Tracking Control using Attitude Measurements for Flexible Spacecraft

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Abstract

In this paper the attitude tracking problem is solved for a flexible spacecraft in the case of measurements of the variables describing the attitude error. Such a control could be applied in failure situations for recovering a spacecraft, so increasing the fault tolerance of the spacecraft control system and the success of the mission. The result is a dynamic controller capable of reconstructing the information regarding the spacecraft angular velocity and those regarding the modal variables, describing the dynamics of the flexible appendages present in the spacecraft.
Keywords: Flexible spacecraft, nonlinear control, perturbation analysis, fault tolerance.

1 Introduction

First examples of application of nonlinear controllers to spacecraft attitude control are the works [11], [12], [15], [16], [18] (for the continuous time case) [17], [3], [7] (for the discrete time case). Further contributions to this topic can be found, among the works constituting a rich literature on the subject, in the papers [27], [28], [20]. The principal advantage of these controllers is the improvement of the control performance, due to the cancellation of the nonlinearities in the model. Moreover, the decoupling between the dynamics brings further improvements in the dynamical behavior. Drawbacks of these control strategies are (among the others) the necessity of the whole state for feedback and the perfect knowledge of the system parameters.

The first of these aforementioned drawbacks is here considered, in order to address a possible solution in the case of attitude tracking. The importance of this problem is easily understood once one considers that the robustness (in the sense of fault tolerance) of the control system in case of sensor failure is an obvious requirement, since the continuation of the spacecraft mission is a crucial prerequisite. From this point of view, the development of control laws which are capable of recovering such failures assumes a significant importance, and can be considered economically competitive with a sensor redundance policy. Previous works, which are in this direction, are [26], [22] for rigid spacecraft, and [2], [3], [6], [8], [9] for flexible spacecraft. Clearly these control strategies have to be considered as "back ups", "remedies", and not controllers to be used in a normal operational mode.

In particular [8] regards the case of feedback from quaternion measurements for rest-to-rest maneuvers. This work represents its extension to the case of attitude tracking. Some of the present results have appeared also in [10]. The paper is organized as follows. After reviewing the mathematical model of a flexible spacecraft in Section 2, some results on output feedback stabilization, based on the perturbation theory, are presented in Section 3. Then, in Section 4 a first dynamic controller is determined, supposing that the attitude parameters and the spacecraft angular velocity are known. Finally, from this controller, in Section 5 a second controller is determined, which does not need the measurements of the spacecraft angular velocity. Final comments conclude the paper.

2 Mathematical Model of Flexible Spacecraft

In this Section the mathematical model of a flexible spacecraft is briefly recalled. The model is the one used in [15], [16], [17], and the reader is referred to these works for the details of the derivation. Considering the equations describing the dynamics of the error quaternion e_0, e, of the error angular velocity rate ω_e, and of the modal variables η, ψ, one obtains the model of a flexible spacecraft

$$\begin{aligned}
\begin{pmatrix} \dot{e}_0 \\ \dot{e} \end{pmatrix} &= \frac{1}{2}\mathcal{Q}^T(e_0,e)\omega_e \\
\dot{\omega}_e &= J_{mb}^{-1}\Bigg[-N(\omega_e,\psi,\omega_r) + \delta^T \begin{pmatrix} K \\ C \end{pmatrix}^T \begin{pmatrix} \eta \\ \psi \end{pmatrix} \\
&\qquad - \delta^T C \delta\omega_e + u\Bigg] - \dot{\omega}_r \\
\begin{pmatrix} \dot{\eta} \\ \dot{\psi} \end{pmatrix} &= \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix}\begin{pmatrix} \eta \\ \psi \end{pmatrix} - \begin{pmatrix} I \\ -C \end{pmatrix}\delta\omega_e - \begin{pmatrix} 0 \\ I \end{pmatrix}\delta\dot{\omega}_r
\end{aligned} \tag{1}$$

with

$$\mathcal{Q}(e_0,e) = (\,-e \quad e_0 I - \tilde{e}\,)$$

$\tilde{e}$ the dyadic expression of e and I the identity matrix. Moreover, J_{mb} is the inertia matrix of the main body,

$$N(\omega_e,\psi,\omega_r) = (\tilde{\omega}_e + \tilde{\omega}_r)(J_{mb}\omega_e + \delta^T\psi + J\omega_r) \tag{2}$$

is the gyroscopic term, $J = J_{mb} + \delta^T\delta$ is the total inertia matrix, δ is the coupling matrix between the rigid and the elastic structure, ω_r is the reference angular velocity (depending on the reference quaternions), K, C are the stiffness and damping matrices, u is the input due to gas jets. Note that the first equation, relative to e_0, is not independent of the others, due to the constraint relation among the four unitary quaternions. Therefore, this equation could be eliminated from the model since redundant. The output of the system will be considered, in the particular problem under study, the error quaternions, namely $y = \begin{pmatrix} e_0 & e \end{pmatrix}^T$. We suppose in what follows that $\sigma\begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix} \in \mathbb{C}^-$, where $\sigma(\cdot)$ denotes the set of eigenvalue. This means that $K > 0$, $C > 0$ and, in particular, that the spacecraft structure has a non-negligible internal damping; this is an acceptable hypothesis in common space applications.

3 Some Results on Output Feedback Stabilization

Let us consider the following nonlinear system with relative degree $r > 1$

$$\begin{aligned} \dot{x} &= f(t,x) + g(t,x)u \\ y &= h(x) \end{aligned} \tag{3}$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, $f(t,0) = 0$, $h(0) = 0$, $f(\cdot,\cdot)$, $g(\cdot,\cdot)$, $h(\cdot) \in C^2$ in their arguments, and a feedback control of the form

$$\begin{aligned} \dot{\hat{x}} &= \varphi_0(t,y,\hat{x}) + \varphi_1(t,y,\hat{x})\dot{y} \\ u &= k_0(t,y,\hat{x}) + k_1(t,y,\hat{x})\dot{y} \end{aligned} \tag{4}$$

with $\hat{x} \in \mathbb{R}^n$ the estimate of x, $\varphi_0(t,0,0) = 0$, $k_0(t,0,0) = 0$, and $\varphi_0(\cdot,\cdot,\cdot)$, $\varphi_1(\cdot,\cdot,\cdot)$, $k_0(\cdot,\cdot,\cdot)$, $k_1(\cdot,\cdot,\cdot) \in C^2$ in their arguments.

The following hypothesis will be used later on.

(H) *The control* (4) *renders the origin of the feedback system* (3), (4) *asymptotically stable in the first approximation.* ◇

Theorem 1. Under hypothesis (H), the controller

$$\begin{aligned} \dot{\hat{x}} &= \varphi_0(t,y,\hat{x}) + \varphi_1(t,y,\hat{x})\Psi \\ \dot{\Psi} &= \frac{1}{\varepsilon}(-\Psi + \dot{y}) \\ u &= k_0(t,y,\hat{x}) + k_1(t,y,\hat{x})\Psi \end{aligned} \tag{5}$$

renders the origin of the feedback system (3), (5) asymptotically stable in the first approximation, for $\varepsilon > 0$ sufficiently small.

Proof. Let us consider the error estimate $\tilde{x} = x - \hat{x}$. In terms of x and $\tilde{x}$, the resulting feedback system (3), (5) is given by

$$\begin{aligned} \begin{pmatrix} \dot{x} \\ \dot{\tilde{x}} \end{pmatrix} &= \begin{pmatrix} \bar{f}(t,x,\tilde{x}) \\ \tilde{f}(t,x,\tilde{x}) \end{pmatrix} + \begin{pmatrix} \bar{g}(t,x,\tilde{x}) \\ \tilde{g}(t,x,\tilde{x}) \end{pmatrix}\Psi \\ \dot{\Psi} &= \frac{1}{\varepsilon}(-\Psi + \theta(t,x)) \end{aligned} \tag{6}$$

with $\theta(t,x) = L_f h(x) = \dfrac{\partial h}{\partial x} f(t,x)$ the Lie derivative of $h(x)$ in the direction of f (since r is greater than 1, $L_g h(x) = 0$), and

$$\begin{aligned} \bar{f}(t,x,\tilde{x}) &= \Big[f(t,x) + g(t,x)k_0(t,h(x),\hat{x})\Big]_{\hat{x}=x-\tilde{x}} \\ \bar{g}(t,x,\tilde{x}) &= \Big[g(t,x)k_1(t,h(x),\hat{x})\Big]_{\hat{x}=x-\tilde{x}} \\ \tilde{f}(t,x,\tilde{x}) &= \Big[\bar{f}(t,x,\tilde{x}) - \varphi_0(t,h(x),\hat{x})\Big]_{\hat{x}=x-\tilde{x}} \\ \tilde{g}(t,x,\tilde{x}) &= \Big[\bar{g}(t,x,\tilde{x}) - \varphi_1(t,h(x),\hat{x})\Big]_{\hat{x}=x-\tilde{x}}. \end{aligned}$$

Let us define

$$\chi = \begin{pmatrix} x \\ \tilde{x} \end{pmatrix}, \quad F(t,\chi,\Psi) = \begin{pmatrix} \bar{f}(t,x,\tilde{x}) \\ \tilde{f}(t,x,\tilde{x}) \end{pmatrix} + \begin{pmatrix} \bar{g}(t,x,\tilde{x}) \\ \tilde{g}(t,x,\tilde{x}) \end{pmatrix}\Psi$$

$$k(t,\chi) = \theta(t,x), \quad G(t,\chi,\Psi) = -\Psi + k(t,\chi)$$

so that system (6) can be rewritten as

$$\dot{\chi} = F(t,\chi,\Psi)$$

$$\varepsilon\dot{\Psi} = G(t,\chi,\Psi)$$

The proof now follows from the application of singular perturbation theory [19]. First, note that for all $t \geq 0$ and for all $\chi \in B_r$ one can verify that

1) $F(t,0,0) = 0$, $G(t,0,0) = 0$.
2) The equation
$$\Psi = k(t,\chi)$$
is an isolated solution of $0 = G(t,\chi,\Psi)$, with $k(t,0) = 0$.
3) The functions $F, G, k \in C^2$ for all $\Psi - k(t,\chi) \in B_\rho$.
4) The origin of the reduced system
$$\dot{\chi} = F(t,\chi,k(t,\chi))$$
is exponentially stable, thanks to (H).
5) Finally, setting
$$\Upsilon = \Psi - k(t,\chi), \qquad \tau = \frac{1}{\varepsilon}(t - t_0)$$
the origin of
$$\frac{\partial\Upsilon}{\partial\tau} = G(t,\chi,\Upsilon + k(t,\chi)) = -\Upsilon$$
is exponentially stable, uniformly in (t,χ).

Then, for $0 < \varepsilon < \varepsilon_0$, ε_0 a small positive quantity, the origin of (6) is exponentially stable [19]. ■

4 Stabilization using Quaternion and Angular Velocity Measurements

In this Section we suppose to measure the error attitude quaternions e_0, e and the error angular velocity ω_e. The following result, obtained in the line of [9], ensures the spacecraft asymptotic stabilization.

Theorem 2. If the modal variables η, ψ are not measured, for all $k_p > 0$ and for $k_d > 0$ large enough the dynamic controller

$$\begin{aligned}\begin{pmatrix}\dot{\hat{\eta}}\\ \dot{\hat{\psi}}\end{pmatrix} &= \begin{pmatrix}0 & I\\ -K & -C\end{pmatrix}\begin{pmatrix}\hat{\eta}\\ \hat{\psi}\end{pmatrix} - \begin{pmatrix}I\\ -C\end{pmatrix}\delta\omega_e - \begin{pmatrix}0\\ I\end{pmatrix}\delta\dot{\omega}_r\\ &\quad + \Gamma\begin{pmatrix}K\\ C\end{pmatrix}\delta(e+\omega_e) + \begin{pmatrix}0\\ \delta(\tilde{e}-\tilde{\omega}_r)\end{pmatrix}(\omega_e+\omega_r) \qquad (7)\end{aligned}$$

$$\begin{aligned}u &= -k_p e - k_d\omega_e - \frac{1}{2}J_{mb}(e_0 I + \tilde{e})\omega_e - \delta^T\begin{pmatrix}K\\ C\end{pmatrix}^T\begin{pmatrix}\hat{\eta}\\ \hat{\psi}\end{pmatrix}\\ &\quad + \delta^T C\delta\omega_e + J_{mb}\dot{\omega}_r - (\tilde{e}-\tilde{\omega}_r)J_{mb}\omega_e \qquad (8)\\ &\quad - \tilde{\rho}(\hat{\psi},\omega_r)(\omega_e+\omega_r)\end{aligned}$$

$\rho(\hat{\psi},\omega_r) = J\omega_r + \delta^T\hat{\psi}$ (the "~" denotes the dyadic expression), solves the attitude tracking problem for system (1) with a reference angular velocity $\omega_r \in \mathcal{L}_\infty$ and derivative $\dot{\omega}_r \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, where $\Gamma = \Gamma^T > 0$ is a gain matrix.

Proof. This result is proved by applying Barbalat theorem [23]. To this aim, it is necessary to prove the boundedness and the square integrability of the state of the system. To this aim, let us consider the following Lyapunov function [19]

$$\begin{aligned}V(t,x) &= \bar{k}\Big[(1-e_0)^2 + e^T e\Big] + \frac{1}{2}(e+\omega_e)^T J_{mb}(e+\omega_e)\\ &\quad + \frac{1}{2}\begin{pmatrix}\eta^T & \psi^T\end{pmatrix}P\begin{pmatrix}\eta\\ \psi\end{pmatrix} + \frac{1}{2}\begin{pmatrix}e_\eta^T & e_\psi^T\end{pmatrix}\Gamma^{-1}\begin{pmatrix}e_\eta\\ e_\psi\end{pmatrix}\end{aligned}$$

$\bar{k} = k_p + k_d$, where $V(t,x) \le \alpha(\|x\|)$, $\alpha \in \mathcal{K}_\infty$,

$$x = \begin{pmatrix}e^T & \omega_e^T & \eta^T & \psi^T & e_\eta^T & e_\psi^T\end{pmatrix}^T$$

is the state vector and $P = P^T > 0$, $\Gamma^{-1} = \Gamma^{-T} > 0$, and $e_\eta = \eta - \hat{\eta}$, $e_\psi = \psi - \hat{\psi}$ are the estimate errors.

Using equations (1), the time derivative along the system trajectories is

$$\begin{aligned}\dot{V}(t,x) &= (k_p+k_d)e^T\omega_e + (e+\omega_e)^T\Bigg[\frac{1}{2}J_{mb}(e_0 I + \tilde{e})\omega_e\\ &\quad - N(\omega_e,\psi,\omega_r) + \delta^T(C\psi + K\eta - C\delta\omega_e) + u - J_{mb}\dot{\omega}_r\Bigg]\\ &\quad + \begin{pmatrix}\eta^T & \psi^T\end{pmatrix}P\mathcal{D} + \begin{pmatrix}e_\eta^T & e_\psi^T\end{pmatrix}\Gamma^{-1}\left[\mathcal{D} - \begin{pmatrix}\dot{\hat{\eta}}\\ \dot{\hat{\psi}}\end{pmatrix}\right]\end{aligned}$$

with $\mathcal{D} = \begin{pmatrix}0 & I\\ -K & -C\end{pmatrix}\begin{pmatrix}\eta\\ \psi\end{pmatrix} - \begin{pmatrix}I\\ -C\end{pmatrix}\delta\omega_e - \begin{pmatrix}0\\ I\end{pmatrix}\delta\dot{\omega}_r$.

Substituting (8) and (7), $\dot{V}(t,x)$ can be written as follows

$$\begin{aligned}\dot{V}(t,x) &= -k_p\|e\|^2 - k_d\|\omega_e\|^2 - \begin{pmatrix}\eta\\ \psi\end{pmatrix}^T Q_P\begin{pmatrix}\eta\\ \psi\end{pmatrix}\\ &\quad - \begin{pmatrix}\eta\\ \psi\end{pmatrix}^T P\begin{pmatrix}I\\ -C\end{pmatrix}\delta\omega_e - \begin{pmatrix}\eta\\ \psi\end{pmatrix}^T P\begin{pmatrix}0\\ I\end{pmatrix}\delta\dot{\omega}_r\\ &\quad - \begin{pmatrix}e_\eta\\ e_\psi\end{pmatrix}^T Q_\Gamma\begin{pmatrix}e_\eta\\ e_\psi\end{pmatrix}\end{aligned} \qquad (9)$$

since $(e+\omega_e)^T(\tilde{e}+\tilde{\omega}_e)J_{mb}\omega_e = 0$ and

$$\begin{aligned}&\tilde{\omega}(J_{mb}\omega_e + \delta^T\psi + J\omega_r) = \tilde{\omega}J_{mb}\omega_e - \tilde{\rho}(\hat{\psi},\omega_r)\omega + \tilde{\omega}\delta^T e_\psi\\ &(e+\omega_e)^T\tilde{\omega}\delta^T e_\psi = \begin{pmatrix}e_\eta\\ e_\psi\end{pmatrix}^T\begin{pmatrix}0\\ \delta(\tilde{e}-\tilde{\omega}_r)\end{pmatrix}\omega\\ &Q_P = -\frac{1}{2}\left[P\begin{pmatrix}0 & I\\ -K & -C\end{pmatrix} + \begin{pmatrix}0 & I\\ -K & -C\end{pmatrix}^T P\right]\\ &Q_\Gamma = -\frac{1}{2}\left[\Gamma^{-1}\begin{pmatrix}0 & I\\ -K & -C\end{pmatrix} + \begin{pmatrix}0 & I\\ -K & -C\end{pmatrix}^T\Gamma^{-1}\right].\end{aligned}$$

Setting

$$Q = \begin{pmatrix}k_p I & 0 & 0 & 0\\ 0 & k_d I & \frac{1}{2}P\begin{pmatrix}I\\ -C\end{pmatrix}\delta^T & 0\\ 0 & \frac{1}{2}P\begin{pmatrix}I\\ -C\end{pmatrix}\delta & Q_P & 0\\ 0 & 0 & 0 & Q_\Gamma\end{pmatrix} \qquad (10)$$

with I the identity matrix, one has

$$\dot{V}(t,x) = -x^T Q x - \begin{pmatrix}\eta\\ \psi\end{pmatrix}^T PB\delta\dot{\omega}_r \le -\lambda_m\|x\|^2 + \Delta\|\dot{\omega}_r\|\|x\|, \qquad (11)$$

$\lambda_m = \min\sigma(Q)$ and $\Delta = \|PB\delta\|$. Once the matrices $Q_P > 0$, $Q_\Gamma > 0$ are fixed and $P > 0$, $\Gamma^{-1} > 0$ are determined as solution of the Sylvester equations, the matrix Q is positive-definite for $k_d > 0$ large enough. Therefore, $\lambda_m > 0$. Since $\dot{\omega}_r \in \mathcal{L}_\infty$, $\dot{\omega}_r$ is bounded, say $\|\dot{\omega}_r(t)\| \le c$, $\forall t \ge t_0$ (this reflects the fact that $\ddot{q}_{r0}$, $\ddot{q}_r$ are bounded, since q, q_r are unitary vectors). Hence, one has that $\dot{V}(t,x) \le 0$ when $\|x(t)\| \ge c\dfrac{\Delta}{\lambda_m}$, namely $x(t)$ is bounded (see [19]).

To prove the square integrability of $x(t)$, let us integrate both sides of (11) and use the Schwarz inequality [1]. Considering the limit as t tends to infinity and denoting with $\|\cdot\|_2$ the $\mathcal{L}_2$-norm, one has

$$V(\infty,x) - V(t_0,x_0) \le -\lambda_m\|x\|_2^2 + \Delta\|\dot{\omega}_r\|_2\|x\|_2. \qquad (12)$$

Moreover, since $V(\infty,x) \ge 0$,

$$\lambda_m\|x\|_2^2 - \Delta\|\dot{\omega}_r\|_2\|x\|_2 \le V(t_0,x_0) - V(\infty,x) \le V(t_0,x_0)$$

and this implies that $x \in \mathcal{L}_2$, since

$$\|x\|_2 \leq \frac{1}{\sqrt{\lambda_m}} \left[V(t_0, x_0) + \frac{\Delta^2}{4\lambda_m} \|\dot{\omega}_r\|_2^2 \right]^{1/2} + \frac{\Delta}{2\lambda_m} \|\dot{\omega}_r\|_2, \tag{13}$$

$\dot{\omega}_r \in \mathcal{L}_2$ by hypothesis, and $V(t,x) \leq \alpha(\|x\|)$, $\alpha \in \mathcal{K}_\infty$, as previously observed.

The application of Barbalat theorem allows one to conclude that $\lim_{t\to\infty} x = 0$. ∎

5 Stabilization from Quaternion Measurements

In this Section the hypothesis of measurability of the angular velocity is removed. The following result solves the attitude tracking problem from quaternion measurements.

Theorem 3. The following controller

$$\begin{aligned}
\begin{pmatrix} \dot{\hat{\eta}} \\ \dot{\hat{\psi}} \end{pmatrix} &= \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix} \begin{pmatrix} \hat{\eta} \\ \hat{\psi} \end{pmatrix} + \Gamma \begin{pmatrix} K \\ C \end{pmatrix} \delta e - \begin{pmatrix} 0 \\ \delta\tilde{\omega}_r \end{pmatrix} e - \begin{pmatrix} 0 \\ I \end{pmatrix} \delta\dot{\omega}_r \\
&\quad + \frac{2}{\varepsilon} \left[\begin{pmatrix} I \\ -C \end{pmatrix} \delta - \Gamma \begin{pmatrix} K \\ C \end{pmatrix} \delta - \begin{pmatrix} 0 \\ \delta(\tilde{e} - \tilde{\omega}_r) \end{pmatrix} \right] \mathcal{Q}(e_0, e)\chi \\
\dot{\chi} &= \frac{1}{\varepsilon} \left(-\chi + \begin{pmatrix} e_0 \\ e \end{pmatrix} \right) \\
u &= -k_p e - \delta^T \begin{pmatrix} K \\ C \end{pmatrix}^T \begin{pmatrix} \hat{\eta} \\ \hat{\psi} \end{pmatrix} + \tilde{\omega}_r (J\omega_r + \delta^T \hat{\psi}) + J_{mb}\dot{\omega}_r \\
&\quad + \frac{2}{\varepsilon} \Big[k_d I + \frac{1}{2} J_{mb}(e_0 I + \tilde{e}) - \delta^T C \delta + (\tilde{e} - \tilde{\omega}_r) J_{mb} \\
&\qquad + \tilde{\rho}(\hat{\psi}, \omega_r) \Big] \mathcal{Q}(e_0, e)\chi
\end{aligned} \tag{14}$$

$\rho(\hat{\psi}, \omega_r) = J\omega_r + \delta^T \hat{\psi}$, renders the origin asymptotically stable in the first approximation, for $k_p > 0$, $k_d > 0$, and $\varepsilon > 0$ small enough, with a reference angular velocity $\omega_r \in \mathcal{L}_\infty$ and derivative $\dot{\omega}_r \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, where $\Gamma = \Gamma^T > 0$ is a gain matrix.

Proof. First, let us write the feedback equation for the spacecraft, using the control law (7), (8). To this aim, let us introduce first the state variable

$$\varrho_0 = 1 - e_0, \qquad \begin{pmatrix} \bar{\eta} \\ \bar{\psi} \end{pmatrix} = \begin{pmatrix} \eta \\ \psi \end{pmatrix} - \begin{pmatrix} \varphi_\eta \\ \varphi_\psi \end{pmatrix}.$$

Here $\begin{pmatrix} \varphi_\eta \\ \varphi_\psi \end{pmatrix}$ is given as the solution of the linear system

$$\begin{pmatrix} \dot{\varphi}_\eta \\ \dot{\varphi}_\psi \end{pmatrix} = \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix} \begin{pmatrix} \varphi_\eta \\ \varphi_\psi \end{pmatrix} - \begin{pmatrix} 0 \\ I \end{pmatrix} \delta\dot{\omega}_r \tag{15}$$

where $\dot{\omega}_r$ is a known and bounded input to this system. Obviously, these dynamics are ISS [24], [25], and physically this means that the value assumed by the modal displacement and velocity depend on the angular acceleration to be tracked. Then, using the state vector $\bar{x} = \begin{pmatrix} \varrho_0 & e^T & \omega_e^T & (\bar{\eta}^T \;\; \bar{\psi}^T) & (e_\eta^T \;\; e_\psi^T) \end{pmatrix}^T$, the feedback system equations are

$$\begin{aligned}
\dot{\varrho}_0 &= \frac{1}{2} e^T \omega_e \\
\dot{e} &= \frac{1}{2}\omega_e + \frac{1}{2}(-\varrho_0 I + \tilde{e})\omega_e \\
\dot{\omega}_e &= -k_p J_{mb}^{-1} e - k_d J_{mb}^{-1}\omega_e - \frac{1}{2}\omega_e + J_{mb}^{-1}\delta^T \begin{pmatrix} K \\ C \end{pmatrix}^T \begin{pmatrix} e_\eta \\ e_\psi \end{pmatrix} \\
&\quad - J_{mb}^{-1}\tilde{\omega}_r \delta^T e_\psi - J_{mb}^{-1}\tilde{\omega}_e (J_{mb}\omega_e + \delta^T e_\psi) \\
&\quad - J_{mb}^{-1}\tilde{e} J_{mb}\omega_e - \frac{1}{2}(-\varrho_0 I + \tilde{e})\omega_e \\
\begin{pmatrix} \dot{\bar{\eta}} \\ \dot{\bar{\psi}} \end{pmatrix} &= \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix} \begin{pmatrix} \bar{\eta} \\ \bar{\psi} \end{pmatrix} - \begin{pmatrix} I \\ -C \end{pmatrix} \delta\omega_e \\
\begin{pmatrix} \dot{e}_\eta \\ \dot{e}_\psi \end{pmatrix} &= \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix} \begin{pmatrix} e_\eta \\ e_\psi \end{pmatrix} - \Gamma \begin{pmatrix} K \\ C \end{pmatrix} \delta(e + \omega_e) \\
&\quad - \begin{pmatrix} 0 \\ \delta\tilde{\omega}_r \end{pmatrix} (e + \omega_e) + \begin{pmatrix} 0 \\ \delta\tilde{e} \end{pmatrix} \omega_e.
\end{aligned} \tag{16}$$

This system is in the form

$$\dot{\bar{x}} = \mathcal{A}(t)\bar{x} + T_{nl}$$

where T_{nl} denotes the nonlinear terms and

$$\mathcal{A}(t) = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2}I & 0 & 0 \\
0 & -k_p J_{mb}^{-1} & -k_d J_{mb}^{-1} - \frac{1}{2}I & 0 & J_{mb}^{-1}\delta^T \begin{pmatrix} K \\ C \end{pmatrix}^T + J_{mb}^{-1} \begin{pmatrix} 0 \\ \delta^T\tilde{\omega}_r \end{pmatrix}^T \\
0 & 0 & -\begin{pmatrix} I \\ -C \end{pmatrix}\delta & \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix} & 0 \\
0 & -\Gamma \begin{pmatrix} K \\ C \end{pmatrix}\delta + \begin{pmatrix} 0 \\ \delta\tilde{\omega}_r \end{pmatrix} & -\Gamma \begin{pmatrix} K \\ C \end{pmatrix}\delta + \begin{pmatrix} 0 \\ \delta\tilde{\omega}_r \end{pmatrix} & 0 & \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix}
\end{pmatrix}.$$

It is easy to show that the origin of the linear system $\dot{\bar{x}} = \mathcal{A}(t)\bar{x}$ is asymptotically stable. To this aim, let us consider the Lyapunov function

$$\begin{aligned}
\mathcal{V} &= (k_p + k_d)\left(\varrho_0^2 + e^T e\right) + \frac{1}{2}(e + \omega_e)^T J_{mb}(e + \omega_e) \\
&\quad + \frac{1}{2}\begin{pmatrix} \bar{\eta}^T & \bar{\psi}^T \end{pmatrix} P \begin{pmatrix} \bar{\eta} \\ \bar{\psi} \end{pmatrix} + \frac{1}{2}\begin{pmatrix} e_\eta^T & e_\psi^T \end{pmatrix} \Gamma^{-1} \begin{pmatrix} e_\eta \\ e_\psi \end{pmatrix}
\end{aligned}$$

and its derivative is $\dot{\mathcal{V}} = -\bar{x}^T Q \bar{x} = -W(\bar{x})$, W a definite positive function, so that $\lim_{t\to\infty} \bar{x} = 0$ [19].

Finally, we show that it is possible to eliminate the use of ω_e in the control law (8) and in the dynamics (7), by invoking Theorem 1. In fact, from equations (1), due to the left invertibility of the matrix $\mathcal{Q}^T(e_0, e)$, and using the state variables

$$\begin{pmatrix} \tilde{\eta} \\ \tilde{\psi} \end{pmatrix} = \begin{pmatrix} \hat{\eta} \\ \hat{\psi} \end{pmatrix} - \begin{pmatrix} \varphi_\eta \\ \varphi_\psi \end{pmatrix}$$

with φ_η, φ_ψ defined as in (15), one obtains

$$\begin{aligned}
\begin{pmatrix} \dot{\tilde{\eta}} \\ \dot{\tilde{\psi}} \end{pmatrix} &= \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix} \begin{pmatrix} \tilde{\eta} \\ \tilde{\psi} \end{pmatrix} + \Gamma \begin{pmatrix} K \\ C \end{pmatrix} \delta e - \begin{pmatrix} 0 \\ \delta\tilde{\omega}_r \end{pmatrix} e \\
&\quad + 2\,\mathcal{D}_1 \mathcal{Q}(e_0, e) \begin{pmatrix} \dot{e}_0 \\ \dot{e} \end{pmatrix} \\
u &= -k_p e - \delta^T \begin{pmatrix} K \\ C \end{pmatrix}^T \begin{pmatrix} \tilde{\eta} \\ \tilde{\psi} \end{pmatrix} - \delta^T \begin{pmatrix} K \\ C \end{pmatrix}^T \begin{pmatrix} \varphi_\eta \\ \varphi_\psi \end{pmatrix} \\
&\quad - \tilde{\rho}(\hat{\psi}, \omega_r)\omega_r + J_{mb}\dot{\omega}_r + 2\,\mathcal{D}_2 \mathcal{Q}(e_0, e) \begin{pmatrix} \dot{e}_0 \\ \dot{e} \end{pmatrix}.
\end{aligned}$$

with

$$\begin{aligned}
\mathcal{D}_1 &= -\begin{pmatrix} I \\ -C \end{pmatrix} \delta + \Gamma \begin{pmatrix} K \\ C \end{pmatrix} \delta + \begin{pmatrix} 0 \\ \delta(\tilde{e} - \tilde{\omega}_r) \end{pmatrix} \\
\mathcal{D}_2 &= -k_d I - \frac{1}{2} J_{mb}(e_0 I + \tilde{e}) + \delta^T C \delta - (\tilde{e} - \tilde{\omega}_r) J_{mb} \\
&\quad - \tilde{\rho}(\hat{\psi}, \omega_r).
\end{aligned}$$

Thanks to Theorem 1, also the controller

$$\begin{aligned}
\begin{pmatrix} \dot{\tilde{\eta}} \\ \dot{\tilde{\psi}} \end{pmatrix} &= \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix} \begin{pmatrix} \tilde{\eta} \\ \tilde{\psi} \end{pmatrix} + \Gamma \begin{pmatrix} K \\ C \end{pmatrix} \delta e - \begin{pmatrix} 0 \\ \delta\tilde{\omega}_r \end{pmatrix} e \\
&\quad + 2\,\mathcal{D}_1 \mathcal{Q}(e_0, e) \Psi \\
\dot{\Psi} &= \frac{1}{\varepsilon}(-\Psi + \dot{y}) \\
u &= -k_p e - \delta^T \begin{pmatrix} K \\ C \end{pmatrix}^T \begin{pmatrix} \tilde{\eta} \\ \tilde{\psi} \end{pmatrix} - \delta^T \begin{pmatrix} K \\ C \end{pmatrix}^T \begin{pmatrix} \varphi_\eta \\ \varphi_\psi \end{pmatrix} \\
&\quad - \tilde{\rho}(\hat{\psi}, \omega_r)\omega_r + J_{mb}\dot{\omega}_r + 2\,\mathcal{D}_2 \mathcal{Q}(e_0, e) \Psi
\end{aligned}$$

ensures the control objective. Note that this controller still depends on $\dot{y}$, so that can not be implemented. However, setting

$$\Psi = \dot{\chi}$$

one can substitute the dynamics of Ψ with

$$\begin{aligned}
\dot{\chi} &= \frac{1}{\varepsilon}(-\chi + y) \\
\Psi &= \frac{1}{\varepsilon}(-\chi + y).
\end{aligned}$$

Therefore, the dynamic controller is finally

$$\begin{aligned}
\begin{pmatrix} \dot{\tilde{\eta}} \\ \dot{\tilde{\psi}} \end{pmatrix} &= \begin{pmatrix} 0 & I \\ -K & -C \end{pmatrix} \begin{pmatrix} \tilde{\eta} \\ \tilde{\psi} \end{pmatrix} + \Gamma \begin{pmatrix} K \\ C \end{pmatrix} \delta e - \begin{pmatrix} 0 \\ \delta\tilde{\omega}_r \end{pmatrix} e \\
&\quad - \frac{2}{\varepsilon} \mathcal{D}_1 \mathcal{Q}(e_0, e) \chi \\
\dot{\chi} &= \frac{1}{\varepsilon}(-\chi + y) \\
u &= -k_p e - \delta^T \begin{pmatrix} K \\ C \end{pmatrix}^T \begin{pmatrix} \tilde{\eta} \\ \tilde{\psi} \end{pmatrix} - \delta^T \begin{pmatrix} K \\ C \end{pmatrix}^T \begin{pmatrix} \varphi_\eta \\ \varphi_\psi \end{pmatrix} \\
&\quad + \tilde{\omega}_r(J\omega_r + \delta^T \hat{\psi}) + J_{mb}\dot{\omega}_r - \frac{2}{\varepsilon} \mathcal{D}_2 \mathcal{Q}(e_0, e) \chi
\end{aligned} \tag{17}$$

where it was noted that $\mathcal{Q}(e_0, e) y = 0$ and $-\tilde{\rho}(\hat{\psi}, \omega_r)\omega_r = \tilde{\omega}_r(J\omega_r + \delta^T \hat{\psi})$. Using the old state variables, one gets the controller (17). ■

Conclusions

In this paper we have proposed a dynamic controller which solves the tracking problem for a flexible spacecraft. This controller does not need the measurements of the modal variables and the spacecraft angular velocity. On the other hand, they rely on the perfect knowledge of the system parameters, in particular those describing the elastic motion (natural frequencies and damping ratios). This is an obvious limitation, since they are not usually known accurately. Future work will regard the design of structurally stable controllers which avoid this drawback.

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