Strictly Positive Real Problem with Observers.

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Abstract

We study the extension of what class of linear time invariant plants may be transformed into SPR systems introducing an observer. It is shown that for the open loop stable systems a cascaded observer achieves the result. For the open loop unstable systems an observer-based feedback is required to success. In general any system stabilizable and observable may be transformed into an SPR system. This overcomes the old conditions of minimum phase and relative degree one. The result is illustrated with some examples.
Keywords: Strictly Positive Real, Kalman Yakubovich Popov Lemma, Minimal Realizations.

1 Introduction

The celebrated Kalman-Yakubovich-Popov (KYP) Lemma gives algebraic equations which are equivalent to an analytic property in the frequency domain of a square transfer matrix $Z(s)$. A system that holds this frequency domain property is called Strictly Positive Real (SPR) system. The original references [8], [18] and [13] express the algebraic equations for a system to be SPR provided that state space realization is minimal, i.e., controllable and observable. It has been recognized for long time (Meyer [10], [12] and [2]) that the minimality of the realization can be weakened to only stabilizability and observability of the system. However a proof has not yet been provided. Implicitly, Rantzer [14], has presented a novel proof based on convexity properties and linear algebra that does not requires minimality. But it was not until recently [4] that the minimality issue was explicitly proved in an algebraic fashion. It has been shown that in some cases a minimal state-space SPR realization can preserve this property even if we introduce uncontrollable modes as long as they are stable. Further properties of SPR systems and comparisons with other related results are presented in [17] and [9].

This paper addresses the problem of transforming a linear time invariant system into an SPR system. Molander and Willems [11] solved the problem for state feedback, i.e., when the state is measurable. In the context of nonlinear systems, Byrnes et. al. [3] solved the problem under smooth state feedback, and they established that the problem may be solved if and only if the system has relative degree 1 and is (weakly) minimum phase. In the linear case Sun et. al. [16] presented a solution using output feedback. The transformed system they obtain is Extended SPR (ESPR) provided that the condition $D + D^T > 0$ is verified. A related result based on auxiliary optimization problem was provided by Haddad and Bernstein [5]. Contrary to the algorithm in [16], our approach is based on the definition of an output that is only a function of the observed state.

The paper is organized as follows. Section II presents some preliminaries. The case of stable systems is presented in section III while section IV is devoted to unstable systems. Some illustratives examples are given in section V. The concluding remarks are finally given in section VI.

2 Preliminaries

Let us consider a linear time-invariant m-inputs m-outputs transfer matrix $Z(s)$ with a minimal realization given by

$$\Sigma_1 \quad \begin{array}{rcl} \dot{x} &=& Ax + Bu \\ y &=& Cx \end{array} \tag{1}$$

where $x \in \mathbb{R}^n, u, y \in \mathbb{R}^m, m \leq n$ and A, B, C are matrices of the approriate dimensions. Denote by $\mathbb{C}, \overline{\mathbb{C}}_-$ and $\mathbb{C}_-$, the complex plane, the closed left hand complex plane and the open left hand complex plane respectively. Denote by $\sigma(T)$ the set of eigenvalues of the square matrix T.

Definition 1 [1], [12] The transfer matrix $Z(s)$ is said to be PR if: i) All elements of $Z(s)$ are analytical in $Re[s] > 0$; and ii) $Z(s) + Z^T(-s) \geq 0$ for all $Re[s] > 0$. $Z(s)$ is said to be SPR if $Z(s - \varepsilon)$ is PR for some $\varepsilon > 0$.

For the scalar case, $m = 1$, the classical interpretation of $z(s)$ being PR (SPR) is that its Nyquist plot lies entirely in the right hand complex plane (open right hand complex plane). We will need in the sequel the following version of the KYP Lemma for strictly proper systems:

Lemma 2 Let $Z(s) = C(sI - A)^{-1}B$ be a $m \times m$ transfer matrix such that $Z(s) + Z^T(-s)$ has normal rank m, where A is Hurwitz, (A, B) is stabilizable, and (C, A) is observable. Then, $Z(s)$ is strictly positive real (SPR) if and only if there exist positive definite symmetric matrices P and Q such that

$$\begin{aligned} PA + A^T P &= -Q \\ PB &= C^T \end{aligned} \tag{2}$$

3 Stable case

Let us consider a linear time invariant system described in standard state-space equations as (1), see [15].

Assumption I: The A matrix is stable [15] or [9], i.e. $\sigma(A) \subset \mathring{\mathbb{C}}_-$, the open left half complex plane.

A full order observer for the system Σ_1 is given by

$$\Sigma_c \begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + LC(x - \hat{x}) \\ z = M\hat{x} \end{cases} \tag{3}$$

where the observer gain matrix L is such that $A - LC$ has its spectrum in the open left half complex plane.

The system (1) and the observer (3) may be written compactly as:

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & 0 \\ LC & A - LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} u \tag{4}$$

Introducing the state estimation error as $e \triangleq \hat{x} - x$, the system (4) may be expressed as:

$$\Sigma_0 \left\{ \begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = A_0 \begin{bmatrix} x \\ e \end{bmatrix} + B_0 u \right. \tag{5}$$

where

$$A_0 \triangleq \begin{bmatrix} A & 0 \\ 0 & A - LC \end{bmatrix} \text{ and } B_0 \triangleq \begin{bmatrix} B \\ 0 \end{bmatrix} \tag{6a}$$

Remark 1 Notice that the system Σ_0 is not minimal, all the modes associated to the block $A - LC$ are not controllable. Since Σ_0 is not minimal, few studies have been made in the past [7] to define an output for system (5) and attempt to obtain an SPR system.

Since A and $A_L \triangleq A - LC$ are stable, then for all positive definite matrices Q_{11} and Q_{22}, there exist positive definite matrices P and P_L solution of the Lyapunov equations:

$$\begin{aligned} A^T P + PA &= -Q_{11} \\ A_L^T P_L + P_L A_L &= -Q_{22} \end{aligned} \tag{7}$$

Let us now define

$$P_0 \triangleq \begin{bmatrix} P & P \\ P & \mu P_L \end{bmatrix} \tag{8}$$

where $\mu > 0$ will be determined later. Then using $A_L \triangleq A - LC$

$$A_0^T P_0 + P_0 A_0 = \begin{bmatrix} A & 0 \\ 0 & A_L \end{bmatrix}^T \begin{bmatrix} P & P \\ P & \mu P_L \end{bmatrix} + \begin{bmatrix} P & P \\ P & \mu P_L \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & A_L \end{bmatrix} = -Q_0 \tag{9}$$

Note that block (1,1) corresponds to the first equation of (7), the block (2,2) is a μ scaled version of the second equation of (7) and the cross term is

$$A^T P + P(A - LC) = A^T P + PA - PLC = -Q_{11} - PLC \tag{10}$$

Then

$$Q_0 = \begin{bmatrix} Q_{11} & Q_{11} + PLC \\ Q_{11} + C^T L^T P & \mu Q_{22} \end{bmatrix} \tag{11}$$

The composite system Σ_0 (5) will satisfy the first equation of the KYP Lemma if $Q_0 > 0$ and $P_0 > 0$. We obtain the following conditions for positiveness of Q_0 and P_0.

I) Conditions that guaranteed the positiveness of P_0 are:

$$P_0 > 0 \iff \begin{cases} \text{I.1} & P > 0 \\ \text{I.2} & \mu P_L - P > 0 \end{cases} \tag{12}$$

Condition I.1 is satisfied due to first equation of (7) and condition I.2 in (12) is obtained by the Schur complement [6], Condition I.2 can also be expressed as $\mu P_L > P$ or equivalently

$$\mu > \mu_1 \triangleq \frac{\|P\|}{\|P_L\|} \tag{13}$$

II) Conditions that guarantee the positiveness of Q_0 in (11) are; i.e. $Q_0 > 0$ if and only if:

$$\begin{aligned} &\text{II.1} \quad Q_{11} > 0 \\ &\text{II.2} \quad \mu Q_{22} > \left(Q_{11} + C^T L^T P\right) Q_{11}^{-1} (Q_{11} + PLC) \end{aligned} \tag{14}$$

Condition II.2 is satisfied if

$$\mu > \mu_2 \triangleq \left\| \left(Q_{11} + C^T L^T P\right) Q_{11}^{-1} (Q_{11} + PLC) \right\| / \|Q_{22}\| \tag{15}$$

Combining conditions I and II, P_0 and Q_0 are positive definite if (see 13 and (15)

$$\mu > \mu^* \triangleq \max\{\mu_1, \mu_2\} \tag{16}$$

Notice that μ can always be chosen to satisfy the above inequality. We have proved the first part of the following:

Theorem 3 Consider the stable transfer matrix $Z(s)$ with m-inputs and m-outputs and state-space realization:

$$\Sigma_1 \begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx \end{cases} \tag{17}$$

where A is stable, the pair (A, B) is stabilizable and the pair (C, A) is observable. Then there exists a gain observer matrix L given in (3) satisfying (9) such that the transfer matrix between u and the new output $z = M_0 \hat{x}$ (with $M_0 = B_0^T P_0$) is characterized by a representation (A_0, B_0, M_0) that is SPR.

Proof. The proof of the first equation of the KYP Lemma is already done provided that $\mu > \mu^*$, now if the output z becomes

$$\begin{aligned} z &= M_0 \begin{bmatrix} x \\ e \end{bmatrix} \\ &= B_0^T P_0 \begin{bmatrix} x \\ e \end{bmatrix} \\ &= \begin{bmatrix} B^T & 0 \end{bmatrix} \begin{bmatrix} P & P \\ P & \mu P_L \end{bmatrix} \begin{bmatrix} x \\ \hat{x} - x \end{bmatrix} \\ &= B^T P x + B^T P (\hat{x} - x) \\ &= B^T P \hat{x} \\ &= M \hat{x} \end{aligned} \tag{18}$$

The composed system (A_0, B_0, M_0), which is not minimal, satisfies the KYP Lemma equations, i.e. $A_0^T P_0 + P_0 A_0 = -Q_0$ and $M_0 = B_0^T P_0 = \begin{bmatrix} B^T P & B^T P \end{bmatrix}$ ■

The system (1) and the observer (3) can be combined to obtain the composite system (5). A_0 in (5) and (6a) satisfies the Lyapunov equation (9) where P_0 and Q_0 are positive definite. Therefore if the new output z is defined as $z = B_0^T P_0 \hat{x}$, the transfer function from u to z is SPR.

4 Unstable case

Let us now study the case when Assumption I is not fulfilled. We will therefore consider the case when system (1) is unstable, i.e. $\sigma(A) \not\subset \overset{\circ}{\mathbb{C}}_-$. It is clear that if the state is measurable we can introduce a state feedback to stabilize the system and then proceed as in the previous section. However, we assume that the state is not measurable and only the output y in (1) is available. Therefore we introduce a stabilizing control law based on the state estimate $\hat{x}$ (3) as follows

$$u = -K\hat{x} + v \tag{19}$$

where v is a new input signal. The composed system becomes

$$\Sigma_K \left\{ \begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = A_1 \begin{bmatrix} x \\ e \end{bmatrix} + B_1 v \right. \tag{20}$$

where $A_1 \triangleq \begin{bmatrix} A - BK & -BK \\ 0 & A - LC \end{bmatrix}$; $B_1 \triangleq \begin{bmatrix} B \\ 0 \end{bmatrix}$

Let us introduce the short hand notation $A_K = A - BK$ and $A_L = A - LC$. Again K and L are such that $\sigma(A - BK), \sigma(A - LC) \subset \overset{\circ}{\mathbb{C}}$. Then for every Q_K and Q_L positive definite, there exist positive definite matrices P_K and P_L solution of the Lyapunov equations

$$\begin{aligned} A_K^T P_K + P_K A_K &= -Q_K \\ A_L^T P_L + P_L A_L &= -Q_L \end{aligned} \tag{21}$$

Let us define P_1 as

$$P_1 \triangleq \begin{bmatrix} P_K & P_K \\ P_K & \mu P_L \end{bmatrix} \tag{22}$$

where again $\mu > 0$ will be defined later. We then have the equation

$$A_1^T P_1 + P_1 A_1 = -Q_{KL} \tag{23}$$

where, if we define $Q_{12} \triangleq -A_K^T P_K - P_K A_L + P_K BK$

$$Q_{KL} = \left[\begin{array}{c|c} Q_K & Q_{12} \\ \hline Q_{12}^T & \mu Q_L + K^T B^T P_K + P_K BK \end{array} \right] \tag{24}$$

Stability of the feedback system will be guaranteed if we can find a value for μ such that $P_1 > 0$ and $Q_{KL} > 0$.

Condition III. As before, positiveness of P_1 is guaranteed if μ is large enough, i.e.

$$P_1 > 0 \iff \begin{array}{ll} \text{III.1} & P_K > 0 \\ \text{III.2} & \mu P_L > P_K \end{array}$$

Condition III.1 is satisfied in view of (21) and Condition III.2 is satisfied if

$$\mu > \mu_3 \triangleq \frac{\|P_K\|}{\|P_L\|} \tag{25}$$

Condition IV. Positiveness of Q_{KL} can be obtained using the Schur complement again. Introducing as $Q_{22} \triangleq \mu Q_L + K^T B^T P_K + P_K BK$ and remebering $Q_{12} \triangleq -A_K^T P_K - P_K A_L + P_K BK$; then $Q_{KL} > 0$ if $Q_K > 0$ and

$$Q_{22} - Q_{12}^T Q_K^{-1} Q_{12} > 0 \tag{26}$$

The above condition (26) will be satisfied if μ is such that

$$\mu > \mu_4 \triangleq \left\| K^T B^T P_K + P_K B K - Q_{12}^T Q_K^{-1} Q_{12} \right\| / \|Q_L\| \tag{27}$$

Combining (25) and (27), it follows that P_1 and Q_1 are positive definite if

$$\mu > \mu_K^* \triangleq \max\{\mu_3, \mu_4\} \tag{28}$$

Notice that μ can always be chosen to satisfy the above inequality. Now we can state the main result of this section:

Theorem 4 Consider a strictly proper square transfer matrix $Z(s)$ not identically zero, with stabilizable and observable realization (A, B, C). There exists a gain observer matrix L as in (3), an estimated state feedback gain matrix K and matrix M_1 which defines a new output $z = M_1 \hat{x}$ (with $M_1 = B_1^T P_1$) such that the transfer matrix from v (see (19)) to the new output z is SPR.

Proof. The first equation of the KYP Lemma is just proved for sufficiently high μ, the second part is similar to the Theorem for stable systems. ■

The system (1), the observer (3) and the controller in (19) can be combined to obtain the composite system (20). A_1 in (20) satisfies the Lyapunov equation (23) where P_1 and Q_1 are positive definite. Therefore if the new output $\bar{z}$ is defined as $\bar{z} = M_1 \hat{x}$ with $M_1 = B_1^T P_1$ then the transfer function from ν to $\bar{z}$ is SPR.

Remark 2 Notice that either in the stable or in the unstable cases, the Lyapunov equations $A_0^T P_0 + P_0 A_0 = -Q_0$ and (23) do not have positive definite solutions P_0 and P_1 respectively for all $Q_0 > 0$ and $Q_1 > 0$. We are imposing a particular structure on the solution matrices P_0 and P_1 and μ sufficiently large in order that P_0, Q_0, P_1, Q_1 are all positive definite.

5 Some illustrative examples.

We will present two detailed examples. The first example deals with an unstable system. The second example is an unstable system with a nonminimal state space representation.

Example 5 Let us consider the following unstable transfer function

$$\begin{aligned} z(s) &= \frac{1}{s^3 + 2s^2 - s - 2} \\ &= \frac{1}{(s+1)(s-1)(s+2)} \end{aligned} \tag{29}$$

which has a minimal representation representation

$$\Sigma_a \begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x \end{cases} \tag{30}$$

A full order observer for Σ_a, with eigenvalues at $\{-2, -3, -4\}$, is

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & -2 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \underbrace{\begin{bmatrix} 7 \\ 13 \\ 7 \end{bmatrix}}_{L} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} (x - \hat{x})$$

If we assign the closed loop eigenvalues at $\{-1, -1 \pm j\}$ we get

$$K = -\begin{bmatrix} 4 & 5 & 1 \end{bmatrix} \tag{31}$$

If we choose $Q_{11} = -2I$ then we get the solution of Lyapunov equation $A_K^T P_K + P_K A_K = -Q_{11}$,

$$P_K = \begin{bmatrix} 3.9 & 2.8 & 0.5 \\ 2.8 & 4.95 & 0.95 \\ 0.5 & 0.95 & 0.65 \end{bmatrix} > 0 \tag{32}$$

For $Q_{22} = -2I$ then the solution of the Lyapunov equation $A_L^T P_L + P_L A_L = -Q_{22}$,

$$P_L = \begin{bmatrix} 3.0958 & -1.2583 & -0.8625 \\ -1.2583 & 0.7500 & 0.2583 \\ -0.8625 & 0.2583 & 0.6292 \end{bmatrix} > 0$$

$$\begin{aligned} \mu^* &> \max\{\mu_3, \mu_4\} \\ &= \max\{3879.1, 1.9124\} = 3879.1 \end{aligned}$$

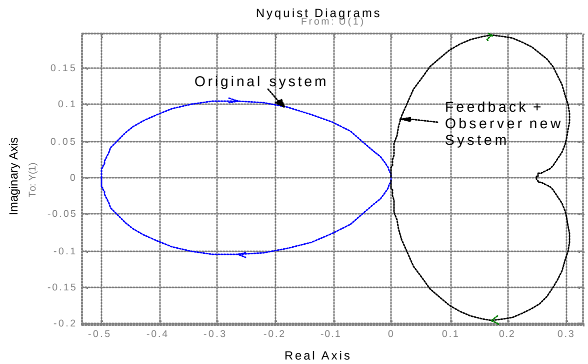
Choosing $\mu = 3900$, then

$$P_0 = \begin{bmatrix} 3.9 & 2.8 & 0.5 & 3.9 & 2.8 & 0.5 \\ 2.8 & 4.95 & 0.95 & 2.8 & 4.95 & 0.95 \\ 0.5 & 0.95 & 0.65 & 0.5 & 0.95 & 0.65 \\ 3.9 & 2.8 & 0.5 & 12074 & -4907.5 & -3363.7 \\ 2.8 & 4.95 & 0.95 & -4907.5 & 2925 & 1007.5 \\ 0.5 & 0.95 & 0.65 & -3363.7 & 1007.5 & 2453.7 \end{bmatrix}$$

$$Q_0 = \begin{bmatrix} 2 & 0 & 0 & 73.2 & 6.3 & 3.1 \\ 0 & 2 & 0 & 96.9 & 11.5 & 4.2 \\ 0 & 0 & 2 & 23.5 & 4.2 & 3.3 \\ 73.2 & 96.9 & 23.5 & 7800 & 0 & 0 \\ 6.3 & 11.5 & 4.2 & 0 & 7800 & 0 \\ 3.1 & 4.2 & 3.3 & 0 & 0 & 7800 \end{bmatrix}$$

the spectra of the matrix P_o and Q_o are:

$$\begin{aligned} \sigma(P_0) &= \{0.4447, 1.618, 7.3511, 607.5, 1679.8, 15165\} \\ \sigma(Q_0) &= \{.0107, 1.9988, 1.9998, 7800, 7800, 7802\} \end{aligned}$$



Nyquist Diagrams for the unstable example.

The output matrix becomes
$M_o = \begin{bmatrix} 0.5 & 0.95 & 0.65 & 0.5 & 0.95 & 0.65 \end{bmatrix}$; and
$M = \begin{bmatrix} 0.5 & 0.95 & 0.65 \end{bmatrix}$
The transfer function from a new input v and new output z becomes

$$\begin{aligned} H_{zv}(s) &= \frac{0.65s^2 + 0.95s + 0.5}{s^3 + 3s^2 + 4s + 2} \\ &= \frac{0.65\,(s + .731 \pm 0.4846j)}{(s+1)\,(s+1 \pm j)} \end{aligned} \quad (33)$$

Example 6 Let us consider the following unstable and non-minimum phase transfer function

$$z(s) = \frac{s - 2}{s^2 + s - 2} = \frac{s - 2}{(s-1)\,(s+2)} \quad (34)$$

which has a nonminimal, but stabilizable and observable, state space representation

$$\Sigma_b \begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -1 & -2 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} -2 & 1 & 4 \end{bmatrix} x \end{cases} \quad (35)$$

A full order observer for Σ_b, with eigenvalues at $\{-2, -3, -4\}$, is

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & -1 & -2 \\ 0 & 0 & -1 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u + \underbrace{\begin{bmatrix} -13 \\ -7 \\ -13 \end{bmatrix}}_{L} \begin{bmatrix} -2 & 1 & 4 \end{bmatrix} (x - \hat{x})$$

If we assign the closed loop eigenvalues at $\{-1, -1 \pm j\}$ we get $K = -\begin{bmatrix} 4 & 1 & -2 \end{bmatrix}$
If we choose $Q_{11} = -2I$ then we get the solution of Lyapunov equation $A_K^T P_K + P_K A_K = -Q_{11}$,

$$P_K = \begin{bmatrix} 2.5 & 0.5 & 0 \\ 0.5 & 0.75 & 0 \\ 0 & 0 & 1 \end{bmatrix} > 0 \quad (36)$$

For $Q_{22} = -2I$ then the solution of the Lyapunov equation $A_L^T P_L + P_L A_L = -Q_{22}$,

$$P_L = \begin{bmatrix} 3.7571 & -3.1286 & -9.8571 \\ -3.1286 & 3.2024 & 7.8619 \\ -9.8571 & 7.8619 & 27.924 \end{bmatrix} > 0$$

$$\begin{aligned} \mu &> \max\{\mu_3, \mu_4\} \\ &= \max\left\{7861.9, 0.4889\right\} = 7861.9 \end{aligned}$$

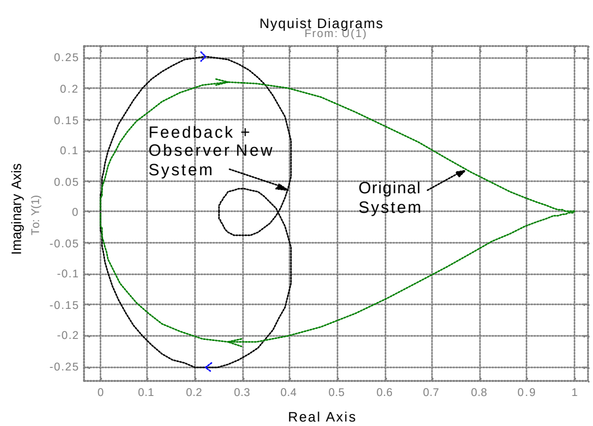
Taking $\mu = 7900$, then

$$P_0 = \begin{bmatrix} 2.5 & 0.5 & 0 & 2.5 & 0.5 & 0 \\ 0.5 & 0.75 & 0 & 0.5 & 0.75 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 2.5 & 0.5 & 0 & 29681 & -24716 & -77871 \\ 0.5 & 0.75 & 0 & -24716 & 25299 & 62109 \\ 0 & 0 & 1 & -77871 & 62109 & 2.2 \times 10^5 \end{bmatrix} \quad (37)$$

$$Q_0 = \begin{bmatrix} 2 & 0 & 0 & 78 & -32.5 & -145 \\ 0 & 2 & 0 & 27 & -8.25 & -48.5 \\ 0 & 0 & 2 & 5 & -4.5 & -10 \\ 73.2 & 96.9 & 23.5 & 15800 & 0 & 0 \\ 6.3 & 11.5 & 4.2 & 0 & 15800 & 0 \\ 3.1 & 4.2 & 3.3 & 0 & 0 & 15800 \end{bmatrix} \quad (38)$$

the spectra of the matrix P_0 and Q_0 are:
$\sigma(P_0) = \{0.6171, .9999, 2.6262, 9.18, 7774.7, 266810\}$
$\sigma(Q_0) = \{.00965, 1.9992, 2, 15800, 15800, 15802\}$
The output matrix becomes
$M_0 = \begin{bmatrix} 0.5 & 0.75 & 0 & 0.5 & 0.75 & 0 \end{bmatrix}$; and
$M = \begin{bmatrix} 0.5 & 0.75 & 0 \end{bmatrix}$ The transfer function from a new input v and new output z becomes

$$H_{zv}(s) = \frac{0.75s + 0.5}{s^2 + 2s + 2} = \frac{0.75\,(s + 0.6667)}{(s + 1 \pm j)} \quad (39)$$



Nyquist Diagrams of a nonminimal unstable system and its SPR transformed.

Nevertheless the Nyquist Diagram of the open loop original system lies in the open right hand complex plane, it fails to be SPR because it is not analytic in the stability domain. ■

6 Concluding Remarks.

This paper has shown that a non SPR but stable linear system can be transformed into an SPR system by introducing a state observer and de¯ning an appropriate output as a function of the observed state. The state of the original system is not assumed to be measurable. In spite of that fact the composite system obtained from the original system and the observer has a non-minimal state space representation, the overall system is SPR.

It has also been proved that an unstable system can also be transformed into an SPR system by introducing an observer, a feedback control using the state estimates and de¯ning an appropriate output as a function of the state estimate.

We have proved that in both cases the transformed system veri¯es the equations of the Kalman-Yakubovich-Popov Lemma. Therefore, the composite system inherits all the properties of SPR systems.

Some examples have been given to illustrate the result in the case of stable or unstable open loop systems with minimal realizations, and in the case of unstable systems with nonminimal realization.

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