Compensation of backlash effects in an Electrical Actuator

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Abstract— **We develop for systems with backlash a non linear observer, based on the estimation of the disturbed torque transmitted due the dead zone. Then we design an adaptive controller using this non linear observer in order to compensate the disturbed torque on line. Simulation and experimantal results applied on an elecrical actuator are given to support the theoretical demonstrations.**

Keywords: ***Backlash effects, Transmitted torque, electrical actuator, dead zone, adaptive control.***

I. Introduction

The presence of the dead zone in a mechanical systems introduces an hysteresis phenomenon between the input and the output positions. It describes the backlash phenomenon, which causes a non stable behavior for the controlled system. Backlash is inherent in mechanical systems, specially when starting the motion. But if it increases due to the wear, it will disturb the performances of the system. In this case, we try to compensate its effects by using a mechanic or an automatic methods. For a long time, the mechanical solutions existed to eliminate these disturber effects by changing all imperfect parts on the system. Also, some control solutions have been proposed, like in: Brandenbourg & Schaffer[1] where they have studied the influence and the partial compensation of simultaneously acting backlash and coulomb friction in a speed and position controller elastic two-mass system. Recker & al [4] and Tao & Kokotovic [2] have worked on the adaptive control of system with backlash. On this subject, different mathematical models are proposed like in: Tao & Kokotovic [2] have melodized the inverse backlash model based on a hysteresis cycle. Cadiou & M'Sirdi [5] have developed a differentiable model based on the dead zone characteristic.

In this paper, we inspire from the last medullization to design the backlash compensation and control. In most applications, the backlash non linearity could not be accurately known and only an estimation of its effects could be possible. First, we are going to design a non linear observer to estimate the disturber torque describing the backlash effects by using a mathematical model representing an inversed sigmoid. And then, we construct an adaptive control using a PD controller associated to the last torque observer to compensate the dead zone effects.

II. Definitions & Backlash Medullization

.1 Description of the bench test

the experimental bench test of Fig. 1 is an electrical actuator, divided on two parts: The first one corresponds to the motor part, driving by a DC motor. the second represents the reducer part which regroups three important mechanical imperfections.

The first imperfection is the static and the viscous friction, where there coefficients could be changed for different applications by using many brake parts like aluminium, metal,..etc., and different lubricious liquids. These coefficients could be also identified approximately by using the classic identification algorithms.

The second imperfection is the backlash and is represented by a variable dead zone from 0 deg to 24 deg. Finally the transmitted motion to the output axis is via a couple of strings with a variable stiffness.

On this bench test, we can measure the input and the output positions of the reducer axis by using two incremental coders as it's shown in Fig. 1.

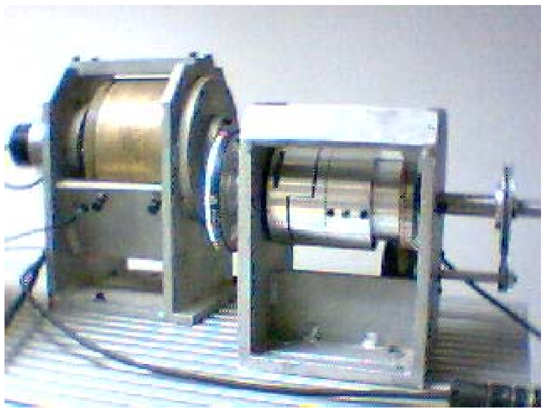

Fig. 1. Bench Test

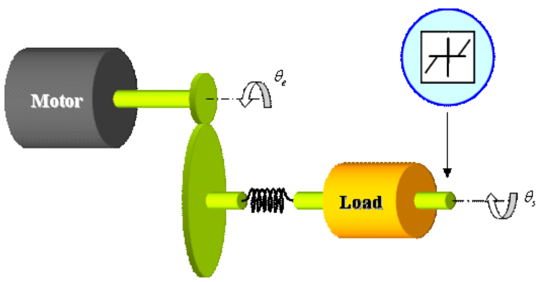

Fig. 2. Bloc scheme of the bench test

.2 Description of the backlash mechanism

We can resume the backlash mechanism by describing the Fig. 3. The *body*1 try to transmit the motion to *body*2 via a dead zone of magnitude $2.j_0$. The transmission will be correct when the two bodies are in contact, in this case there positions are identical. Out of the contact, the transmission will be delayed by the presence of the dead zone where the relation between the bodies positions will describe an hysteresis cycle behavior.

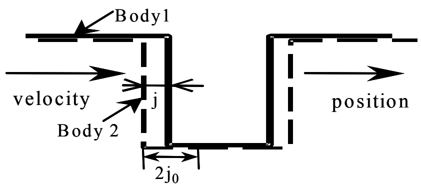

Fig. 3. The backlash mecanism

Now, referring to Fig. 4, we can notice that the transmitted torque inside a dead zone could have the representation of the torque Cr. This simple medullization of the transmitted torque via a dead zone of magnitude $2.j_0$ is constant and equal to zero inside this zone but it is linear outside.

Let's put $\Delta\theta$, the difference between the input position θ_e and the output position $N_0.\theta_s$ of the reducer part, with N_0 the reduction constant. Then we can formulate mathematically this transmitted torque as follows:

$$\begin{aligned} \text{if} \quad &: \quad -j_0 < \Delta\theta < j_0 \text{ then: } Cr = 0 \\ \text{else} \quad &: \quad Cr = K.(\Delta\theta - j_0.sign(\Delta\theta)) \end{aligned} \tag{1}$$

This model describes the expression of the transmitted torque inside and outside a dead zone with a flexible parts where K corresponds to the *rigidity constant* and gives the linear representation to the last torque Fig. 4.

The presence of the *signum* function make the last formulation of the transmitted torque not derivable. For that, we have chosen to write the characteristic of the transmitted torque inside and outside the dead zone in function of a *sigmoid* function, as follows:

$$C = K.\left(\Delta\theta - 4.j_0.\frac{1 - e^{-\gamma.\Delta\theta}}{1 + e^{-\gamma.\Delta\theta}}\right) \tag{2}$$

where C is the approximative transmitted torque represented in Fig. 4.

We can decompose the last expression of the transmitted torque C to two parts as follows:

$$C = C_0 + w \tag{3}$$

with C_0 the linear transmitted torque, given by:

$$C_0 = K.\Delta\theta \tag{4}$$

and w is the disturber and nonlinear transmitted torque, given by:

$$w = -4.K.j_0.\frac{1 - e^{-\gamma.\Delta\theta}}{1 + e^{-\gamma.\Delta\theta}} \tag{5}$$

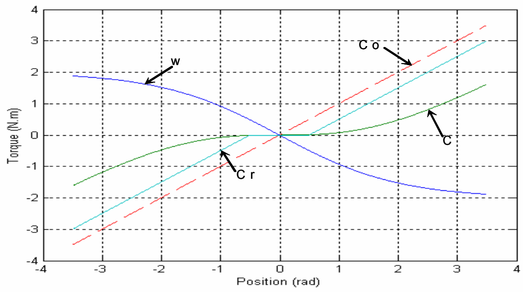

Fig. 4. The dead zone and the backlash torque

We also notice that the representation of w depends on the value of the constant γ, which it depends on the magnitude j_0. The value $'4'$ given in the last' representation, corresponds to the best approximation of the torque to be equal to *zero* inside $[-j_0, j_0]$ and varying exponentially outside Fig. 4.

According to the simulation and experimental tests, we obtain a best decreasing of w inside $[-j_0, j_0]$ after putting:

$$\gamma = \frac{1}{2.j_0} \tag{6}$$

For the next sections, the parameter γ is calculated on or line by giving an approximative and initial value to j_0. Then we estimate the torque w after observing the evolution of the magnitude j_0 on line.

III. Control of the bench test including backlash

We suppose that the static friction in neglected, then the mechanical model of the bench test including the backlash is describing by the equations system as follows:

$$\begin{cases} J_s.\ddot{\theta}_s + f_s.\dot{\theta}_s = C \\ J_m.\ddot{\theta}_e + f_m.\dot{\theta}_e + C = U \end{cases} \tag{7}$$

J_s, f_s, $\ddot{\theta}_s$, $\dot{\theta}_s$ are successively: the inertia of the reducer part, the viscous output friction which is supposed known, the output reducer acceleration and velocity.

J_m, f_m, $\ddot{\theta}_e$, $\dot{\theta}_e$ are successively: the inertia of the motor part, the viscous input friction which is supposed known, the input reducer acceleration and velocity.

U is the control torque, θ_e and θ_s are the input and output measured positions of the reducer part.

C represents the transmitted torque to the load via a flexible axis and a dead zone, it's expressed as follows:

$$C = K.\Delta\theta + w \tag{8}$$

K is the stiffness of the flexible parts and w is the disturber torque described before.

We take:

$$\Delta\theta = \theta_e - N_0.\theta_s \tag{9}$$

which represents the difference between the input and output positions of the reducer part. N_0 is the redaction constant.

Then, the system (7) will be expressed by the system (10):

$$\begin{cases} J_s.\ddot{\theta}_s + f_s.\dot{\theta}_s = K.\theta_e - K.N_0.\theta_s + w \\ J_m.\ddot{\theta}_e + f_m.\dot{\theta}_e = U - K.\theta_e + K.N_0.\theta_s - w \end{cases} \tag{10}$$

We put:

$$\begin{cases} \varepsilon_s = \theta_s - \theta_s^d \\ \varepsilon_e = \theta_e - \theta_e^d \\ z = K.\theta_e - K.N_0.\theta_s \\ \varepsilon_z = z - z_d \\ \tilde{w} = w - \hat{w} \end{cases} \tag{11}$$

θ_s^d is the desired output, θ_e^d is the desired input, z is the difference between the input and output positions of the reducer part and z_d its desired value. $\hat{w}$ is the estimated disturber torque and $\tilde{w}$ corresponds to the estimated error of the disturber torque.

Now, the system (10) could be expressed as follows:

$$\begin{cases} J_s.\ddot{\theta}_s + f_s.\dot{\theta}_s = K.\theta_e - K.N_0.\theta_s + w \\ J_m.\ddot{\theta}_e + f_m.\dot{\theta}_e + J_s.\ddot{\theta}_s + f_s.\dot{\theta}_s = U \end{cases} \tag{12}$$

The control scheme to the bench test is given by Fig. (5).

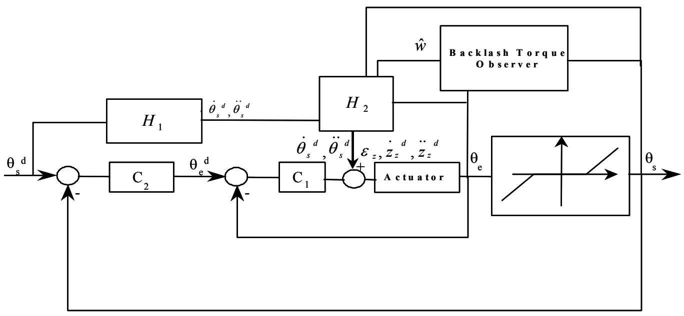

Fig. 5. Control scheme of the bench test including backlash

In order to linearize the first equation of system (12), we replace the z and w values by there estimated values. So, we will obtain the following equation:

$$J_s.\ddot{\theta}_s + f_s.\dot{\theta}_s = \varepsilon_z + z_d + \tilde{w} + \hat{w} \tag{13}$$

For that, we choose z_d equal to:

$$z_d = J_s.\ddot{\theta}_s^d + f_s.\dot{\theta}_s^d - \hat{w} \tag{14}$$

If we replace expression (14) in (13), we will obtain:

$$J_s.\ddot{\varepsilon}_s + f_s.\dot{\varepsilon}_s = \varepsilon_z + \tilde{w} \tag{15}$$

which is the equation deduced after linearization of the reducer part model.

Now, to linearize the second equation of the system (12), we do these variables affectations:

$$\begin{cases} \theta_e = \frac{z}{K} + N_0.\theta_s \\ \dot{\theta}_e = \frac{\dot{z}}{K} + N_0.\dot{\theta}_s \\ \ddot{\theta}_e = \frac{\ddot{z}}{K} + N_0.\ddot{\theta}_s \end{cases} \tag{16}$$

So, the second equation of the system (12) will become:

$$\frac{J_m}{K}.\ddot{z} + \frac{f_m}{K}.\dot{z} + (J_m.N_0 + J_s).\ddot{\theta}_s + (f_m.N_0 + f_s).\dot{\theta}_s = U \tag{17}$$

Thus, we define a control law U for the global system expressed as follows:

$$\begin{aligned} U \;=\; & \frac{J_m}{K}.\ddot{z}^d + \left(\frac{f_m}{K} + \frac{K_{D1}}{K}\right).\dot{z}^d + \frac{K_{P1}}{K}.z^d & (18) \\ & + (J_m.N_0 + J_s).\ddot{\theta}_s^d + (f_m.N_0 + f_s + K_{D1}.N_0).\dot{\theta}_s^d \\ & + K_{P1}.N_0.\theta_s^d - k_1.\varepsilon_z - K_{D1}.\dot{\varepsilon}_e - K_{P1}.\varepsilon_e & (19) \end{aligned}$$

K_{P1} and K_{D1} are the PD constants of the controller C_1 of Fig. 5.

According to the control scheme of Fig. 5, we have:

$$\theta_e^d = K_{D2}.\dot{\varepsilon}_s + K_{P2}.\varepsilon_s \tag{20}$$

with K_{P2} and K_{D2} represent the controller C_2 constants of Fig. 5.

We replace expression of U in equation (17), we obtain:

$$\begin{aligned} & \tfrac{J_m}{K}.\ddot{\varepsilon}_z + \left(\tfrac{f_m}{K} + \tfrac{K_{D1}}{K}\right).\dot{\varepsilon}_z + \left(k_1 + \tfrac{K_{P1}}{K}\right).\varepsilon_z \\ & + (J_m.N_0 + J_s - K_{D1}.K_{D2}).\ddot{\varepsilon}_s \\ & + (f_m.N_0 + f_s - K_{P1}.K_{D2} - K_{D1}.K_{P2} + K_{D1}.N_0).\dot{\varepsilon}_s \\ & + (K_{P1}.N_0 - K_{P1}.K_{P2}).\varepsilon_s = 0 \end{aligned} \tag{21}$$

This last equation describes the linearization of the motor part of the bench test.

the torque estimator has the same formulation as its model and given by the following expression:

$$\hat{w} = -4.K.\hat{j}_0.\frac{1 - e^{-\gamma.\Delta\theta}}{1 + e^{-\gamma.\Delta\theta}} \tag{22}$$

with γ supposed known.

Let take the case where $\Delta\theta > 0$, then $\tilde{w}$ will take the following expression:

$$\tilde{w} = -4.K.\tilde{j}_0 \tag{23}$$

Now we choose a backlash magnitude model given by:

$$\frac{dj_0}{dt} = 0 \tag{24}$$

with: j_0 the magnitude constant.

Thus, we define an observer for this magnitude depending on the output position and velocity errors, as follows:

$$\frac{d\hat{j}_0}{dt} = -k_2.\dot{\varepsilon}_s - k_3.\varepsilon_s \tag{25}$$

k_2 and k_3 are a constants.

The estimation error of the backlash magnitude is given as follows:

$$\frac{d\tilde{j}_0}{dt} = \frac{dj_0}{dt} - \frac{d\hat{j}_0}{dt} = k_2.\,\dot{\varepsilon}_s + k_3.\varepsilon_s \tag{26}$$

So, the global system will be the combination of all these equations:

$$\begin{cases} J_s.\,\ddot{\varepsilon}_s + f_s.\,\dot{\varepsilon}_s = \varepsilon_z + \tilde{w} \\ \frac{J_m}{K}.\,\ddot{\varepsilon}_z + \left(\frac{f_m}{K} + \frac{K_{D1}}{K}\right).\,\dot{\varepsilon}_z + \left(k_1 + \frac{K_{P1}}{K}\right).\varepsilon_z \\ +(J_m.N_0 + J_s - K_{D1}.K_{D2}).\,\ddot{\varepsilon}_s \\ +(f_m.N_0 + f_s - K_{P1}.K_{D2} - K_{D1}.K_{P2} + K_{D1}.N_0).\,\dot{\varepsilon}_s \\ +(K_{P1}.N_0 - K_{P1}.K_{P2}).\varepsilon_s = 0 \\ \frac{d\tilde{j}_0}{dt} = k_2.\,\dot{\varepsilon}_s + k_3.\varepsilon_s \\ \tilde{w} = -4.K.\tilde{j}_0 \end{cases} \tag{27}$$

Then, the characteristic equation of the global system (27), is defined by the following equation:

$$\begin{aligned} &4.K.J_s.a_3.p^6 + 4.K.\left(a_3.f_s + J_s.a_2\right).p^5 \\ &+ \left(4.K.f_s.a_2 + k_2.a_3 + 4.K.J_s.a_1 + 4.K.b_2\right).p^4 \\ &+ \left(a_3.k_3 + a_2.k_2 + 4.K.f_s.a_1 + 4.K.b_1\right).p^3 \\ &+ \left(a_1.k_2 + a_2.k_3 + 4.k.b_0\right).p^2 + a_1.k_3.p = 0 \end{aligned} \tag{28}$$

with $b_2 = J_m.N_0 + J_s - K_{D1}.K_{D2}$,
$b_1 = f_m.N_0 + f_s - K_{P1}.K_{D2} - K_{D1}.K_{P2} + K_{D1}.N_0$,
$b_0 = K_{P1}.N_0 - K_{P1}.K_{P2}$,
$a_3 = \frac{J_m}{K}$, $a_2 = \frac{f_m}{K} + \frac{K_{D1}}{K}$,
$a_1 = k_1 + \frac{K_{P1}}{K}$.

Now, we establish the imposed conditions on the controllers constants and on k_1, k_2 and k_3, for the global system to converge to the equilibrium state ($\varepsilon_s \to 0$, $\varepsilon_z \to 0$, $\tilde{w} \to 0$), after using the *Routh* criterium.

So, we obtain:

$$\begin{cases} k_1 > 0 \\ k_2 > \frac{a_3.C_6.k_3 + C_3.C_6 - C_5.C_4 - C_5.C_4}{C_5.a_3 - a_2.C_6} \end{cases} \tag{29}$$

with $C_6 = 4.K.J_s.a_3$,
$C_5 = 4.K.\left(a_3.f_s + J_s.a_2\right)$
$C_4 = 4.K.f_s.a_2$,
$C_4 = 4.K.J_s.a_1 + 4.K.b_2$,
$C_3 = 4.K.f_s.a_1 + 4.K.b_1$,
$C_2 = 4.k.b_0$,
$C_1 = a_1.k_3$,

A. Simulation results

The simulation tests are done on a mechanical model representative of the bench test of Fig. 1. The parameters of the model for the simulation tests are:

$K_{P1} = 15$, $K_{D1} = 0.5$, $K_{P2} = 15$, $K_{D2} = 0.3$, $K = 1$ $N.m/rad$, $J_m = 0{,}000972$ $N.m^2$, $f_m = 0{,}00043$ $N.m.s/rad$, $J_s = 7{,}5$ $N.m^2$, $f_s = 16$ $N.m.s/rad$, $j_0 = 0.1$ rad, $k_1 = 1$, $k_2 = 0{,}01$, $k_3 = 1$ and $N_0 = 59$.

For a desired output signal $\theta_s^d(t) = 0{,}5.\sin(0{,}2.\pi.t)$ applied on system Fig. 5, we obtain figure Fig. 6(top) which describes the tracking of the output signal before the compensation of the disturber torque. We notice that the real position signal is less deformed at the peak area due to the presence of the dead zone and flexibility effects. This deformation is compensated after introducing of the torque observer as it is shown in Fig. 6(low), where the output real position signal is approached the desired one with a magnitude difference due the flexibility effects. Fig. 7 describes the hysteresis cycle between the input and output reducer positions. After compensating the dead zone effect, the width of the cycle is reduced. knowing that the flexibility effect is present in the medullization, so the relation between the input and output positions is not exactly linear after compensation. Fig. 8 describes the tracking of the input signal before (top) and after (low) the disturber torque compensation. The tracking is perfect in the two cases but without the perturbations and more clear in the case after compensation. Fig. 9 shows the evolution of the errors corresponding to the output and input positions tracking. After adding the torque observer, the output tracking error is reduced and uniformed for each period. But a static error is still present, due to the presence of the flexibility effects. For the input tracking error, the convergence to zero is verified in the two cases. The presence of the controller $C1$ and $C2$ in the system of Fig. 5 are sufficient to make an error convergence in a finite time, but in the case after compensation, the error signal is less perturbed. Fig. 10 represents the control signals before and after compensation. The signal before compensation is less clean than the after compensation one. This is due to the backlash effects, defined by the disturber torque of Fig. 11, where the dead zone is presented on the transmitted torque curve. After the compensation, we obtain a linear representation of the torque, transmitted via a flexible axis.

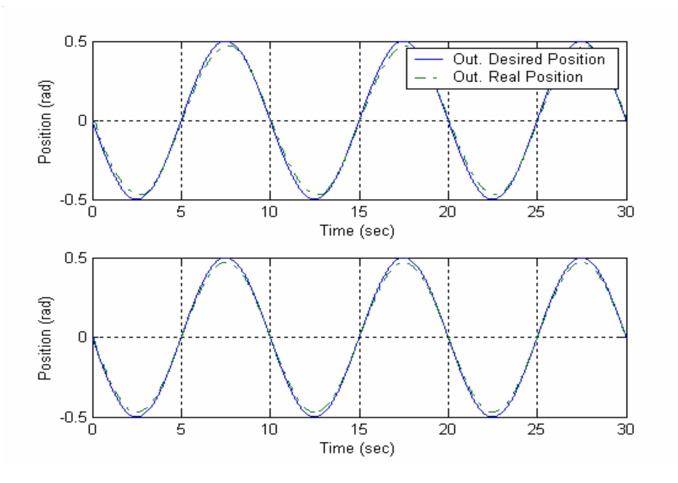

Fig. 6. Desired and Reel output signals before and after compensation

IV. Experimental results

The experimental tests have been applied on the bench test of Fig. 1, with the following control parameters:

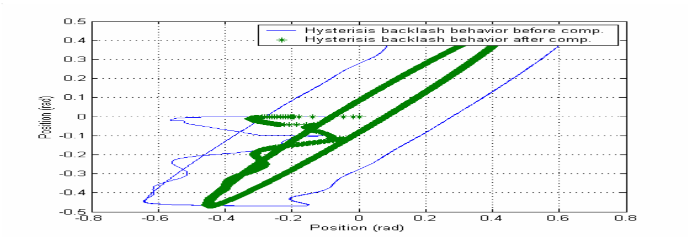

Fig. 7. Hysterisis backlash behaviour before and after compensation

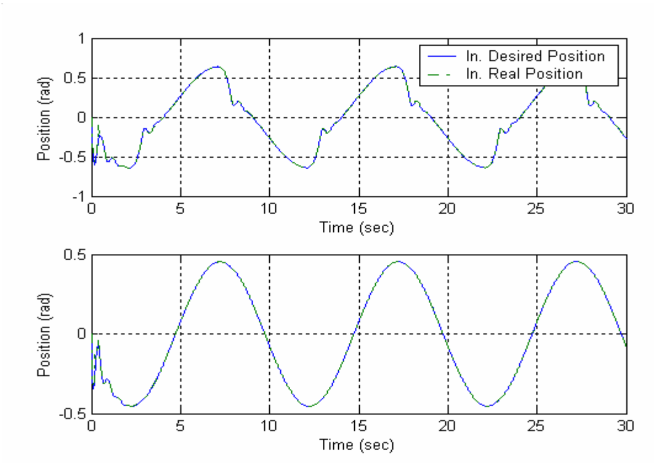

Fig. 8. Desired and Reel input signals before and after compensation

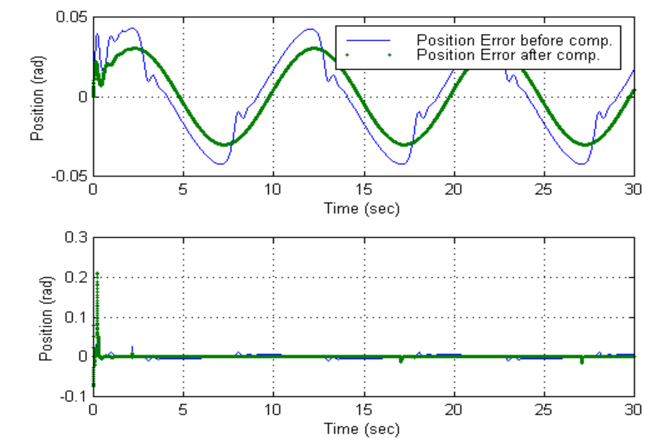

Fig. 9. Position output and input error signals before and after compensation

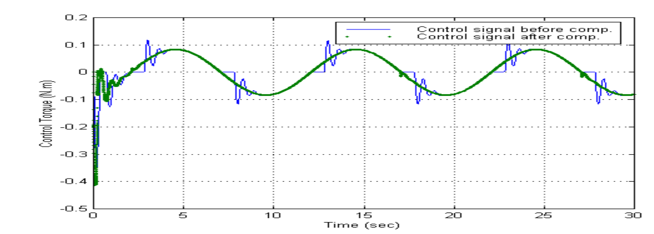

Fig. 10. Control Torque before and after compensation

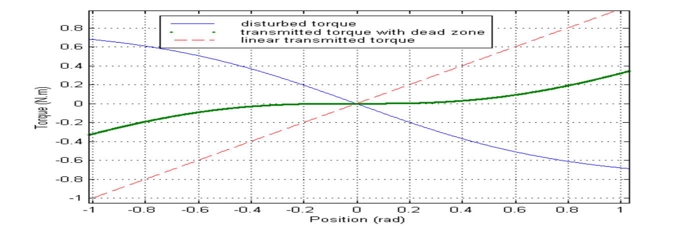

Fig. 11. Compensation of the dead zone after compensation of the disturbed torque

$K_{P1} = 1$, $K_{D1} = 0.01$, $K_{P2} = 10$, $K_{D2} = 10$, $K = 1$ $N.m/rad$, $j_0 = 0.16$ rad, $k_1 = 1$, $k_2 = 0,01$, $k_3 = 1$.

In these tests, the motor-reducer was required to move from the initial static output position $\theta_s(0) = \pi/2\ rad$, and output velocity $\dot{\theta}_s\ (0) = 0\ rad/\sec$ to the origin $\theta_s(0) = 0$ rad, $\dot{\theta}_s\ (0) = 0\ rad/\sec$

Fig.12 represents the tracking output position before (i.e. the regulation is made by only a PD controller) and after applying the adaptive compensation. The static position error is about $0,32$ rad.and it's compensated after adding the estimators of the undesired dead zone torque. Fig.13 shows the output velocity signals, before and after the adaptive compensation. So, before the compensation case, an undesired oscillations around $\dot{\theta}_s\ (0) = 0$ $rad/\sec$ are present and represent the non linearities effects. These imperfection effects are compensated after applying the estimated disturber torque. The control signals before and after compensating the backlash effects are shown in Fig.14. In the case after compensation, the control signal is cleaner than its equivalent before the compensation due to the adaptive compensation of the disturbance effects and backlash. Finally, the supposed disturber torque which introduced the dead zone is represented in Fig. 15.

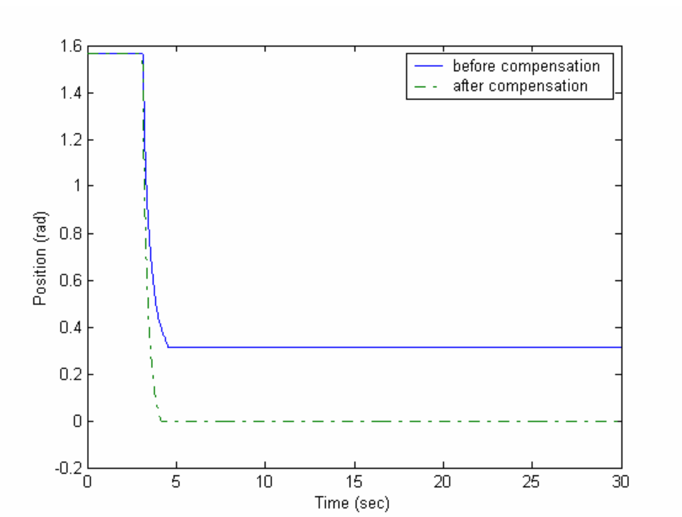

Fig. 12. Output position before and after adaptive compensation

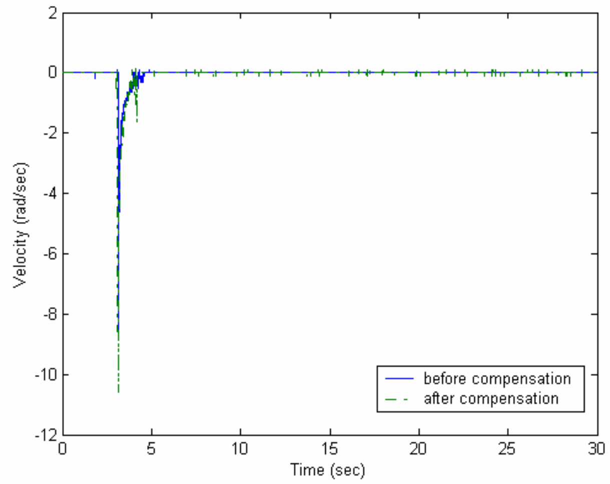

Fig. 13. Output velocity before and after adaptive compensation

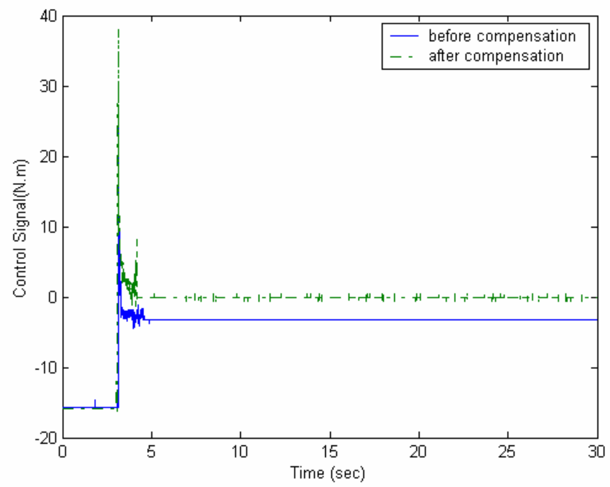

Fig. 14. Control signal before and after adaptive compensation

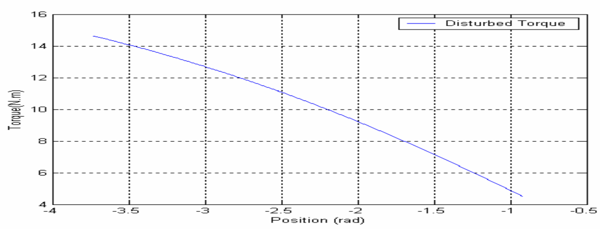

Fig. 15. The estimtion of the disturbed transmitted torque

V. Conclusion

The presence of backlash in mechanical system make difficult its control with high accuracy. So, we could reduce the effects of this last imperfection by estimating the necessary disturber torque inside the dead zone, then adding it in the control law in order to compensate the effect of the real torque. For that, a nonlinear and derivable mathematical model for the disturber torque has been chosen and the dead zone magnitude has been supposed constant value. The estimation of the magnitude variation is observed in function of the output position and velocity errors. A good choice of the control system parameters make the convergence of the global system to the origin state, as it has been shown in the simulation and experimental results.

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