THE ISS SMALL GAIN APPROACH TO STABILIZATION OF BILATERALLY CONTROLLED TELEOPERATORS WITH COMMUNICATION DELAY

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Keywords: Teleoperators, Communication delay, ISS small-gain theorem

Abstract

The ISS small gain approach to the stabilization of bilaterally controlled teleoperators in the presence of time delay in the communication chanel is presented. A control scheme is proposed that makes the teleoperator system stable regardless of the delay in the communication channel. The central idea is to make both the master and the slave manipulators input-to-state stable with prescribed ISS gains, so that the stability of the overall system can be guaranteed by ISS small gain theorem.

1 Introduction

Teleoperation can be defined as the extension of a persons sensing and manipulation capability to a remote location [13]. A standard teleoperator system consists of two manipulators called master and slave, and a communication chanel between them. The master is moved by the human operator, and the information about master's trajectory is sent throw the communication chanel to the remotely located slave. The slave is controlled to follow the motion of the master. In order to make the human operator feel the contact force of the slave, the information about the contact force is reflected back to the motors of the master. In this case the teleoperator is said to be controlled bilaterally [2]. If time delay is present in the communication chanel, the force reflection can make the system unstable [6]. The stabilization problem for such a system was considered, among other papers, in [2, 3, 11, 9, 10, 1] (see also [4], and the bibliography therein). The first solution of this problem was presented in [2], where the case of linear one-degree-of-freedom master/slave manipulators was considered. The solution is based on passivity properties of master and slave manipulators. The key idea of the approach is to use a feedback bilinear transformation which transforms a passive system into a system with gain less than or equal to one [5]. This transformation is applied for both master and slave subsystems. As a result, feedback configuration of two systems with gain less than or equal to one is obtained, which is stable regardless of time delay in the communication chanel. In [3] this result was extended to the case of nonlinear multi-degrees-of-freedom manipulators, and asymptotic stability of the system was proved. For the case of linear manipulators an alternative control strategy which is also based on passivity arguments is presented in [1]. However, the passivity based approach has several shortcomings. First, stability of the bilaterally controlled teleoperator is proved under the assumption that both human operator and environment can be modeled as passive systems, an assumption which appears to be restrictive. Another disadvantage of this approach is the following: in order to preserve passivity of the slave block, a specially designed coordinating torque term rather, than environment contact force, must be send to the master. This coordinating torque is insensitive to changes in the contact force. Thus, specially designed local force feedback around the slave needs to be implemented [2]. Even in the presence of such a feedback, however, the coordinating torque term cannot provide precise information about contact force to the human operator, thus leading to a deterioration of the performance of the teleoperation.

An alternative approach to the stabilization of bilaterally controlled teleoperators in the presence of delay in the communication chanel was presented in [12]. In that paper, a control law was proposed that makes both the master and the slave subsystem input-to-state stable with respect to external forces. Using the properties of input-to-state stable systems, it is then shown that the overall system is stable for any delay in the communication channel. A possible drawback of the results in [12] is that a restrictive model of the environmental dynamics has been utilized. Namely, it is assumed that the environmental force is uniformly bounded with unknown bound. In this paper, we address the stabilization problem under essentially more general assumptions on environmental dynamics. Specifically, we assume that the environmental dynamics satisfy a weak form of finite-gain assumption with respect to slave variables. Using an appropriate form of the ISS small-gain theorem [8], we show that the proposed control law makes the overall system stable regardless of delay in the communication channel.

The paper is organized as follows. In section 2 the necessary preliminary material is given. In section 3 we consider a control scheme which makes the teleoperator system input-to-state stable independently of the communication delay, and formulate the main result. Proof of the main result is given in section 4. Due to space reasons, we do not present computer simulation results, which will be published in the full version of the paper.

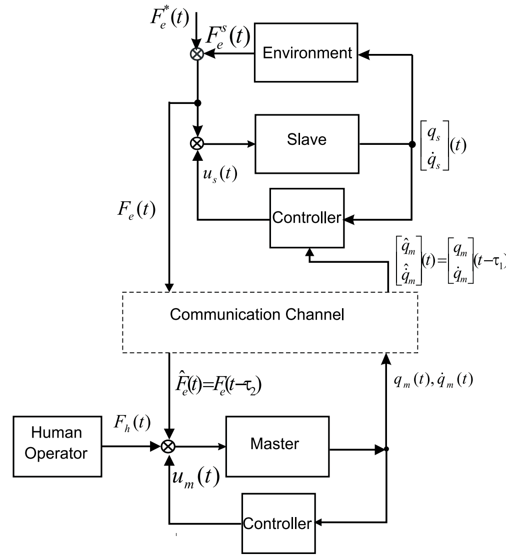

Figure 1: Structure of the teleoperator system

2 Preliminaries

2.1 Euler-Lagrange equations of manipulators

For simplicity we will consider the manipulators with revolute (rotational) joints. Let the configuration of a robotic manipulator be described by n generalized joint angles $q = (q_1, \ldots, q_n)^T \in T^n$, where T^n is n-dimensional torus. Suppose that in these coordinates the dynamics of the manipulator are described by Euler-Lagrange equations of the following standard form

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau. \tag{1}$$

Here $\tau \in R^n$ is the vector of external forces, $H(q) \in R^{n\times n}$, $C(q,\dot{q}) \in R^{n\times n}$, and $G(q) \in R^n$ are smooth matrix-valued (vector-valued) functions of their arguments, $H(q)$ represents the inertia matrix of the manipulator, $C(q,\dot{q})\dot{q}$ is the vector of centrifugal and Coriolis forces, and $G(q)$ is the vector of potential forces.

It is well-known, that the dynamic model (1) has the following properties [17].

Property 1. The inertia matrix $H(q)$ is symmetric and positive definite.

Property 2. The (i,j)-entry of the matrix $C(q,\dot{q})$ has the following structure

$$C_{ij}(q,\dot{q}) = \sum_{i=1}^{n} \Gamma_{ijk}(q)\dot{q}_k,$$

where $\Gamma_{ijk}(q)$ are so called Christoffel symbols,

$$\Gamma_{ijk}(q) = \frac{1}{2}\left(\frac{\partial H_{ij}(q)}{\partial q_k} + \frac{\partial H_{ik}(q)}{\partial q_j} - \frac{\partial H_{kj}(q)}{\partial q_i}\right). \tag{2}$$

A direct consequence of property 2 is the following property.

Property 3. The matrix $\dot{H}(q) - 2C(q,\dot{q})$ is skew-symmetric.

2.2 Teleoperation with time delay

The structure of the bilateral teleoperation system is presented in figure 1. The following notations will be used. Let $q_m \in T^n$, $\dot{q}_m \in R^n$ be position and velocity of the master, $q_s \in T^n$, $\dot{q}_s \in R^n$ position and velocity of the slave, F_h is a force applied by the human operator to control the motion of the master, and $F_e \in R^n$ is the contact force due to environment applied to the slave. Throughout the paper we impose the following assumption on the dynamics of environment.

Assumption 1. The contact force F_e can be represented as follows

$$F_e(t) = F_e^s(t) + F_e^*(t), \tag{3}$$

where F_e^s satisfies the following "finite-gain" condition with respect to the slave variables

$$|F_e^s(t)| \leq \gamma_e\left(|\dot{q}_s(t)| + |q_s(t)|\right) \tag{4}$$

for some $\gamma_e > 0$ and for almost all $t \geq 0$, and $F_e^*(t)$ is an arbitrary measurable essentially bounded function. The term F_e^* represents disturbances as well as the external environmental forces which do not depend on the teleoperator dynamics.

Further, by $\hat{q}_m \in T^n$, $\hat{\dot{q}}_m \in R^n$ we denote position and velocity of the master transmitted to the slave via the communication chanel with some delay $\tau_1 \geq 0$, so that

$$\hat{q}_m(t) = q_m(t-\tau_1), \tag{5}$$
$$\dot{\hat{q}}_m(t) = \dot{q}_m(t-\tau_1), \tag{6}$$

and $\hat{F}_e \in R^n$ represents the contact force transmitted back to the master with some delay $\tau_2 \geq 0$,

$$\hat{F}_e(t) = F_e(t-\tau_2). \tag{7}$$

The dynamics of the bilaterally controlled teleoperator system are described as follows

$$\begin{aligned} H_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m + G_m(q_m) \\ = F_h + \hat{F}_e + u_m, \end{aligned} \tag{8}$$

$$H_s(q_s)\ddot{q}_s + C(q_s,\dot{q}_s)\dot{q}_s + G(q_s) = F_e + u_s, \tag{9}$$

where $u_m, u_s \in R^n$ are the control inputs of the master and the slave respectively.

2.3 Input-to-state stability

Recently, the notion of input-to-state stability has been studied extensively in the nonlinear control literature (see [14], and the bibliography therein). Since the teleoperator system contains delay blocks, it is natural to describe such a system by functional-differential equations, so in this case the standard definition of ISS is not directly applicable. In this paper, we will utilize the following extension of the ISS notion which was proposed by Teel in [18]. Consider a functional-differential equation of the form

$$\dot{x}(t) = F(x_d(t), w_d(t)), \tag{10}$$

where $x_d(t)(\cdot)$ is a function $[0,t_d] \to R^n$ for some $t_d \geq 0$, defined as $x_d(t)(s) = x(t-s)$. Similarly, $w_d(t)(s) = w(t-s)$.

Following [18], denote

$$|x_d(t)| = \max_{t-t_d \leq s \leq t} |x(s)|, \quad \|x_d\|_{t_0} = \sup_{s \geq t_0} |x_d(s)|,$$

and analogously for $|w_d(t)|$.

Definition 1. The system (10) is said to be input-to-state stable with ISS gain $\gamma \in \mathcal{K}$, if $|x_d(t_0)| < \infty$ and $\|w_d\|_{t_0} < \infty$ imply the solutions of (10) are defined for all $t \in [t_0 - t_d, +\infty)$, and the following two properties hold uniformly in $t_0 \geq 0$:

i) uniform boundedness: there exists a function $\delta \in \mathcal{K}_\infty$ such that

$$\|x_d\|_{t_0} \leq \max\{\delta(|x_d(t_0)|), \gamma(\|w_d\|_{t_0})\}; \tag{11}$$

ii) uniform convergence: for each $\epsilon, \eta > 0$ there exists $T > 0$ such that

$$|x_d(t_0)| \leq \eta \Rightarrow \|x_d\|_{t_0+T} \leq \max\{\epsilon, \gamma(\|w_d\|_{t_0})\}. \tag{12}$$

3 Input-to-state stability of the teleoperator system

In this section we address the problem of stabilization of the bilateral teleoperator system (3)–(9), (13), (14). Consider the following control law

$$\begin{aligned} u_m = -H_m(q_m)\dot{q}_m - C_m(q_m,\dot{q}_m)q_m \\ +G_m(q_m) - K_m(\dot{q}_m + q_m), \end{aligned} \tag{13}$$

$$\begin{aligned} u_s = H_s(q_s)\left(\dot{\hat{q}}_m - \dot{q}_s\right) + C_s(q_s,\dot{q}_s)(\hat{q}_m - q_s) \\ +G_s(q_s) - K_s(\dot{q}_s + (q_s - \hat{q}_m)), \end{aligned} \tag{14}$$

where $K_m, K_s \in R^{n\times n}$ are symmetric positive definite matrices. In the following, for given symmetric matrix K, the minimal (maximal) eigenvalue of K will be denoted by $\lambda_{\min}(K)$ ($\lambda_{\max}(K)$). Our main result is presented in the following theorem.

Theorem 1. There exist $\eta_m, \eta_s > 0$ such that if $\lambda_{\min}(K_m) \geq \eta_m$, $\lambda_{\min}(K_s) \geq \eta_s$, then for any communication delays $\tau_1, \tau_2 \geq 0$ the controlled bilateral teleoperator system (3)–(9), (13), (14) is input-to-state stable with respect to the input $\left(F_h^T, F_e^{*T}\right)^T$.

4 Proof of Theorem 1

We start from standard definition of the input-to-state stability property for systems described by ordinary differential equations.

Definition 2. A system of the form

$$\dot{x} = F(x,w), \tag{15}$$

$x \in R^n$, $w \in R^m$, is said to be input-to-state stable (ISS), if there exists $\gamma \in \mathcal{K}$ such that the following two properties hold.

i) There exist $\delta \in \mathcal{K}_\infty$ such that

$$\sup_{t\geq t_0} |x(t)| \leq \max\left\{\delta(|x(t_0)|), \gamma\left(\sup_{t\geq t_0}|w(t)|\right)\right\}. \tag{16}$$

ii) For each $\eta, \epsilon > 0$ there exists $T = T(\eta, \epsilon) \geq 0$ such that $|x(t_0)| \leq \eta$ implies

$$\sup_{t\geq t_0+T} |x(t)| \leq \max\left\{\epsilon, \gamma\left(\sup_{t\geq t_0}|w(t)|\right)\right\}. \tag{17}$$

Properties i) and ii) are referred as uniform boundedness and uniform convergence respectively. Thus defined ISS property admits several equivalent characterizations [15, 16]. In particular, the system (15) is ISS if and only if there exist a smooth ISS-Lyapunov function $V\colon R^n \to R^+$ with the following properties:

i) there exist $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ such that

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|)$$

for all $x \in R^n$;

ii) there exist $\alpha_3, \chi \in \mathcal{K}$ such that $\chi(|w|) \leq |x|$ implies

$$\frac{\partial V}{\partial x} F(x,w) \leq -\alpha_3(|x|). \tag{18}$$

Moreover, if there exists an ISS Lyapunov function satisfying the above two properties, then the function $\alpha_1^{-1} \circ \alpha_2 \circ \chi \in \mathcal{K}$ is an ISS gain for the system (15).

The idea of our proof is to show that under suitable choice of matrices K_m, K_s, the proposed control law (13), (14) makes both the master and the slave subsystems input-to-state stable in the sense of definition 2 with arbitrary prescribed gains $\gamma_m, \gamma_s > 0$. Then, the application of ISS small gain type arguments [8] completes the proof.

Proposition 1. For any $\gamma_m^* > 0$ there exists $\lambda(\gamma_m^*) > 0$ such that if $\lambda_{\min}(K_m) \geq \lambda(\gamma_m^*)$, then the closed-loop master subsystem (8), (13) is ISS with respect to the state $\left(q_m^T, \dot{q}_m^T\right)^T$ and input $F_h + \hat{F}_e$ with ISS gain less than or equal to γ_m^*.

Proof. Denote $e_m = \dot{q}_m + q_m$. Substituting the control law (13) into the equation (8), we get that the closed-loop system (8), (13) is described as follows

$$\begin{aligned} H_m(q_m)\dot{e}_m + C_m(q_m,\dot{q}_m)e_m + K_m e_m \\ = F_h - \hat{F}_e, \end{aligned} \tag{19}$$

$$\dot{q}_m = -q_m + e_m. \tag{20}$$

Take an ISS-Lyapunov function candidate

$$V_m(e_m, q_m) = \frac{1}{2}\left(e_m^T H_m(q_m) e_m + q_m^T q_m\right). \tag{21}$$

By Property 1, $H_m(q_m)$ is positive definite smooth matrix function on compact configuration space T^m, therefore there exist $\upsilon_1, \upsilon_2 > 0$ such that

$$\upsilon_1 |x|^2 \leq x^T H_m(q_m) x \leq \upsilon_2 |x|^2 \quad \text{for all } x \in R^n, q \in T^n.$$

Consequently,

$$\nu_1\left(|e_m|^2 + |q_m|^2\right) \leq V(q_m, e_m) \leq \nu_2\left(|e_m|^2 + |q_m|^2\right)$$

for some $\nu_1, \nu_2 > 0$. Using Property 3, it is easy to see that the time derivative of V_m along the trajectories of (19), (20) admits the following upper estimate

$$\frac{d}{dt}V_m \leq -e_m^T K_m e_m + |e_m|\left|F_h - \hat{F}_e\right| - |q_m|^2 + |q_m|\,|e_m|.$$

Using quadratic Young's inequality, we get

$$\begin{aligned} \tfrac{d}{dt}V_m \leq -e_m^T K_m e_m + \tfrac{\lambda_{\min}(K_m)}{2}|e_m|^2 \\ + \tfrac{1}{2\lambda_{\min}(K_m)}\left|F_h - \hat{F}_e\right|^2 - |q_m|^2 + \tfrac{1}{2}|q_m|^2 + \tfrac{1}{2}|e_m|^2. \end{aligned}$$

Assuming $\lambda_{\min}(K_m) > 2$, we have

$$\frac{d}{dt}V_m \leq -\frac{1}{2}\left(|e_m|^2 + |q_m|^2\right) + \frac{1}{2\lambda_{\min}(K_m)}\left|F_h - \hat{F}_e\right|^2.$$

We see that if

$$|e_m|^2 + |q_m|^2 \geq \frac{2}{\lambda_{\min}(K_m)}\left|F_h - \hat{F}_e\right|^2,$$

then

$$\frac{d}{dt}V_m \leq -\frac{1}{4}\left(|e_m|^2 + |q_m|^2\right).$$

Therefore, the closed-loop master subsystem (8), (13) is ISS, and the corresponding ISS gain from the input $F_h - \hat{F}_e$ to the state $\left(q_m^T, e_m^T\right)^T$ is less than or equal for

$$\tilde{\gamma}_m = \sqrt{\frac{2\upsilon_2}{\upsilon_1 \lambda_{\min}(K_m)}}.$$

Further, since

$$\left|\begin{pmatrix} q_m \\ \dot{q}_m \end{pmatrix}\right|^2 = |q_m|^2 + |\dot{q}_m|^2$$

$$\leq 3|q_m|^2 + 2|\dot{q}_m + q_m|^2 \leq 3\left|\begin{pmatrix} q_m \\ e_m \end{pmatrix}\right|^2,$$

we see that the ISS gain from the input $F_h - \hat{F}_e$ to the state $\left(q_m^T, \dot{q}_m^T\right)^T$ is less than or equal to

$$\gamma_m = \sqrt{\frac{6\upsilon_2}{\upsilon_1 \lambda_{\min}(K_m)}}.$$

Choosing matrix K_m such that $\lambda_{\min}(K_m) > 2$ is sufficiently large, we get the result. The proof is complete. •

Now consider the "slave-environment" subsystem (3), (4), (9), (14). Denote $\tilde{q} = q_s - \hat{q}_m$, $\dot{\tilde{q}} = \dot{q}_s - \hat{\dot{q}}_m$ (note that $\hat{\dot{q}}_m = \dot{\hat{q}}_m$). The following proposition is valid.

Proposition 2. For any $\gamma_s^* > 0$ there exists $\lambda(\gamma_s^*) > 0$ such that if $\lambda_{\min}(K_s) \geq \lambda(\gamma_s^*)$, then the closed-loop "slave-environment" subsystem (3), (4), (9), (14) is ISS with respect to to state $\left(\tilde{q}^T, \dot{\tilde{q}}^T\right)^T$, and input $\left(\hat{q}_m^T, \hat{\dot{q}}_m^T, F_e^{*T}\right)$ with ISS gain less than or equal to γ_s^*.

Proof of proposition 2 is similar to the proof of proposition 1. It is omitted here due to space reasons and will be published in the full version of the paper.

In the next proposition a simple fact is formulated that if the system is ISS in the sense of definition 2, then the same system with delays in some of the input channels, being considered as a system of FDE, is input-to-state stable in the sense of definition 1.

Proposition 3. Suppose the system

$$\dot{x}(t) = F\left(x(t), u(t), v(t)\right) \tag{22}$$

is ISS with respect to input $\left(u^T, v^T\right)^T$ in the sense of definition 2 with ISS gain less than or equal to $\gamma > 0$. Then for any $\tau \geq 0$ the system

$$\begin{aligned}\dot{x}(t) &= F\left(x(t), u(t-\tau), v(t)\right)\\ &:= F^*\left(x_d(t), u_d(t), v_d(t)\right)\end{aligned} \tag{23}$$

is ISS in the sense of definition 1 for any $t_d \geq \tau$, and the corresponding ISS gain is less than or equal to 2γ.

Proof of proposition 3 is omitted due to space reasons.

Now take $t_d \geq \tau_1 + \tau_2$. Combining propositions 1 and 3, and taking into account (3), (4), we have the following fact.

Fact A. For any $\gamma_m > 0$ there exists a symmetric positive definite matrix K_m such that the "environment + backward communication channel + controlled master" subsystem (3), (4), (7), (8), (13) has the following properties:

i) there exists a function $\delta_1 \in \mathcal{K}_\infty$ such that

$$\left\| \begin{pmatrix} q_m \\ \dot{q}_m \end{pmatrix}_d \right\|_{t_0} \leq \delta_1 \left(\left| \begin{pmatrix} q_m \\ \dot{q}_m \end{pmatrix}_d (t_0) \right| \right)$$

$$+\gamma_m \left(\gamma_e \left\| \begin{pmatrix} q_s \\ \dot{q}_s \end{pmatrix}_d \right\|_{t_0+\tau_1} + \|(F_e^*)_d\|_{t_0} + \|(F_h)_d\|_{t_0} \right);$$

ii) for each $\epsilon, \eta > 0$ there exists $T > 0$ such that

$$\left| \begin{pmatrix} q_m \\ \dot{q}_m \end{pmatrix}_d (t_0) \right| \leq \eta$$

implies that

$$\left\| \begin{pmatrix} q_m \\ \dot{q}_m \end{pmatrix}_d \right\|_{t_0+T} \leq \epsilon$$

$$+\gamma_m \left(\gamma_e \left\| \begin{pmatrix} q_s \\ \dot{q}_s \end{pmatrix}_d \right\|_{t_0+\tau_1} + \|(F_e^*)_d\|_{t_0} + \|(F_h)_d\|_{t_0} \right).$$

On the other hand, combining propositions 2 and 3, and taking into account that $\tilde{q}(t) = q_s(t) - q_m(t-\tau_1)$, $\dot{\tilde{q}}(t) = \dot{q}_s(t) - \dot{q}_m(t-\tau_1)$, one can get the following statement.

Fact B. For any $\gamma_s > 0$ there exists a symmetric positive definite matrix K_s such that the "forward communication channel + controlled slave + environment" subsystem (3), (4), (5), (6), (9), (14) has the following properties:

i) there exists a function $\delta_2 \in \mathcal{K}_\infty$ such that

$$\left\| \begin{pmatrix} q_s \\ \dot{q}_s \end{pmatrix}_d \right\|_{t_0+\tau_1} \leq \delta_2 \left(\left| \begin{pmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{pmatrix}_d (t_0) \right| \right)$$

$$+ (\gamma_s + 1) \left\| \begin{pmatrix} q_m \\ \dot{q}_m \end{pmatrix}_d \right\|_{t_0} + \gamma_s \|F^*_{e\,d}\|_{t_0};$$

ii) for each $\epsilon, \eta > 0$ there exists $T > 0$ such that

$$\left| \begin{pmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{pmatrix}_d (t_0) \right| \leq \eta$$

implies that

$$\left\| \begin{pmatrix} q_s \\ \dot{q}_s \end{pmatrix}_d \right\|_{t_0+T} \leq \epsilon + (\gamma_s + 1) \left\| \begin{pmatrix} q_m \\ \dot{q}_m \end{pmatrix}_d \right\|_{t_0} + \gamma_s \|F^*_{e\,d}\|_{t_0}.$$

Fact A means that the "environment + backward communication channel + controlled master" subsystem is input-to-state stable, while fact B implies that the "forward communication channel + controlled slave + environment" subsystem is input-to-output stable. Since

$$\left| \begin{pmatrix} q_s \\ \dot{q}_s \end{pmatrix} \right| \leq \left| \begin{pmatrix} \tilde{q} \\ \dot{\tilde{q}} \end{pmatrix} \right| + \left| \begin{pmatrix} q_m \\ \dot{q}_m \end{pmatrix}_d \right|,$$

we see that the "forward communication channel + controlled slave + environment" subsystem has the unboundedness observability property [8]. Therefore, using facts A, B, one can apply the ISS small gain type arguments [8] to derive the following sufficient conditions for the input-to-state stability of the telerobotic system (3)–(9), (13), (14).

Proposition 4. The telerobotic system (3)–(9), (13), (14) is input-to-state stable, if

$$\gamma_m \gamma_e \left(\gamma_s + 1\right) < 1. \tag{24}$$

Proof of proposition 4 follows standard line of reasoning (see, for example [8, 7] where the proofs for more general case of nonlinear gain functions are presented), and is omitted here.

Note that it follows from propositions 1, 2 that the condition (24) can always be satisfied by suitable choice of matrices K_m, K_s. This completes the proof of Theorem 1.

5 Concluding remarks

We have presented a new approach to the stabilization of bilaterally controlled teleoperation systems with communication delay. The central idea of this approach is to make both the master and the slave manipulators input-to-state stable with prescribed ISS gains, and then apply the ISS small gain theorem to prove the input-to-state stability of the overall system. The important feature of this approach is that the stability of the telerobotic system is guaranteed for any communication delay. To fulfill the small-gain condition (24), it may be necessary to choose the master gain K_m large enough, which may lead to deterioration of compliance of the system for the human operator. This fact reflects the trade-off between stability and compliance. To achieve better compliance, one can take K_m smaller than it is necessary to guarantee the fulfillment of the small gain condition. In this case the stability of the overall telerobotic system is not guaranteed, however, the motions of the master and the slave will remain synchronized (see proposition 2), which may be sufficient for successful teleoperation.

Acknowledgments

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

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