SMOOTH VARIABLE STRUCTURE OBSERVER CONTROLLER WITH ADAPTIVE GAINS: APPLICATION TO ROBOT MANIPULATORS CONTROL

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Abstract

In this paper, an observer based controller is developed, both of them working in sliding mode, applied to the square MIMO non-linear systems control. An application to the rigid robots with n links and n revolute joints (n-degree of freedom) is presented. In order to reduce the chattering, a parameterised smooth switching function is used: the tangent hyperbolic function both in the observer and controller. The gain of the switching functions is adaptively updated, depending on the state estimation error and tracking error, respectively. Using the adaptive gains, the transient and tracking response are improved. Closed loop simulations with a 2-degree of freedom robot manipulator, in the presence of parameter uncertainties, are presented in order to show the robustness of the approach.

1 Why adaptive gain smooth variable structure control?

The structured (parameter) and/or unstructured uncertainties of MIMO nonlinear systems lead to difficulties in parameter identification. Therefore, the controller and/or the observer have to be designed in order to assure closed loop robustness. The robustness to uncertainties and external disturbances of the closed loop with a variable structure controller is well known. Maintaining the system on a sliding surface weakens the influence of the uncertainties in the closed loop and quickly leads to an equilibrium point. In [3], an adaptive variable structure controller with parameterised sigmoid switching function (denoted k-sigmoid) and adaptive gain (denoted λ-modification), instead of a pure relay with constant gain, is proposed. In this paper, a parameterised hyperbolic function (denoted k-tanh) is used as a switching function. By using k-tanh as the switching function, the chattering can be alleviated. The chattering is a high frequency oscillation on the control input, and/or on the state and on the plant output. When decreasing the parameter k in the switching function, the gain around zero becomes smaller and the un-modelled dynamics are less excited at high frequency. Also, the delay due to the control input calculus and the finite rate of switching can lead to chattering. Using the λ-modification into the gain of the k-tanh switching function smoothes the response and increases the robustness to parameter uncertainties and external disturbances. The adaptive gain is time varying, with the norm of the corresponding sliding surface as an input. The sliding surface can be the estimation error of measurable states for the observer and the tracking error for the controller.

We develop a variable structure observer-controller based on the works [7, 9]. Extension of sliding control to MIMO non-linear systems has been studied in [4, 10]. Application to robot manipulators has been presented in [2, 8]. Extension to MIMO non-linear affine systems can be found in [5].

With the k-tanh switching function and the λ-modification in the observer-controller gains, the closed-loop system behaves as an approximate sliding mode, in the neighbourhood of the corresponding sliding surface.

2 Adaptive gain smooth sliding observer for square MIMO nonlinear systems

The considered square MIMO non-linear system is:

$$\begin{aligned} &\dot{x}_1 = x_2; \quad x_1 \in \Re^n \\ &\dot{x}_2 = h(x_1, p)^{-1}\left[f(x_1, x_2) + g(x_1, x_2)u\right] \\ &y = x_1 \quad x_2 \in \Re^n \quad u \in \Re^n, \end{aligned} \tag{1}$$

where only the vector x_1 is available for measurement, u and y are control input and measured output, respectively. The functions f g and h may be partially unknown, with some parameter uncertainties. If one assumes the partial knowledge of the model parameters, then one can define $\hat{h}$,

$\hat{f}$ and $\hat{g}$ as the estimates of the functions h, f and g. Moreover, the system is assumed feedback linearizable, ($g(x_1, x_2) \neq 0$ and $\hat{g}(x_1, x_2) \neq 0$ for all x).

This is the most general model of the robotic manipulator. The physical robot may have gears and clutches, inside the joint, through the torque, supplied by the DC motor, is transmitted in order to move the arm.

Considering $S_o = \hat{x}_1 - x_1 = 0$ as the observer sliding surface and the k-tanh as switching function, the sliding observer can be written as

$$\begin{aligned} \dot{\hat{x}}_1 &= -\Gamma_1(\hat{x}_1 - x_1) + \Theta_1(t)\tanh(k_o S_o) + \hat{x}_2 \\ \dot{\hat{x}}_2 &= -\Gamma_2(\hat{x}_1 - x_1) + \Theta_2(t)\tanh(k_o S_o) \\ &+ \hat{h}(x_1)^{-1}\left[\hat{f}(x_1, \hat{x}_2) + \hat{g}(x_1, \hat{x}_2)u(x_1, \hat{x}_2)\right] \end{aligned} \tag{2}$$

where

$$\Gamma_1 = \mathrm{diag}[\gamma_{11} \cdots \gamma_{1n}],\ \Gamma_2 = \mathrm{diag}[\gamma_{21} \cdots \gamma_{2n}] \tag{3}$$

with $\gamma_{ij} > 0, i = 1,2$ and $j = 1, n$. $k_o > 0$ is a design parameter. The gains of the switching function

$$\Theta_1 = \mathrm{diag}[\theta_{11} \cdots \theta_{1n}],\ \Theta_2 = \mathrm{diag}[\theta_{21} \cdots \theta_{21}] \tag{4}$$

are time varying and defined by (the λ-modification included)

$$\dot{\Theta}_1(t) = -\lambda_1\Theta_1(t) - \rho_1 \mathrm{diag}[|\hat{x}_{11} - x_{11}| \cdots |\hat{x}_{1n} - x_{1n}|] \tag{5}$$

$$\dot{\Theta}_2(t) = -\lambda_2\Theta_2(t) - \rho_2 \mathrm{diag}[|\hat{x}_{11} - x_{11}| \cdots |\hat{x}_{1n} - x_{1n}|] \tag{6}$$

where $\lambda_1 = \mathrm{diag}[\lambda_{11} \cdots \lambda_{1n}]$, $\lambda_2 = \mathrm{diag}[\lambda_{21} \cdots \lambda_{2n}]$, $\rho_1 = \mathrm{diag}[\rho_{11} \cdots \rho_{1n}]$, $\rho_2 = \mathrm{diag}[\rho_{21} \cdots \rho_{2n}]$, with λ_{1i}, λ_{2i}, ρ_{1i} ρ_{2i}, $i = 1, \cdots, n$ positive constants.

Remark 1. The dynamics (5) and (6) of the switching gains force the matrices Θ_1 and Θ_2 to negative values. They are almost zero if the estimation error $\hat{x}_1 - x_1$ is almost zero. The negative values of the matrices Θ_1 and Θ_2, given by the dynamics (5) and (6), lead to the changed sign in the observer equation (2). They become almost zero whilst the observer evolutes in the neighbourhood of the sliding surface.

Remark 2. In order to satisfy the attractiveness condition $\dot{\hat{S}}_{oi}\hat{S}_{oi} < 0,\quad i = 1, \ldots, n$, the gain Θ_1 must be chosen such that

$$-\theta_{1i}(t) > |\hat{x}_{2i}(t) - x_{2i}(t)|,\ i = 1, \ldots, n,\ \forall t \in [t_o \quad \infty) \tag{7}$$

By an appropriate choice of the matrices λ_1 and ρ_1, the above condition at $t = t_o$ remains satisfied for any $t \geq t_o \geq 0$.

3. Smooth sliding controller with adaptive gain

The sliding surface, corresponding to the n-dimensional control input, is defined as

$$\hat{S}_c(\hat{x}, t) = \hat{x}_2(t) - \dot{y}_r(t) + \psi(x_1(t) - y_r(t)) \tag{8}$$

where $y_r(t)$ represents the trajectory to be tracked. The matrix $\psi = \mathrm{diag}\{\psi_1, \ldots, \psi_n\}$, with positive constants ψ_i, $i = 1, \cdots, n$ determines the dynamics in the sliding mode. The controller is defined assuming that the state x_1 is known and that the state x_2 is provided by the observer. The sliding surface is attractive if the following condition holds

$$\dot{\hat{S}}_{ci}\hat{S}_{ci} < 0, \quad i = 1, \ldots, n \tag{9}$$

The time derivative of the sliding surface can be expressed as

$$\begin{aligned} \dot{\hat{S}}_c &= \dot{\hat{x}}_2 - \ddot{y}_{1r} + \psi(\hat{x}_2 - \dot{y}_r) \\ &= \hat{h}(x_1)^{-1}\left[\hat{f}(x_1, \hat{x}_2) + \hat{g}(x_1, \hat{x}_2)\hat{u}(x_1, \hat{x}_2)\right] \\ &- \ddot{y}_r + \psi(\hat{x}_2 - \dot{y}_r) \end{aligned} \tag{10}$$

To fulfil the sliding condition ($\dot{\hat{S}}_c = 0$), the controller has to be expressed as follows

$$\begin{aligned} \hat{u} &= -\hat{f}(x_1, \hat{x}_2) + \hat{g}^{-1}(x_1, \hat{x}_2)\hat{h}(x_1) \\ &\left[-\psi\hat{S}_c + \eta(t)\tanh(k_c\hat{S}_c) + \ddot{y}_r - \psi(\hat{x}_2 - \dot{y}_r)\right] \end{aligned} \tag{11}$$

where the gain of the switching function, $\eta = \mathrm{diag}[\eta_1 \quad \cdots \quad \eta_n]$ is also time varying (the λ-modification included)

$$\dot{\eta}(t) = -\lambda_c\eta(t) - \rho_c \mathrm{diag}\left[|\hat{S}_{c1}| \quad \cdots \quad |\hat{S}_{cn}|\right] \tag{12}$$

with $\lambda_c = \mathrm{diag}[\lambda_{c1} \cdots \lambda_{cn}]$, $\rho_c = \mathrm{diag}[\rho_{c1} \quad \rho_{cn}]$ and $\lambda_{ci}, \rho_{ci} > 0,\ i = 1, n$.

As in [5] and [6], the term $-\psi\hat{S}_c$ is introduced to reduce the controller to a classical feedback linearization one if the switching term is set to zero.

Remark 3. The observer error is nonzero if the k-tanh function is used as a switching function in the observer equations. The controller sliding surface $\hat{S}_c$ can still be attractive by choosing sufficiently large initial values for the switching gains Θ_1 and Θ_2. Moreover, the tracking error does not go to zero on the controller sliding surface, due to the smooth controller (k -tanh switching function).

Remark 4. In order to reduce the influence of the velocity estimation error in the control input, the relative weight of the states $\hat{x}_2$ in the definition of the sliding surface should be decreased. This explains the introduction of the supplementary term $-\psi\hat{S}_c$ in the control input. The increase of the parameter ψ is limited by the switching frequency and possible measurement noise.

4. Smooth sliding observer and controller with adaptive gains applied to a robot manipulator

Consider the n-degree of freedom robot manipulator model:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + V_F\dot{q} + G(q) = u \tag{13}$$

where $q = [q_1 \ \dots \ q_n]^T$ is the vector of link angles, $H(q) \in \Re^{nxn}$ is the positive definite inertia matrix, $C(q,\dot{q}) \in \Re^{nxn}$ is the Coriolis and centripetal force matrix, $V_F \in \Re^{nxn}$ is the positive semi-definite diagonal matrix with the viscous friction coefficients, u is the vector of driving torques. Define y as the measurements vector. The positions of the robot links, $q_i, i = 1,\dots,n$, are the elements of y. Define the state $q = x_1 = [x_{11} \cdots \ x_{1n}]$, $\dot{q} = x_2 = [x_{21} \dots \ x_{2n}]$, the angular position and velocity vector, respectively. The state space representation can written as:

$$\begin{aligned}&\dot{x}_1 = x_2\\&\dot{x}_2 = -H(x_1)^{-1}[C(x_1,x_2)x_2 + +G(x_1) + V_F x_2 - u]\\&y = x_1\end{aligned} \tag{14}$$

Taking into account the parameter uncertainties and the presence of observer, $\hat{H}(x_1)$, $\hat{C}(x_1,\hat{x}_2)$, $\hat{G}(x_1)$, $\hat{V}_F$, can be defined as the available estimates of the function matrices $H(x_1)$, $C(x_1,x_2)$, $G(x_1)$, V_F.

The dynamics of the smooth sliding observer (k -tanh as switching function), with gains adaptively updated (the λ-modification included) can be expressed as:

$$\begin{aligned}&\dot{\hat{x}}_1 = -\Gamma_1(\hat{x}_1 - x_1) + \Theta_1(t)\tanh(k_o S_o) + \hat{x}_2\\&\dot{\hat{x}}_2 = -\Gamma_2(\hat{x}_1 - x_1) + \Theta_2(t)\tanh(k_o S_o)\\&-\hat{H}^{-1}\left[\hat{C}\hat{x}_2 + \hat{V}_F\hat{x}_2 + \hat{G} - \hat{u}\right]\end{aligned} \tag{15}$$

The smooth switching function allows to consider that the conditions: $\tilde{x}_1 = 0$, $\dot{\tilde{x}}_1 = 0$ are satisfied during sliding. The state estimate error equation is

$$\begin{aligned}&\dot{\tilde{x}}_2 = -\Theta_2\Theta_1^{-1}\tilde{x}_2 -\\&-\hat{H}(x_1)^{-1}\left[\hat{C}(x_1,\hat{x}_2)\hat{x}_2 + \hat{G}(x_1) - u + \hat{V}_F\hat{x}_2\right] +\\&+H(x_1)^{-1}[C(x_1,x_2)x_2 + G(x_1) - u + V_F x_2]\end{aligned} \tag{16}$$

By an appropriate choice of the gain Θ_2, the stability of the observer and exponential convergence rate can be achieved, as how is proved in [7]. Let $Q \in \Re^{nxn}$ be the positive definite matrix defined as:

$$Q = \hat{M}(x_1)\Theta_2\Theta_1^{-1} + \hat{\pi}(x_1,\hat{x}_2) + \hat{V}_F \tag{17}$$

where

$$\hat{\pi}(x_1,\hat{x}_2) = \frac{\partial}{\partial x_2}[C(x_1,x_2)\ \hat{x}_2]\Big|_{x_2=\hat{x}_2} \tag{18}$$

$$\hat{C}(x_1,x_2)\hat{x}_2 = \hat{C}(x_1,\hat{x}_2)\hat{x}_2 - \hat{\pi}(x_1,\hat{x}_2)\tilde{x}_2 \tag{19}$$

The matrix Q determines the robustness of the observer to the parameter uncertainties. Choosing large eigenvalues for the matrix Q, the observation error can be globally ultimately bounded, (Corollary 5.3 from [4]). Defining V_2 as a Lyapunov function candidate:

$$V_2 = \frac{1}{2}\tilde{x}_2^T\hat{H}(x_1)\tilde{x}_2, \tag{20}$$

one obtains the derivative:

$$\begin{aligned}&\dot{V}_2 = \tilde{x}_2^T\hat{H}(x_1)\dot{\tilde{x}}_2 + \frac{1}{2}\tilde{x}_2^T\dot{\hat{H}}(x_1)\tilde{x}_2 =\\&-\tilde{x}_2^T\begin{Bmatrix}\hat{H}(x_1)\Theta_2\Theta_1^{-1}\tilde{x}_2 + \hat{\pi}(x_1,\hat{x}_2)\tilde{x}_2 + \hat{V}_F\tilde{x}_2 - \tilde{G}\\-\tilde{V}_F x_2 - \tilde{C}x_2 + \hat{H}\tilde{H}^{-1}[G + Cx_2 - u + V_F x_2]\end{Bmatrix}\end{aligned} \tag{21}$$

Let define the vector $\mu = \mu(x_1,x_2,\hat{x}_2)$ as:

$$\begin{aligned}&\mu = -\tilde{G} - \tilde{V}_F x_2 - \tilde{C}x_2\\&-\hat{H}\tilde{H}^{-1}[G + Cx_2 - u(x_1,\hat{x}_2) + V_F x_2]\end{aligned} \tag{22}$$

and assume that μ is linearly bounded by $\tilde{x}_2$

$$\|\mu\| \le \beta + \gamma\|\tilde{x}_2\|, \quad \forall t \tag{23}$$

for some β, $\gamma > 0$, then the derivative of the Lyapunov function is bounded by:

$$\begin{aligned}&\dot{V}_2 \le -\lambda_{\min Q}\|\tilde{x}_2\|^2 + \|\tilde{x}_2\|\|\mu\|\\&\le (-\lambda_{\min Q} - \gamma)\|\tilde{x}_2\|^2 + \beta\|\tilde{x}_2\| \le -\varepsilon\|\tilde{x}_2\|^2\end{aligned} \tag{24}$$

where $\varepsilon \le \lambda_{\inf Q} - \gamma$. If at $t = 0$, the switching gain Θ_1 satisfies (7), both gains: Θ_1 and Θ_2 follow the adaptation laws (3) and (4), respectively, and the vector μ is bounded, then there exists $t_1 \ge 0$ such that the observer velocity estimation error satisfies the inequality:

$$\|\tilde{x}_2(t)\| \le \sqrt{\frac{\lambda_{\max \hat{H}}}{\lambda_{\min \hat{H}}}} \|\tilde{x}_2(0)\| e^{\frac{-\varepsilon}{2\lambda_{\max \hat{H}}} t}, \quad \forall t < t_1 \quad (25)$$

Moreover, the estimation error converges to the ball $B(0, r)$ (centred in zero and with radius r):

$$\|\tilde{x}_2(t)\| \le r, \quad \forall t \ge t_1 \quad (26)$$

where the radius satisfies the inequality:

$$r \ge \sqrt{\frac{\lambda_{\max \hat{H}}}{\lambda_{\min \hat{H}}}} \frac{\beta}{\lambda_{\min Q} - \gamma - \varepsilon} \quad (27)$$

The calculus of the control input for n-degree of freedom robotic manipulator follows. In order to fulfil the attractiveness condition (9), it is necessary to express the derivative of the sliding surface (8):

$$\begin{aligned} \dot{\hat{S}}_c &= \dot{\hat{x}}_2 - \ddot{y}_r + \psi(\hat{x}_2 - \dot{y}_r) \\ &= -\hat{H}(x_1)^{-1}\left[\hat{C}(x_1, \hat{x}_2)\hat{x}_2 + \hat{V}_F \hat{x}_2 + \hat{G}(x_1) - \hat{u}\right] \\ &- \ddot{y}_r + \psi(\hat{x}_2 - \dot{y}_r) \end{aligned} \quad (28)$$

Similarly as for the observer, by the using k-tanh as a switching function and the λ-modification into the gain, the sliding condition is fulfilled if the control input is chosen as:

$$\begin{aligned} \hat{u} &= \hat{C}\hat{x}_2 + \hat{V}_F \hat{x}_2 + \hat{G} \\ &+ \hat{H}\left[-\psi \hat{S}_c + \eta(t)\tanh(k_c \hat{S}_c) + \ddot{y}_r - \psi(\hat{x}_2 - \dot{y}_r)\right] \end{aligned} \quad (29)$$

The controller switching gain $\eta(t)$ is adaptively updated as in (12). Using (16), the derivative of the sliding surface (8) can be expressed as:

$$\dot{\hat{S}}_c = \eta(t)\tanh(k_c \hat{S}_c) - \psi \hat{S}_c + \left(\Theta_2(t)\Theta_1^{-1}(t) - \psi\right)\tilde{x}_2 \quad (30)$$

If the gain η of the switching function satisfies the inequality:

$$\psi_i \left|\hat{S}_{ci}\right| - \eta_i(t) \ge \left|\left[\left(\Theta_2(t)\Theta_1^{-1}(t) - \psi\right)\tilde{x}_2\right]_i\right|, \forall t, i = 1, \ldots, n \quad (31)$$

then the attractiveness condition is verified. Because Θ_1 and Θ_2 are diagonal matrices, the inequality (31) can be written as:

$$\psi_i \left|\hat{S}_{ci}\right| - \eta_i(t) \ge \left|\left(\frac{\theta_{2i}(t)}{\theta_{1i}(t)} - \psi_i\right)\tilde{x}_{2i}\right|, \forall t, i = 1, \ldots, n \quad (32)$$

Remark 5. The initial value of the switching controller gain has to be defined to guarantee the sliding condition after the convergence of the observer, when the error in state estimates is bounded by (26). The term $\psi \hat{S}_c$ maintains the sliding variable bounded during the observer transient. This leads to:

$$-\eta_i(t_o) \ge \left|\frac{\theta_{2i}(t_o)}{\theta_{1i}(t_o)} - \psi_i\right| \frac{\lambda_{\max \hat{H}}}{\lambda_{\min \hat{H}}} \frac{\beta}{(\lambda_{\min Q} - \gamma)} \quad (33)$$

With an appropriate choice of λ_c and ρ_c with respect to λ_1, λ_2 ρ_1 and ρ_2, the above condition can be satisfied all the time.

Remark 6. Expressing the control input sliding condition as:

$$x_2 - \dot{y}_{1r} + \psi(x_1 - y_{1r}) = -\tilde{x}_2 \quad (34)$$

where the true velocity state is introduced, and taking into account (26), the following bound of the tracking error can be obtained:

$$\left|x_{1i} - y_{1ri}\right| \le \frac{1}{\psi_i} \frac{\lambda_{\max \hat{H}}}{\lambda_{\min \hat{H}}} \frac{\beta}{(\lambda_{\min Q} - \gamma)}, \forall t > t_1 \quad (35)$$

Remark 7. The actual value of t_1 depends on the convergence rate of the observer and on the time defined by the gain matrix ψ. The observer and the controller, both of them into a smoothed form, can achieve high performance. For values of the constant k_o greater than k_c, the smooth switching function of the observer is closer to a pure relay than the smooth switching function of the controller. Therefore, the observer converges faster than the controller with small state estimate error. The state estimates could be chattering-free, independently of the value of the gains Θ_1 and Θ_2. Moreover, by choosing the matrices Θ_1 and Θ_2 adaptively updated as in (5) and (6), the magnitudes of the switching function go to small values while link position errors go to small values.

Remark 8. During sliding, the error $S_o = \hat{x}_1 - x_1$. is approximately zero. The derivate is not exactly zero, but it is a high frequency signal, of average approximately zero, with an amplitude depending on Θ_1. If the gain Θ_1 goes to zero, the derivative of the velocity estimation error goes to zero or becomes very small. That means a reduced observation error even in the presence of parameter uncertainties. Also, the behaviour of the controller is similar to that with the full state measurement if the switching is based on a smooth variable. The smooth controller means a reduced or free chattering for the control input and/or the output.

5. Simulation results

In order to test the proposed smooth variable structure observer-controller, the robotic manipulator model from [1], is considered. It is 2-degree of freedom vertical robot with two rigid revolute joints and two rigid links and a mass m_p as the load. The vectors of position and velocities are:

$$y = [y_1 \quad y_2]^T = x_1 = [x_{11} \quad x_{12}]^T \tag{36}$$

$$x_2 = [x_{21} \quad x_{22}]^T, \tag{37}$$

The robot parameters are the following:

$$H(y) = \begin{bmatrix} 9.77 + 2.02\cos(y_2) & 1.26 + 1.01\cos(y_2) \\ 1.26 + 1.01\cos(y_2) & 1.12 \end{bmatrix} + \begin{bmatrix} 2 + 2\cos(y_2) & 1 + \cos(y_2) \\ 1 + \cos(y_2) & 1 \end{bmatrix} m_p \tag{38}$$

$$C(y, x_2) = \sin(y_2) \left\{ \begin{bmatrix} -x_{22} & -x_{21} - x_{22} \\ x_{21} & 0 \end{bmatrix} 1.01 + \begin{bmatrix} -x_{22} & -x_{21} + x_{22} \\ x_{21} & 0 \end{bmatrix} m_p \right\} \tag{39}$$

$$G(y) = g \begin{bmatrix} 8.1\sin(y_1) + 1.13\sin(y_1 + y_2) \\ 1.13\sin(y_1 + y_2) \end{bmatrix} + g \begin{bmatrix} \sin(y_1) + \sin(y_1 + y_2) \\ \sin(y_1 + y_2) \end{bmatrix} m_p \tag{40}$$

$$V_F(x_2) = \mathrm{diag}[10 \quad 10] \tag{41}$$

The initial conditions are:

$$y_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; x_2(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \hat{x}_1(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \hat{x}_2(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \tag{42}$$

$$\Theta_1(0) = \mathrm{diag}[-10 \quad -10]; \Theta_2(0) = \mathrm{diag}[-100 \quad -200]$$
$$\eta(0) = \mathrm{diag}[-5 \quad -10] \tag{43}$$

The following observer-controller constants are chosen:

$$\boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_2 = \boldsymbol{\lambda}_c = \mathrm{diag}[1 \quad 1], \tag{44}$$

$$\Gamma_1 = \mathrm{diag}[10 \quad 10], \Gamma_2 = \mathrm{diag}[5000 \quad 5000], \tag{45}$$

$$\rho_1 = \rho_2 = \rho_c = \mathrm{diag}[1 \quad 1], \quad \psi = \mathrm{diag}[20 \quad 20] \tag{46}$$

The mass m_p is assumed to be unknown. For the simulation, a mass of 3kg is supposed for the controller and observer design, while the real one used in the model is 5kg. Small parameter uncertainties (2%) are considered. The trajectory to be tracked is

$$\mathbf{y}_r = [-0.5 + 0.3\sin(t - 0.3) \quad 0.7\sin(2t + 0.3)]^T \tag{47}$$

The control input is constrained by:

$|u_1 \le 150|, |u_2 \le 75|$.

In order to compare the convergence rates of the observer and of the controller, two closed loop simulations are presented. For the first one (figure 1 and figure 2), the smooth switching function of the controller is closer to a pure relay than that of the observer. Hence, the sliding observer converges slower than the sliding controller. A small chattering for the control input and oscillations on the sliding surfaces can be observed.

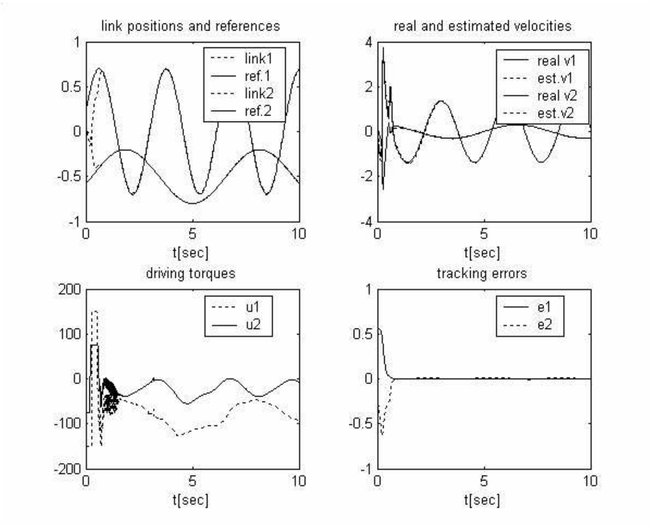

Figure 1. Closed loop robot response, smooth sliding observer, $k_o = 300$, smooth sliding controller, $k_c = 500$.

For the second one (figure 3 and figure 4), the observer converges faster than the controller. Therefore, the control input is based on a small estimation error and then, is chattering free. Obviously, for both simulations, the smooth switching function k-tanh is used. For a larger parameter k, the gain around zero is greater and the observer or/and the controller converge faster. However, smooth switching function leads to the system evolution in a neighbourhood sliding surface. The initial conditions are chosen in order to satisfy the inequalities (32) and (33). The closed-loop system behaves well to parameter uncertainties. The gains adaptively updated, depending of the corresponding sliding surface, make small the influence of the variable structure part into the state estimate error or tracking error.

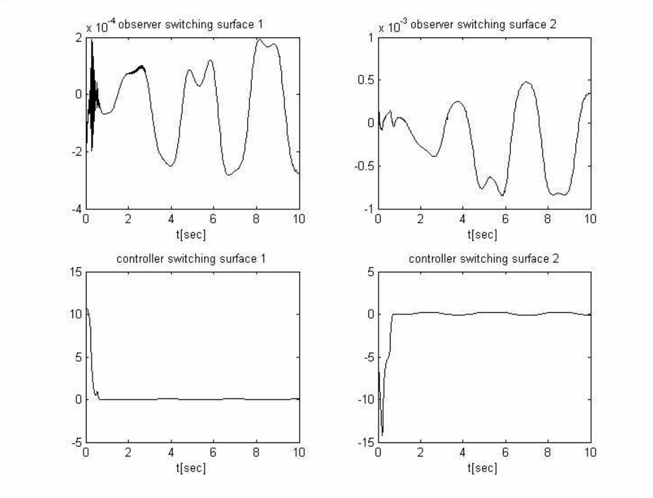

Figure 2. Closed loop robot response, observer sliding surface for $k_o = 300$, controller sliding surface for $k_c = 500$.

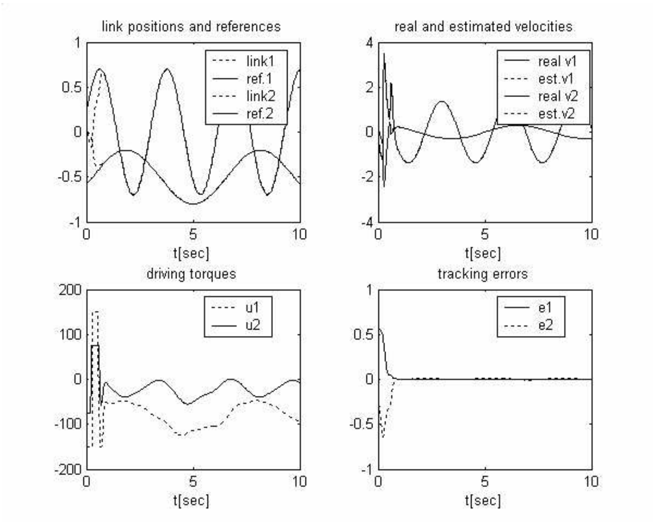

Figure 3. Closed loop robot response, smooth sliding observer, $k_o = 10$, smooth sliding controller, $k_c = 1$.

6. Conclusions

A smooth variable structure observer-controller, with modulation functions gains adaptively updated, was designed and checked by simulation in closed-loop. The general MIMO model for a n-degree of freedom robotic manipulator was used in the sliding observer-controller design. An application to a 2-degree of freedom robotic manipulator control is presented. The output tracking and robustness are increased in the presence of parameter uncertainties. The parameterised tangent hyperbolic, used as a switching function, assures a reduced or free chattering. An appropriate choice of the parameters in the observer and in the controller switching functions allows a faster convergence of the observer than that of the controller. The time varying gains of the switching functions lead to small state estimate and tracking error with an improved transitory response.

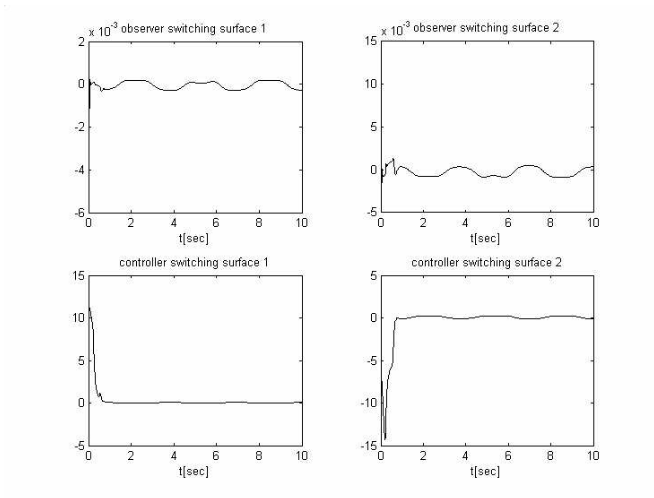

Figure 2. Closed loop robot response, observer sliding surface for $k_o = 10$, controller sliding surface for $k_c = 1$.

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