17)
$$

Equation (17) introduces two new constant states, θ_9 and θ_{10}. Finally, for $\Omega\dot{z}_2$ we have:

$$
\Omega\dot{z}_2 = -y\cdot\theta_2\Omega + u\cdot\theta_4\Omega \quad (18)
$$

which completes the set of the new state variables with $\theta_2\Omega$ and $\theta_4\Omega$. Their differential equations are:

$$
\begin{aligned}
\widehat{\dot{\theta_2\Omega}} &= \theta_2\dot{\Omega} \quad (19) \\
&= -\frac{f_v}{J_m}\Omega\theta_2 + \frac{1}{J_m}py^T Jz\theta_2 - \frac{1}{J_m}\tau_l\theta_2 \\
&= -\frac{f_v}{J_m}\cdot\theta_2\Omega + \frac{1}{J_m}py^T J\cdot z_3 - \frac{1}{J_m}\cdot\theta_9
\end{aligned}
$$

and

$$
\begin{aligned}
\widehat{\dot{\theta_4\Omega}} &= \theta_4\dot{\Omega} \quad (20) \\
&= -\frac{f_v}{J_m}\Omega\theta_4 + \frac{1}{J_m}py^T Jz\theta_4 - \frac{1}{J_m}\tau_l\theta_4 \\
&= -\frac{f_v}{J_m}\cdot\theta_4\Omega + \frac{1}{J_m}py^T J\cdot z_2 - \frac{1}{J_m}\cdot\theta_{10}
\end{aligned}
$$

which indeed only depend on already defined state variables.

Equations (5)…(20), and the differential equations of the constant states give a new representation of the system, which is now *affine* w.r.t. the new state vector X, in the following sense:

$$
\begin{aligned}
\dot{X} &= A(u,y)X + Bu \quad (21) \\
y &= CX \quad (22)
\end{aligned}
$$

Some elements in the new state vector X are made up of two components (y, z, $z_{1...3}$, Ωz_2, $\tau_l z_2$), while all others are scalars (Ω, $\theta_4 z(1)^2$, $\theta_4 z(2)^2$, $\theta_4 z(1)z(2)$, $\theta_2\Omega$, $\theta_4\Omega$, $\theta_{1...7}$, τ_l, $\tau_l\theta_2$, $\tau_l\theta_4$). In order to estimate all these state variables, one idea is to design a Kalman observer as in (23) and (24) below, with $S(0) = I$, ensuring arbitrarily fast exponential convergence by modifying the tuning parameter $\lambda > 0$, under usual excitation conditions, for any $\hat{X}(0)$ [4].

$$
\begin{aligned}
\dot{\hat{X}} &= A(u,y)\hat{X} + Bu - S^{-1}C^T(C\hat{X} - y) \quad (23) \\
\dot{S} &= -\lambda S - A^T(u,y)S - SA(u,y) + C^T C \quad (24)
\end{aligned}
$$

The main advantage when using this kind of observer is that there is *no need to perform any linear approximation*, which guarantees the global convergence of the estimator under sufficiently exciting signals. However, a drawback is that the observer performs *redundant* estimations. The order of the resulting system is indeed thirty, that is six times greater than the order of the original one. Consequently, there are more than one possibilities to recover the original states and the parameters of interest. For instance, in order to recover the parameters, one could use any of the following solutions:

$$
L_s = \frac{\theta_1}{\theta_5}; \; R_s = \frac{\theta_2}{\theta_4}; \; \sigma = \frac{1}{L_s\theta_4}; \; T_r = \frac{1}{\sigma\theta_1};
$$

$$
R_s = \frac{\theta_3}{\theta_5}; \; L_s = \frac{R_s\theta_1}{\theta_3}; \; \sigma = \frac{R_s}{L_s\theta_2}; \; T_r = \frac{1}{\sigma\theta_1};
$$

$$
R_s = \theta_6; \; T_r = \frac{\theta_7}{R_s\theta_3}; \; \sigma = \frac{1}{T_r\theta_1}; \; L_s = \frac{1}{\sigma\theta_4}
$$

4   Simulation results

The simulations were performed with Matlab in the continuous time domain. The response of the asynchronous motor was simulated through (1) and (3), with the electrical parameters chosen as:

$$
L_r = 0.0323H; \; L_s = 0.0317H; \; M = 0.031;
$$

$$
R_r = 0.052\Omega; \; R_s = 0.07\Omega,
$$

and the mechanical parameters as:

$$
J_m = 2kg\cdot m^2; \; f_v = 0.003.
$$

The mechanical parameters were assumed to be available at any time, and thus they were directly used in the estimation process. Instead, for the electrical parameters, significant errors with respect to the real values were simulated in the initial guesses.

The input of the model was a PWM-like voltage waveform generated by the torque/flux controller through the ideal model of a voltage source inverter. As regards the set points for the electromagnetic torque and squared norm of the flux, the first one was generated by the speed controller, while the second one was set to $1Wb$. Two simulations were performed: in the first one, the speed was driven from zero to $50rad/s$ and back under the action of an *active* resistant torque. In the second one, the speed was set to zero, in the absence of any resistant torque. Obviously, due to the initial estimation errors, the voltage initially applied to the motor is not the appropriate one, and the purpose here was to check in simulation how this voltage can make the estimations to converge towards their actual values (*sufficient excitation*), and in turn make the control itself to tend towards the appropriate one.

In order to emphasize the efficiency of the method, the initial errors on the electrical parameters were chosen to be quite large:

$$L_r : -80\%; \; L_s : -50\%; \; M : +70\%;$$

$$R_r : +100\%; \; R_s : +80\%$$

while for the first simulation the initial guess for the resistant torque was $7.5Nm$ (50% off the actual value). The same value was used for the second simulation, where the real value was $0Nm$, as already stated.

The corresponding results in the presence of a constant resistant torque are shown to be fairly good in figures 1 to 3, where it can be seen that all estimation errors go to zero. Notice the accurate speed estimation during the second transitory state ($t = 4 \ldots 4.5s$).
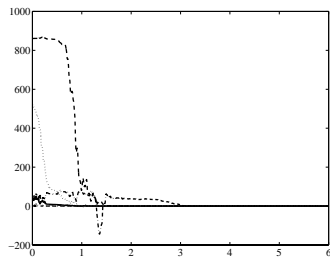

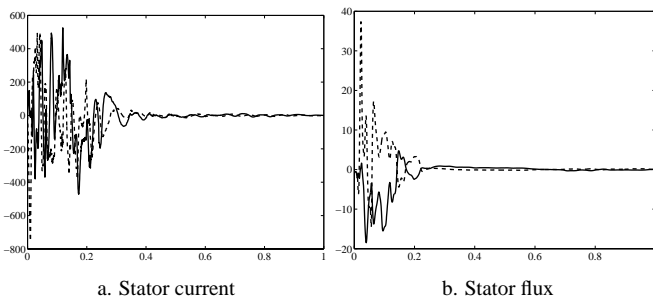Figure 1: Auxiliary parameter $\theta_{1 \ldots 7}$ estimation errors

a. Stator current   b. Stator flux

Figure 2: State estimation errors in the electrical equation

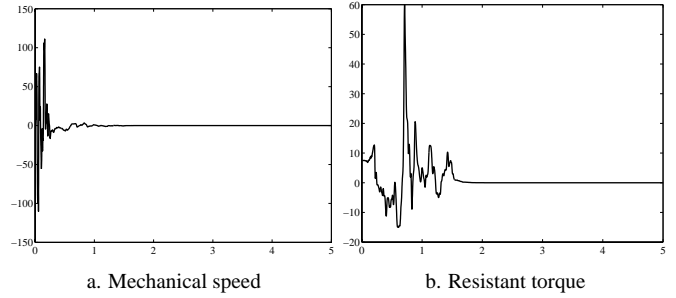a. Mechanical speed   b. Resistant torque

Figure 3: State estimation errors in the mechanical equation

In the same time, it can be checked on figures 4 and 5 that the control achieves the desired tracking (with some oscillations during the first transitory state, which can be of large magnitude due to the large initial errors on the parameters).
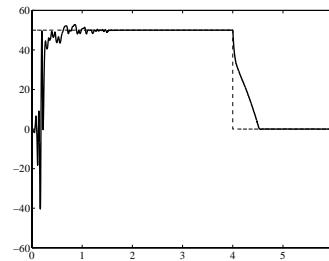

Figure 4: Speed tracking

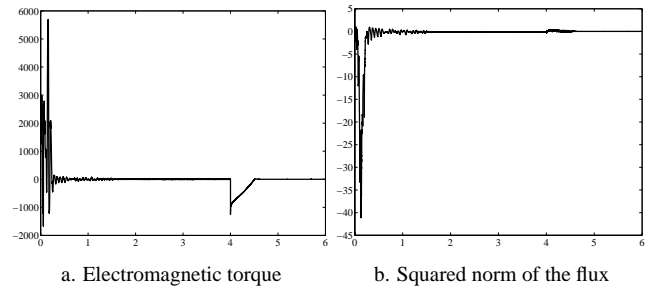a. Electromagnetic torque   b. Squared norm of the flux

Figure 5: Tracking errors in the torque/flux controller

Keeping the motor at standstill is an issue of particular interest in the considered scheme, since the asynchronous motor is known to become "unobservable at zero speed" when it does not produce any electromagnetic torque (notice that in a speed control loop this condition implies the absence of an external torque acting upon the rotor). This is an actual consequence of the physics of such motors: if there is no relative speed between the stator flux and the rotor, the induction of currents in the last one cannot take place, meaning no electromagnetic torque and no information from the rotor side in the stator currents. See [6] for a theoretical observability study focused on the induction motor.

In our case, as long as there is an external torque acting upon the rotor, the machine produces an electromagnetic torque and the speed oscillates around zero (fig. 6a) with amplitudes large enough to ensure that the system remains observable. As a difference, in the absence of the resistant torque the speed approaches zero (fig. 6b), no electromagnetic torque is required, and the observability conditions are no longer fulfilled. As a consequence, estimations that had previously
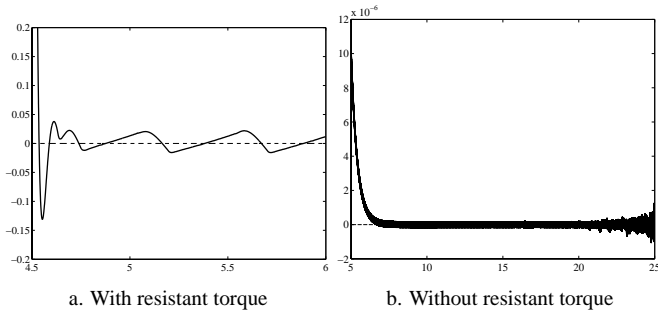
Since from those simulations, the harmonics in the PWM voltage waveform are not enough to keep the system observable at standstill in the absence of a resistant torque, a possible solution can be the use of an additional excitation in the control signal: simulations indeed show that a small magnitude excitation signal is enough to make the speed to oscillate around zero and prevent the system from entering an unobservable state (see fig. 9).

| a. With resistant torque | b. Without resistant torque |

Figure 6: Speed tracking at standstill

Figure 9: Speed tracking at standstill with additional excitation signals

converged towards the corresponding actual values, start to worsen, and at some point a divergence occurs in the auxiliary parameter estimations (fig. 7). The errors become large enough to determine the speed to deviate from the set point. As the rotor turns, the motor produces an electromagnetic torque, thus it becomes observable and the estimation errors as well as the speed converge again towards zero (fig. 8).
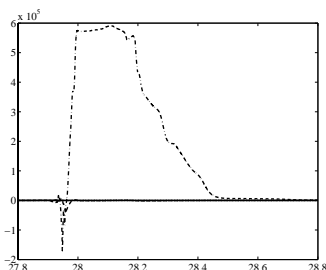
5   Conclusions

In this paper an exact and global method to estimate the flux, the mechanical speed, the resistant torque, as well as all the identifiable electrical parameters was proposed. This method gives accurate estimates provided that the motor is "excited enough", and it was shown in simulation how this can indeed be the case in practice-like operating conditions, due to the presence of the inverter feeding the motor. In particular, simulations have been performed under estimate-based control, and the problem of regulation at zero speed has been considered.

Figure 7: Auxiliary parameter estimation errors at standstill with no resistant torque (detail)

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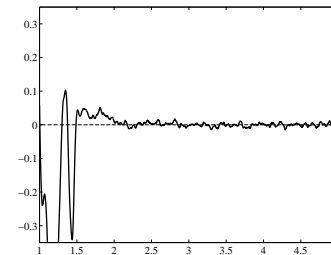[1] K. Akatsu, A. Kawamura. Sensorless very low-speed and zero-speed estimations with online rotor resistance estimation of induction motor without signal injection *IEEE Transaction on Industry Applications*, **36(3)**, pp. 764-771, (2000).

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Figure 8: Speed tracking with no resistant torque

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