## ADAPTIVE NONLINEAR CONTROL OF A DISTRIBUTED COLLECTOR SOLAR FIELD

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## Abstract

This paper is concerned with an application of nonlinear adaptive techniques to temperature control in a distributed collector solar field. In the approach followed, the partial differential equation describing the field is approximated by a lumped parameter bi-linear model, whose states are the temperature values along the field. By using feedback exact linearisation together with a Lyapunov's approach, an adaptive controller is designed. This paper improves on previous work by using a better approximated model which takes into account that, in the field considered, temperature measures are only made at the input and at the output and not along the pipe. The design based on the improved simplified model allows faster convergence of parameter estimate and improved transient responde. The advantages of the new algorithm proposed are illustrated by means of simulations performed in a detailed physical model of the plant. (c) IEE - ECC 2003.

## **1** Introduction

This paper is concerned with an application of nonlinear adaptive techniques to temperature control in a distributed collector solar field. Standard methods for nonlinear control design are used. The contribution of the paper stands on showing that an improved simplified plant model, which comply with the actual sensor positions, can be used to achieve a significantly increase of performance with respect to [1].

#### 1.1 The plant.

Distributed collector solar fields are spatially distributed engineering systems which aim at collecting and storing energy from solar radiation. They are formed by mirrors which concentrate direct incident sun light in a pipe where an oil able to accumulate thermic energy flows. The oil flowing in the pipe is an incompressible fluid, able to support temperatures up to  $300^{\circ}C$ . Since this oil is a very poor thermal conductor, heat diffusion effects in it may be neglected. The oil is extracted at low temperature from the bottom of a storage tank, passed through the field where it is heated by solar radiation and returns to the tank, where it is injected at the top.

#### 1.2 The control problem.

The control objective considered in this paper consists of making the average of the loop outlet oil temperatures (hereafter simply referred as "outlet oil temperature") to track a reference signal by manipulating the oil flow in the presence of fast acting disturbances caused by passing clouds. Other main disturbances are changes in radiation due to atmosphere scattered water steam, in the temperature of the inlet oil coming from the bottom of the storage tank and in ambient temperature. Dust deposition and other factors such as wind, changing collectors shape, also act as disturbances because they alter mirror reflectivity, calling for adaptive methods.

#### **1.3 State of the art review.**

The above control problem may not adequately be solvable with a constant gain linear controller relying on a simple design. [1] provides an example in which a PID controller tuned for higher values of the flow (low temperatures) yields unacceptable oscillatory responses in set-point changes. This motivated research on more sophisticated controllers of which [1, 3, 4, 6, 7, 8, 9, 10, 12] are significant examples.

While adaptive control already provides some form of accommodation of nonlinear behavior by adjusting the controller gains according to the operating point, explicit recognition of plant nonlinearities and their exploitation is expected to lead to performance and robust stability improvements. First steps in this direction were made by employing gain scheduled constant parameter GPC [4] and switched multiple model supervisory controllers [9]. In [1] a nonlinear controller is developed which explicitly takes into account the distributed parameter model of the solar plant.

## 2 Field models

The solar field considered is a spatially distributed system and, as such, it is best described by a partial differential equation. From this distributed parameter model a bilinear state-space model is obtained.

#### 2.1 Distributed parameter model.

Each of the field loops is approximately modelled by the PDE

$$\frac{\partial}{\partial t}T(z,t) = -\frac{1}{S}F(t)\frac{\partial}{\partial z}T(z,t) + \alpha R(t)$$
(1)

where T(z, t) is the increment with respect to the ambient temperature of the oil temperature at location z (measured along the pipe) and at time t, F(t) is the oil flow, R(t) is the corrected solar radiation, S is the area of the inner cross section of the pipe and  $\alpha$  is a parameter measuring collector optical efficiency. On the r. h. s. of eq. (1) the first term (first order space derivative) reflects the change of temperature due to oil flow. The second term models oil heating by solar radiation and the third term models losses along the pipe. No losses or diffusion effects are assumed. Temperature measurements are made at z = 0 and z = L (beginning and end of the active part of the loops).

#### 2.2 Bilinear model.

Assuming a smooth variation of oil temperature along the pipe, it is possible to approximate the temperature distribution by a piecewise linear curve, so that the following finite difference approximation holds:

$$\frac{\partial T}{\partial z}\Big|_{z\in(z_{i-1},z_i]} \cong \frac{T_i - T_{i-1}}{h}, \quad i = 1,\dots, n$$
 (2)

where h is the length of each segment, n is the number of segments,  $z_i = ih$ , L = nh is the pipe length and  $T_i = T(ih, t) - T(0, t)$ .

Defining the state variables

$$x_i(t) = T_i(t) \quad i = 1, \dots, n \tag{3}$$

process dynamics is thus approximately described by the system of nonlinear ordinary differential equations:

$$\dot{x}_i = -u \frac{1}{h} (x_i - x_{i-1}) + \alpha R, \quad i = 1, \dots, n$$
 (4)

where the dot denotes derivative with respect to time t and  $x_0 = 0$ .

Defining the state  $x = [x_1 \dots x_n]^T$  and the vector fields

$$f(x) = \alpha R \begin{bmatrix} 1\\1\\\vdots\\1 \end{bmatrix} \qquad g(x) = -\frac{1}{h} \begin{bmatrix} x_1\\x_2 - x_1\\\vdots\\x_n - x_{n-1} \end{bmatrix}$$

system (4) is written in the form

$$\dot{x} = f(x) + g(x)u \tag{5}$$

It is remarked that f(x) is actually independent of x and, in particular,  $f(0) \neq 0$ . Furthermore, the field g(x) may be written as

$$g(x) = Bx$$

with matrix B given by

$$B = -\frac{1}{h} \begin{bmatrix} 1 & 0 & \dots & 0\\ -1 & \ddots & & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \dots & -1 & 1 \end{bmatrix}$$

For n high enough, the piecewise linear approximation of the spatial distribution of temperature is acceptable and model (4) (or, equivalently, (5)) describes reasonably well the transport and heating phenomena inside the pipe.

## **3** Approximate feedback linearization

In order to design a temperature controller for the field, an approach relying on the following steps is considered [11]:

- First a control transformation is performed in order to linearize the model. The resulting model is driven by the parameter estimation error of mirror reflectivity.
- An adaptation rule is obtained by minimizing a joint Lyapunov function for control and parameter estimation.

#### 3.1 Feedback linearization.

Consider the system defined by (4) with the output defined by

$$y = h(x) = x_n \tag{6}$$

The relative degree is given by the number of times the output y has to be differentiated at a given time  $t_0$  so that  $u(t_0)$  explicitly appears. Differentiating the output given by (6) one obtains:

$$\dot{y} = \dot{x}_n = \alpha R - \frac{x_n - x_{n-1}}{h}u$$

and it is concluded that the relative degree of this system is r = 1. The linearizing control law is given by

$$u = \frac{-L_f h(x) + v}{L_g h(x)} = \frac{\alpha R - v}{x_n - x_{n-1}}h$$
(7)

where v is the input of the linearised system, hereafter called the "virtual control".

Eq. (7) provides a transformation such that, from the transformed input v to the output y, the dynamics reduces to a pure integrator. It is remarked that this computation requires the values of the states  $x_n$  and  $x_{n-1}$  which must be available for measurement. The measure of  $x_{n-1}$  is not available in the plant considered. According to the approach of [1], the temperature  $x_{n-1}$  is replaced by the inlet oil temperature, T(0, t). The effect of this approximation is shown in Example 1.

#### 3.2 Example 1 – Feedback linearization.

Fig. 1 shows the time evolution of the outlet oil temperature when the feedback linearising transform (7) is applied. Since the virtual input v (top of fig. 1) is a square wave, and the linearised system is an integrator, the output will be a triangular wave, as seen on the bottom of fig. 1. In this case,  $x_{n-1}$  is the temperature 1 m before the end of the pipe.

Consider now the case in which the exact linearising control is approximated by replacing  $x_{n-1}$  with the inlet oil temperature, T(0,t). The result is seen on fig. 2. As is apparent,



Figure 1. Example 1: Exact feedback linearization.



Figure 2. Example 1: Approximate feedback linearization.

the edges of the triangular wave are not as sharp. The system is better approximated by a first order filter in series with an integrator. This key feature will be explored below.

#### **3.3** Adaptive control with an approximate model.

In [1] the following simplified model is assumed:

$$\dot{y} = -u(y - y_0)\frac{1}{L} + \alpha R \tag{8}$$

where y is the outlet oil temperature,  $y_0$  is the inlet oil temperature and u is the oil flow velocity. This model may be interpreted by taking n = 1.

An adaptive control law is yielded by considering a quadratic candidate Lyapunov function, amounting to the following equations:

Virtual control computation:

$$v = k_p e - k_d \dot{y} \tag{9}$$

where  $k_p$  and  $k_d$  are controller gains and e is the tracking error given by:

$$e(t) = r - y(t) \tag{10}$$



Figure 3. Example 2: Adaptive control with approximate feedback linearization,  $\gamma = 2 \times 10^{-10}$ . Output temperature.

*r* being a constant reference to track. Parameter adaptation:

$$\dot{\hat{\alpha}} = -\frac{\gamma}{1+k_d}Re\tag{11}$$

Actual control applied to the plant:

$$u = \frac{\hat{\alpha}R - v}{y - y_0}L\tag{12}$$

where L = 180 is the length of the active part of the field.

## **3.4 Example 2 – Adaptive control with approximate feedback linearization.**

Example 2 presents a simulation of the above described adaptive controller, in which feedback linearization is approximated by using the inlet oil temperature instead of the temperature measure located at L/n before the pipe outlet. This example is presented as a base-line to which other algorithms can be compared. Fig. 3 shows the oil output temperature, and fig. 4 shows  $\hat{\alpha}$ . The value of the adaptation gain is  $\gamma = 2 \times 10^{-10}$ .

As seen in fig. 4, the adaptation of  $\hat{\alpha}$  is slow and sensitive to the tracking error *e*. This results in noticeable overshoot when the reference changes. Increasing the adaptation gain  $\gamma$ does not improve the situation. The speed of convergence of the estimate  $\hat{\alpha}$  does not improve in a significant way but the response of the output temperature becomes more oscillatory. This is such that further increases on the value of  $\gamma$  may result in an unstable behavior (fig. 5).

## 4 Control with an improved model

If the exact linearizing control transformation (7) is applied to the bilinear model given by (4), the relation between the virtual input v and the output y becomes an integrator. As shown in Example 1, if the input oil temperature is used in the linearizing transform instead of  $x_{n-1}$ , this results in an extra filtering. Therefore, when (12) is applied to the plant, this results in the following model for the relation between v and y:

$$\tau \ddot{y} + \dot{y} = v + R\tilde{\alpha} \tag{13}$$



Figure 4. Example 2: Adaptive control with approximate feedback linearization,  $\gamma = 2 \times 10^{-10}$ .  $\hat{\alpha}$ .



Figure 5. Example 2: Adaptive control with approximate feedback linearization,  $\gamma = 10 \times 10^{-10}$ . Output temperature. The effect of further increasing  $\gamma$ .

where

$$\tilde{\alpha} := \alpha - \hat{\alpha} \tag{14}$$

is the estimation error of the optical efficiency parameter  $\alpha$ . The time constant  $\tau$  is a parameter modelling the filtering effect seen on example 1 and R is solar radiation.

Assume that a PD defined by (9) is used to generate the virtual input v. Use (9) in (13) to get the following equation for the closed loop system:

$$\frac{\tau}{k_p}\ddot{y} + \frac{k_d + 1}{k_p}\dot{y} + y = r + \frac{1}{k_p}R\tilde{\alpha}$$
(15)

In order to obtain an adaptation rule for the estimate  $\hat{\alpha}$  define the reference model

$$\tau_m^2 \ddot{y}_m + 2\xi_m \tau_m \dot{y}_m + y_m = r \tag{16}$$

where  $y_m$  is the output of the reference model and  $\tau_m$  and  $\xi_m$  are parameters to select, and the error of the actual output with respect to the output of the reference model

$$e_m = y_m - y \tag{17}$$

Let the parameters in the reference model be selected as

$$a := \tau_m^2 = \frac{k_d + 1}{k_p} \quad b := 2\xi_m \tau_m = \frac{k_d + 1}{k_p}$$
(18)

As may be shown using standard arguments [2], the minimization of a suitable Lyapunov function yields the following adaptation law:

$$\dot{\hat{\alpha}} = -k_1 R \dot{e}_m - k_2 R e_m \tag{19}$$

in which

$$k_1 = -\frac{2\gamma\tau}{(k_d+1)^2}$$
  $k_2 = -\frac{\gamma}{k_d+1}$  (20)

The term  $-k_1 R\dot{e}_m$  causes a stabilizing effect, allowing to improve the velocity of convergence and reducing sensitivity of the estimate to the transients of e. By making  $\tau = 0$  the adaptation law (11) of [1] is recovered. Furthermore, the adaptation law (19) can be also obtained by using an argument based on passivity.

#### 4.1 Example 3 – Passivity based adaptation.

Example 3 shows the results obtained with the above adaptive control algorithm, based on an improved model. Figs. 6 and 7 show, respectively, the oil output temperature and  $\hat{\alpha}$ . Compare fig. 7 with fig. 4. As seen in these figures, with the algorithm based on the improved model, convergence of  $\hat{\alpha}$  is much faster and this estimate is much less sensitive to *e*.

#### 4.2 Example 3 – Passivity based adaptation: Large set-point jumps.

Example 3 is concerned with the performance of the modified adaptive controller in the presence of large set point changes. Since  $\hat{\alpha}$  is much less sensitive to *e*, a sudden set point change of 50° can be performed with acceptable overshoot, as seen in fig. 8.



Figure 6. Example 3: Passivity based adaptation. Output temperature.



Figure 7. Example 3: Passivity based adaptation.  $\hat{\alpha}$ .



Figure 8. Example 4: Passivity based adaptation with large set-point jumps. Output temperature.



Figure 9. Example 5: Passivity based adaptation: Fast disturbance rejection. Output temperature.



Figure 10. Example 5: Passivity based adaptation: Fast disturbance rejection. Radiation R divided by 100, control variable u and virtual control v.

# 4.3 Example 5 – Passivity based adaptation: Fast disturbance rejection.

Example 3 is concerned with the performance of the modified adaptive controller in the rejection of fast, large, disturbances cause by passing clouds. Results are shown in figs. 9-11. As seen in fig. 10, at 4.5 h after the begining of the experiment, a cloud causes a drastic drop in solar radiation, reducing it to about half its value in a short period of time. Feedback action reduces the manipulated variable u in order for the oil to be heated for a longer time, and the disturbance is rejected. As seen in fig. 11 the occurrence of the disturbance causes little impact on the adaptation of  $\hat{\alpha}$ .

## 5 Conclusions

Nonlinear adaptive control of distributed collector solar fields is considered assuming that there are no temperature measurements taken along the pipe where the oil to be heated flows.



Figure 11. Example 5: Passivity based adaptation: Fast disturbance rejection.  $\hat{\alpha}$ .

This is a common technological constraint, present *e. g.* in the ACUREX field of Plataforma Solar de Almeria (Spain). An algorithm able to cope with this limitation is derived and shown, by simulation in a detailed physical model of the plant, to improve the results of [1], still preserving the simplicity of modelling.

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