STATE FEEDBACK CONTROL OF A CLASS OF POSITIVE SYSTEMS: APPLICATION TO GAS-LIFT STABILIZATION

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Abstract

A set-stabilizing constrained state feedback controller for a class of nonlinear positive systems is presented. The controller is applied to stabilization of a gas-lifted oil well, and simulations using a rigorous multi-phase flow simulator (OLGA®2000) illustrates the use of the controller.

1 Introduction

Positive systems are dynamical systems that are described by ODEs where the state variables are non-negative. Such systems have been studied for a long time, see for instance [14] for an introduction. It appears *linear* positive systems has gained most interest, see for instance [8] for a recent overview. However, physical systems subject to control will often be described by *nonlinear* positive systems from first principles modeling.

Since mass is an inherently positive quantity, systems modeled by mass balances [1] are perhaps the most natural example of positive systems. Another example is the widely studied class of compartmental systems [9, 12], used in biomedicine, pharmacokinetics, ecology, etc. Compartmental systems, which are often derived from mass balances, are (nonlinear or linear) systems where the dynamics are subject to strong structural constraints. Each state is a measure of some material in a compartment, and the dynamics consists of the flow of material into (*inflow*) or out of (*outflow*) each compartment. If these flows fulfill certain criteria, the system is called compartmental.

Similar to compartmental systems, we will herein assume that each state can be interpreted as the "mass" (or measure of mass; concentration, level, pressure, etc.) of a compartment. However, we do not make the same strong assumptions on the structure on the flows between the compartments. Instead, we make other (strong) assumptions related to the system being controllable according to the control objective under input saturations.

We assume that the compartments that constitute the state can be divided into groups of compartments, which we will call *phases*. Each phase will have a controlled inflow or outflow associated with it. The control objective will be to steer the mass of each phase (the sum of the compartment masses in that phase) to a constant, prespecified value.

The developed state feedback controller is inspired by [2]. However, a larger class of systems is treated, especially since the phase concept allows us to consider multiple input systems. We also allow saturated inputs and more general flows. Similar to [2], the inputs are positive. The controller in [2] can be viewed as a special case of the controller herein.

A related controller for a similar class of systems is developed in [5]. Instead of controlling the system to a constant mass, more general *first integrals* are considered. The controller of [5] is different from the one considered herein, in particular the input can take on negative values. However, the stability properties of the closed loop are similar, in the sense that they both achieve convergence to a certain *set*.

The paper is structured as follows: In Section 2 the system class is presented, while the controller and a convergence result from a general invariant domain of attraction are presented in Section 3. An application to stabilization of gas-lifted oil wells is presented in Section 4.

2 Model class

We consider input affine nonlinear positive systems

$$\dot{x} = f(x,u) = \Phi(x) + \Psi(x) + B(x)u, \tag{1}$$

that is, the state is positive ($x \in \mathbb{R}^n_+$), and the input is positive and upper bounded, $u \in U := \{u \in \mathbb{R}^m_+ \mid 0 \leq u_j \leq \bar{u}_j\}$. Each state can be interpreted as the "mass" (amount of material, or some measure of amount) in a compartment. Loosely speaking, $\Phi(x)$ represents "interconnection structure" between compartments, $\Psi(x)$ represents *uncontrolled* external inflows to and outflows from compartments and $B(x)u$ represents *controlled* external inflows to and outflows from compartments. Systems of this type often arise when developing mechanistic models based on conservation laws, where the states are conserved quantities like mass and energy.

We will assume that the state can be divided into m different parts, which will be denoted *phases*. Phase j will consist of r_j states, and have the control u_j associated with it, corresponding to *either* controlled inflow *or* outflow to compartments of that phase. The states in phase j will be denoted z^j, such that $x = [(z^1)^\top, (z^2)^\top, \ldots, (z^m)^\top]^\top$, and it follows that necessarily, $\sum_{j=1}^m r_j = n$. Corresponding to this structure, the vector functions $\Phi(x)$, $\Psi(x)$ and the matrix function $B(x)$ are on the form

$$\begin{aligned}
\Phi(x) &= \left[\phi^1(x)^\top, \phi^2(x)^\top, \ldots, \phi^m(x)^\top\right]^\top \\
\Psi(x) &= \left[\psi^1(x)^\top, \psi^2(x)^\top, \ldots, \psi^m(x)^\top\right]^\top \\
B(x) &= \text{blockdiag}\left(b^1(x), b^2(x), \ldots, b^m(x)\right).
\end{aligned}$$

Note that element j is (in general) function of x, not (only) z^j. Also note that the partitioning into phases need not be unique.

We will state the assumptions on these functions on the set $D \subseteq \mathbb{R}^n_+$. In the case of global results, $D = \mathbb{R}^n_+$.

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A1. **(Interconnection structure)** The function $\Phi : D \to \mathbb{R}^n$ is locally Lipschitz, $\phi_i^j(x) \geq 0$ for $z_i^j = 0$, and

$$\sum_{i=1}^{r_j} \phi_i^j(x) = 0,\ j = 1, \ldots, m.$$

A2. **(Controlled external flows)** The block diagonal matrix function $B(x) : D \to \mathbb{R}^{n\times m}$ is locally Lipschitz and satisfies:

a. Phase j has controlled inflow:

$$b_i^j(x) \geq 0 \text{ for all } x \in D$$
$$b_i^j(x) > 0 \text{ for all } x \in D \text{ for at least one } i$$

b. Phase j has controlled outflow:

$$b_i^j(x) \leq 0 \text{ for all } x \in D$$
$$\text{if } \exists x \in D \text{ such that } z_i^j = 0, \text{ then } z_i^j = 0 \Rightarrow b_i^j(x) = 0$$
$$b_i^j(x) < 0 \text{ for all } x \in D \text{ with } z_i^j \neq 0, \text{ for at least one } i$$

The uncontrolled external flows must satisfy some "controllability" assumption in relation to the controlled flows. Before we define this, it is convenient to define the "mass" of each phase, being the sum of the compartment masses of that phase: $M_j(x) := \sum_{i=1}^{r_j} z_i^j$. Our control objective will be to control $M_j(x)$ to some prespecified desired mass of phase j, denoted M_j^*, from initial conditions in D. For the control problem to be meaningful, the intersection of the set where $M_j(x) = M_j^*$ and D should be nonempty.

A3. **(Uncontrolled external flows)** For given $M^* = [M_1^*, M_2^*, \ldots, M_m^*]^\top$, $\Psi(x) : D \to \mathbb{R}^n$ is locally Lipschitz and satisfies that $\psi_i^j(x) \geq 0$ for $z_i^j = 0$, and in addition, if:

a. Phase j has controlled inflow:

1. For $x \in \{x \in D \mid M_j(x) > M_j^*\}$, $\sum_{i=1}^{r_j} \psi_i^j(x) \leq 0$ and the set $\{x \in D \mid \sum_{i=1}^{r_j} \psi_i^j(x) = 0$ and $M_j(x) > M_j^*\}$ does not contain an invariant set.
2. For $x \in \{x \in D \mid M_j(x) < M_j^*\}$, $-\sum_{i=1}^{r_j} \psi_i^j(x) < \sum_{i=1}^{r_j} b_i^j(x)\bar{u}_j$.

b. Phase j has controlled outflow:

1. For $x \in \{x \in D \mid M_j(x) < M_j^*\}$, $\sum_{i=1}^{r_j} \psi_i^j(x) \geq 0$ and the set $\{x \in D \mid \sum_{i=1}^{r_j} \psi_i^j(x) = 0$ and $M_j(x) < M_j^*\}$ does not contain an invariant set.
2. For $x \in \{x \in D \mid M_j(x) > M_j^*\}$, $\sum_{i=1}^{r_j} \psi_i^j(x) < -\sum_{i=1}^{r_j} b_i^j(x)\bar{u}_j$.

We briefly note that the upper saturations $\bar{u}_j$ can be state dependent, without affecting the main results. The chosen notation will not reflect this.

Proposition 1 (Positivity) *For $x(0) \in \mathbb{R}_+^n$, the state of the system (1) fulfilling A1-A3 with $D = \mathbb{R}_+^n$, satisfies $x(t) \in \mathbb{R}_+^n$, $t > 0$.*

Proof. If suffices to notice that for $x_i = 0$, $\dot{x}_i \geq 0$. ∎

3 State feedback total mass controller

In this section, the state feedback controller is defined, and a general convergence result is given for a general invariant set D that the assumptions hold on. The set D could then be considered a region of attraction. Corollaries of the main result specifies different set D that could be chosen, for instance $D = \mathbb{R}_+^n$.

3.1 The controller and a convergence result

As mentioned in the previous section, our control objective is to control the total mass $M_j(x)$ of each phase to a prespecified value M_j^*.

To this end, the following constrained, positive state feedback control is proposed:

$$u_j(x) = \begin{cases} 0 & \text{if } \tilde{u}_j(x) < 0 \\ \tilde{u}_j(x) & \text{if } 0 \leq \tilde{u}_j(x) \leq \bar{u}_j \\ \bar{u}_j & \text{if } \tilde{u}_j(x) > \bar{u}_j \end{cases} \tag{2}$$

where

$$\tilde{u}_j(x) = \frac{1}{\sum_{i=1}^{r_j} b_i^j(x)} \left(-\sum_{i=1}^{r_j} \psi_i^j(x) + \lambda_j (M_j^* - M_j(x)) \right) \tag{3}$$

and λ_j is a positive constant. In the unconstrained case, the controller linearizes the phase mass (M_j) dynamics.

The controller can be seen as a generalization of the controller in [2] to systems with multiple inputs, to systems with controlled outflow and to systems with upper constraints on the input. Apparently, we can run into situations where the control is not defined if phase j is outflow controlled, since the term $\sum_{i=1}^{r_j} b_i^j(x)$ then might be zero. However, the continuity of the involved functions and the upper bound on the control ensures that the control in these cases unambiguously is defined by $u_j(x) = \bar{u}_j$.

Define the set

$$\Omega = \{x \in \mathbb{R}_+^n \mid M_1(x) = M_1^*, \ldots, M_m(x) = M_m^*\}. \tag{4}$$

Assumption 1 *There exists a set D that is invariant for the dynamics (1) under the closed loop with control (2), and has a nonempty intersection with Ω.*

Assumption 2 *For $x \in \Omega \cap D$, $0 < \tilde{u}_j(x) < \bar{u}_j$.*

Theorem 1 *Under the given assumptions, the state of the system (1), controlled with (2) and starting from some initial condition $x(0) \in D$, stays bounded and converges to the positively invariant set $\Omega \cap D$.*

Proof. The set D is by Assumption 1 invariant, hence Assumptions A1-A3 hold along closed loop trajectories.

Define the positive semidefinite function

$$V(x) := \frac{1}{2} \sum_{j=1}^{m} \left(M_j(x) - M_j^*\right)^2, \tag{5}$$

with time derivative

$$\dot{V}(x) = \sum_{j=1}^{m} \left[M_j(x) - M_j^*\right] \left(\sum_{i=1}^{r_j} \psi_j^i(x) + \sum_{i=1}^{r_j} b_j^i(x) u_j(x) \right).$$

For $M_j(x) \neq M_j^*$, we have one of the following cases:

1. If $0 \leq \tilde{u}_j \leq \bar{u}_j$, summand j is

$$\left[M_j(x) - M_j^*\right] \left(\sum_{i=1}^{r_j} \psi_j^i(x) + \sum_{i=1}^{r_j} b_j^i(x) u_j(x) \right) = -\lambda_j \left[M_j(x) - M_j^*\right]^2 < 0.$$

2. If $\tilde{u}_j < 0$, then $u_j(x) = 0$ and summand j is

$$\left[M_j(x) - M_j^*\right] \sum_{i=1}^{r_j} \psi_j^i(x).$$

   Assumption A3.a.1 and A3.b.1 ensures that this is negative for both inflow and outflow controlled phases.

3. If $\tilde{u}_j \geq \bar{u}_j$, then $u_j(x) = \bar{u}_j$ and summand j is

$$\left[M_j(x) - M_j^*\right] \left(\sum_{i=1}^{r_j} \psi_j^i(x) + \sum_{i=1}^{r_j} b_j^i(x) \bar{u}_j \right).$$

   Assumption A3.a.2 and A3.b.2 ensures that this is negative for both inflow and outflow controlled phases.

For the details of point 2 and 3, check [11]. We can conclude that $\Omega \cap D$ is invariant, since for $x \in \Omega \cap D$, $V(x) = 0$ and by Assumption 2 and the above, $\dot{V}(x) < 0$ in the intersection between a neighborhood of $\Omega \cap D$ and the invariant set D.

Moreover, since $\dot{V}(x) \leq 0$, $V(x(t)) \leq V(x(t_0))$ along system trajectories. From the construction of $V(x)$ and invariance of $(D \subseteq)\mathbb{R}_+^n$, it is rather easy [11] to see that for $x \in \mathbb{R}_+^n$, $\|x\| \to \infty$ if and only if $V(x) \to \infty$, hence $V(x(t))$ bounded implies that $\|x(t)\|$ is bounded. This allows us to conclude from LaSalle's invariance principle that $x(t)$ converges to the largest invariant set contained in $\{x \mid \dot{V}(x) = 0\} \cap D$. By the above and Assumption A3.a.1 and A3.b.1, there is no other invariant set for which $\dot{V}(x) = 0$ other than $\Omega \cap D$. ■

Although this theorem merely shows convergence to the set Ω, it is possible to prove that Ω is asymptotically stable [11].

To use this theorem, we need to find invariant sets D. In some cases, the assumptions hold globally and we can use $D = \mathbb{R}_+^n$. In other cases, it is possible to choose sets of the shape $D = D_1$ or $D = D_2$, where

$$D_1 := \{x \in \mathbb{R}_+^n | M_j^* - \underline{c}_j \leq M_j(x) \leq M_j^* + \overline{c}_j,\ j = 1, \ldots, m\}$$

and

$$D_2 := \{x \in \mathbb{R}_+^n \mid \underline{z}_j^i \leq z_j^i \leq \overline{z}_j^i,\ i = 1, \ldots, r_j \text{ and } M_j^* - \underline{c}_j \leq M_j(x) \leq M_j^* + \overline{c}_j,\ j = 1, \ldots, m\}.$$

For further details and examples, we refer to [11].

Theorem 1 shows that the state converges to the subset Ω, which often (somewhat inaccurate) is referred to as set stability. In many applications, stability of equilibria is arguably more interesting. It is thus interesting to note that the controller (2) often (but not always, as the counterexample in [11] reveals) leads to a stable equilibrium. A sufficient condition for an asymptotically stable equilibrium can be found from the theory of semidefinite Lyapunov functions, see e.g. [4]. Here, we state the following Theorem which can be proved in a similar way as Theorem 5 in [5]:

Theorem 2 *Let the conditions of Theorem 1 hold. If the closed loop (1) has a single equilibrium in the interior of $\Omega \cap D$ that is asymptotically stable with respect to initial conditions in $\Omega \cap D$ and attractive for all initial conditions in $\Omega \cap D$, the equilibrium is asymptotically stable for the closed loop with a region attraction (of at least) D.*

The proposed feedback scheme is independent of the interconnection structure and hence robust[1] to model uncertainties in $\Phi(x)$ (as long as Assumption A1 holds). As mentioned in [2], the interconnection terms are in practical examples often the terms that are hardest to model. Moreover, the unconstrained controller also has some robustness-properties with respect to bounded uncertainties in $\Psi(x)$ and $B(x)$. For details on this, we refer to [11], but briefly note that convergence holds to a set containing Ω, where the parameters λ_j decides the size of the set.

In the next section, the controller will be used for stabilizing a gas-lifted oil well.

4 Stabilization of flow in gas-lifted oil wells

4.1 Gas-lifted oil wells

The use of hydrocarbons is essential in modern every-day life. In nature, hydrocarbons are typically found in petroleum-bearing geological formations (*reservoirs*) situated under the earth's crust, and hydrocarbons from these reservoirs are produced by means of an *oil well*.

An oil well is made by drilling a hole (wellbore) into the ground. A metal pipe (casing) is placed in the wellbore to secure the well, before "downhole well completion" is performed by running the production pipe (tubing), packing and possibly valves and sensors into the well and perforate the casing to make the reservoir fluid flow into the well. Detailed information on wells and well completion can e.g. be found in [10], see also Figure 1.

If the reservoir pressure is high enough to overcome the back pressure from the flowing fluid column in the well and the surface (topside) facilites, the reservoir fluid can flow to the surface. In some cases, the reservoir pressure is not high enough to make the fluid flow freely, at least not at the desired rate. A remedy is then to inject gas close to the bottom of the well, which will mix with the reservoir fluid, see Figure 1. The gas is transported from the topside through the gas-lift choke into the annulus (the space between the casing and the tubing), and enters the tubing through the injection valve close to the bottom of the well. The gas will help to "lift" the oil out of the

[1] Robust in the sense that convergence to Ω still holds. Note that changes in $\Phi(x)$ will typically move the equilibria on Ω.

tubing, through the production choke into the topside process equipment (separator). This is the type of oil well we will consider herein. A problem with these type of wells, is that they can become (open loop) unstable, characterized by highly oscillatory well flow (*casing heading*). The flow regime of the well (tubing) in this case is denoted *slug flow*. The two main factors that induce casing heading, is high compressibility of gas in the annulus, and gravity dominated pressure drop in the two-phase flow in the tubing.

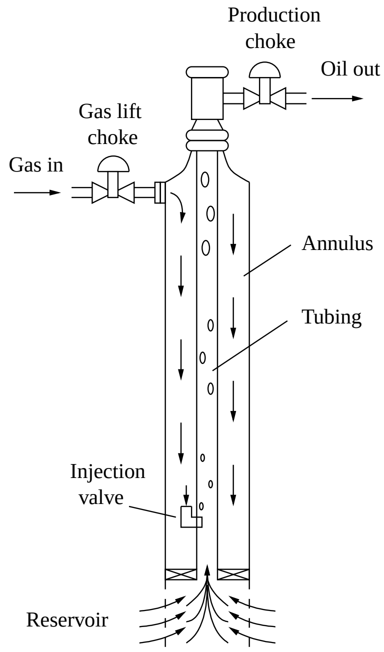

Figure 1: A gas-lifted oil well.

The oil production for a typical oscillating well can be seen in Figure 2. This slug flow is undesirable since it creates operational problems for downstream processing equipment. Further, stabilizing the slug flow in the well leads to increased production, as illustrated in Figure 3. The casing heading problem is industrially important, as a considerable amount of such wells exhibit slug flow. This (or similar) control problems are considered in e.g. [13, 6].

For simplicity, we will assume that the reservoir contains only oil, which is a good approximation if the fraction of gas and water is low. However, the same procedure as taken herein can be taken for wells with higher gas and water production, assuming the amounts are (approximately) known. We assume realistic boundary conditions, that is, constant separator pressure (downstream the production choke), constant gas injection pressure (upstream the gas injection choke) and constant reservoir pressure (far from the well). The (vertical) well is 2km deep and the high fidelity model is modelled in OLGA 2000 dividing both the tubing and the annulus into 25 volumes.

In the sequel we develop a simplified model for control design and analysis, and finally assess the controller on the high-fidelity OLGA model.

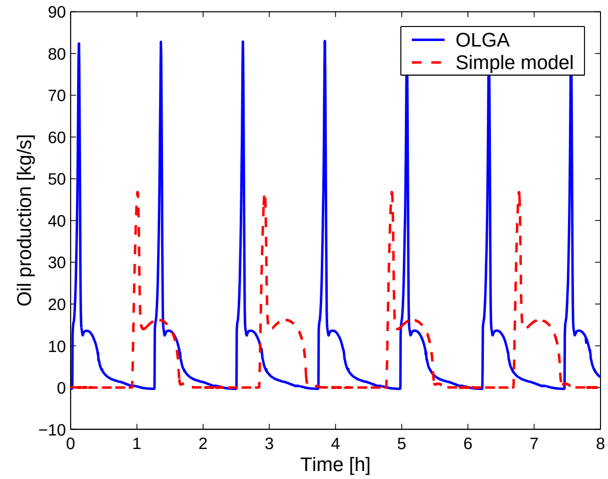

Figure 2: Comparison of open loop (gas-lift choke is 50% open, production choke is 80% open) behavior between simple model and the rigouros multiphase flow simulator OLGA®2000 [3, 15].

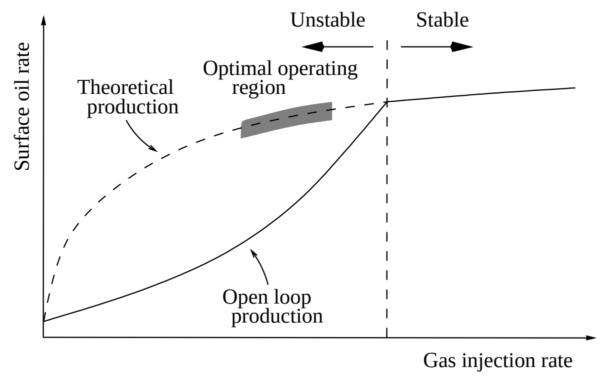

Figure 3: Oil production as a function of gas injection rate. The dotted line is based on steady state calculations, while the solid line is based on dynamic simulations.

4.2 A model of a gas-lifted oil well

As discussed above, the mechanisms that makes the well produce in slugs, are related to the mass of gas in the annulus (compressibility) and the mass of fluid in the tubing (gravity). Consequently, it is reasonable to believe that an ODE based on mass balances will give a good description of the dynamic behavior of the well,

$$\begin{aligned} \dot{x}_1 &= -w_{gc}(x) + w_{iv}(x, u_1) && \text{mass of gas, annulus} \\ \dot{x}_2 &= w_{iv}(x) - w_{pg}(x, u_2) && \text{mass of gas, tubing} \\ \dot{x}_3 &= w_r(x) - w_{po}(x, u_2) && \text{mass of oil, tubing} \end{aligned}$$

where w_{gc} is the flow of gas through the gas injection choke, w_{iv} is the flow of gas through the injection valve, w_{pg} and w_{po} are the flow of gas and oil through the production choke and $w_r(x)$ is the inflow of oil from the reservoir. The challenge in making such a model, is to find the relation between the system masses (x) and the pressures in the system that determines the flows (w). For reasons of space we do not go into this, but refer to [11], and note that the three state model gives a reasonable approximation to the OLGA model as shown in Figure 2.

4.3 State feedback control

The system written as above can fulfill (a slightly modified) Assumption A2. However, as the expressions for the flows are rather inaccurate (especially for the multiphase flow through the production choke) we will assume that the flow of gas through the gas-lift choke and the flow of oil through the production choke are measured, and that fast control loops control these measured variables. The setpoints for these loops will be the new manipulated variables. This will, in addition to being a more sensible engineering approach, simplify the equations.

The dynamic model with the manipulated flows as controls, is

$$\dot{x}_1 = -w_{iv}(x) + v_1 \tag{6a}$$
$$\dot{x}_2 = w_{iv}(x) - w_{pg}(x, u_2(x)) \tag{6b}$$
$$\dot{x}_3 = w_r(x) - v_2 \tag{6c}$$

We choose as phases the sum of gas in the tubing and annulus ($x_1 + x_2$, phase 1) and the oil in the tubing (x_3, phase 2). The upper saturations on both v_1 and v_2 (the maximum flows through the gas-lift choke and the production choke) depend on the state (through the pressures). Noting that the maximum flows are always obtained when the chokes are maximally open, Assumption A3 can be checked for these saturations. Denote the maximum flows as $\bar{v}_1(x)$ and $\bar{v}_2(x)$, which are given by inserting $u_1 = u_2 = 1$ into the expressions for $w_{iv}(x, u_1)$ and $w_{po}(x, u_2)$.

Then, for $j \in \{1, 2\}$, the controller is given by

$$v_j(x) = \begin{cases} 0 & \text{if } \tilde{v}_j(x) < 0 \\ \tilde{v}_j(x) & \text{if } 0 \le \tilde{v}_j(x) \le \bar{v}_j(x) \\ \bar{v}_j(x) & \text{if } \tilde{v}_j(x) > \bar{v}_j(x) \end{cases} \tag{7}$$

where

$$\tilde{v}_1(x) = w_{pg}(x, u_2(x)) + \lambda_1(M_g^* - x_1 - x_2) \tag{8}$$
$$\tilde{v}_2(x) = w_r(x) - \lambda_2(M_o^* - x_3). \tag{9}$$

4.4 Analysis

For a detailed analysis of stability and some notes on performance, we refer to [11]. Here, we briefly note that for the simple mass balance model of the oil well, asymptotic stability of an equilibrium follows from Theorem 1 and 2 for $M_g^* = 4400$ kg and $M_o^* = 4600$ kg, and with the set D chosen as

$$3640 \le x_1 \le 4240, \quad 510 \le x_2 \le 590, \quad 4550 \le x_3 \le 4650.$$

Simulations show that the real region of attraction is larger than the one found above, but not global. For instance, if the system is started in a "no production" state (tubing filled with oil – $x_2 = 0$), the system must be brought to a producing condition before the controller is turned on. This is due to the saturation of the chokes. If the tubing is filled with oil, the casing can be filled with enough gas such that $x_1 + x_2 = M_g^*$, without gas being inserted into the tubing. The "oil controller" tries to decrease the amount of oil, but is unsuccessful since the well cannot produce oil with no gas inserted. Increasing M_g^* (temporarily) might be a solution in this case.

4.5 OLGA simulations

Using the OSI[2] link between OLGA and Matlab, the controller, implemented in Matlab, was used on a well modeled in OLGA. The simulation results are shown in Figure 5 and 4. Note that these are state feedback simulation results, the masses and flows were assumed measured.

In the simulations, the well is operated in open loop the two first hours. In this period, the well is stabilized by using a high opening of the gas-lift choke ($u_1 = 0.7$) and a low opening of the production choke ($u_2 = 0.4$). Then, the controller (with $M_g^* = 3450$ kg and $M_o^* = 9400$ kg) is switched on, and remains on for three hours. We see that the controller stabilizes the well at a higher production, and with a significantly lower use of injection gas. The controller is switched off after 5 hours, keeping the inputs constant. It is seen that this operating point is open loop unstable. In Figure 5, we see that the controller does not quite reach the mass setpoints. This is due to the fact that the OLGA simulator takes into account the flashing phenomena, hence there is mass leaving the oil phase which enters the gas phase, which the controller does not account for. This can be interpreted as errors in the external flows, which the controller is robust to as discussed in Section 3. The influence is more pronounced in the gas phase, since the external flow in the oil phase is larger than in the gas phase. Simulations indicate that larger λ's ($\lambda_1 = \lambda_2 = 0.001\text{s}^{-1}$ was used in the simulations shown) reduces the steady state error. Choosing too high λ's leads to problems with saturations, and also numerical problems may occur. However, increasing the λ's by, say, a decade do not introduce problems. Another remedy for reducing this offset is by including an estimate of the flashing in the equations.

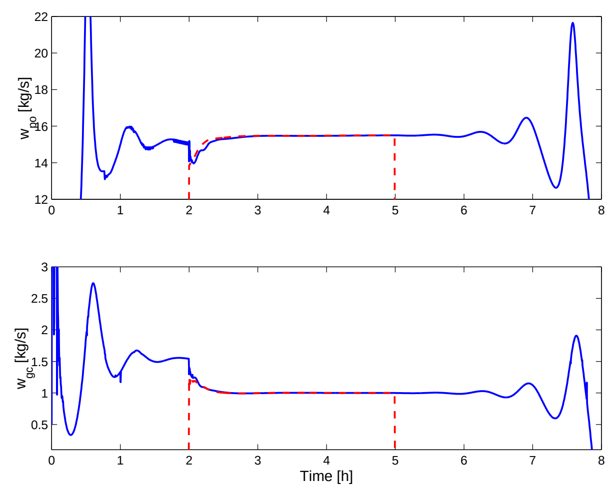

Figure 4: Desired oil production and gas injection calculated by controller (- -) and the "real" values (-), OLGA simulation

4.6 Discussion of the gas-lift stabilization controller

Both analysis on the simple model and simulations on the multiphase flow simulator OLGA, confirm that the developed controller stabilizes the flow in the gas-lifted well.

[2] OLGA Server Interface (OSI) toolbox, for use with Matlab, developed by ABB Corporate Research.

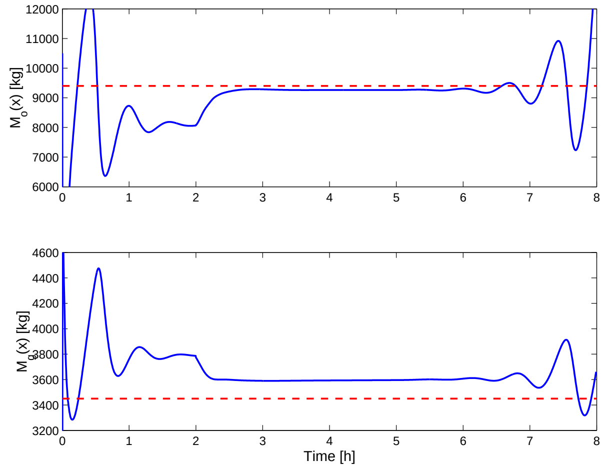

Figure 5: Mass of phase 1 (gas) and 2 (oil) vs. setpoint, OLGA simulation

The controller calculates the desired inflow of gas to the annulus, and the desired outflow of oil from the tubing. We choose to use inner control loops to obtain these desired flows, which means the total control structure can be seen as a cascaded design. Because of choke rate saturation (the choke stroke time in the OLGA simulations was 7 min.), these inner control loops cannot be infinitely fast, but simulations show that the delays introduced by these rate saturations do not have significant influence on the closed loop behavior. The inner control loops cannot both be integral controllers, since the steady state value of the controlled variables are dependent.

Note that the developed state feedback controller is independent of the flow through the injection valve, $w_{iv}(x)$, and hence is robust to modeling errors in this flow. This is in contrast to the fact that the system can be open loop stabilized (or destabilized) by the characteristics of this valve. In some cases this valve is designed to always be in a critical flow condition, effectively decoupling the annulus dynamics from the tubing dynamics. Even though this takes care of the instability problem, operational degrees of freedom are lost compared to the approach herein since it implies a constant, given at the design stage, gas injection into the tubing.

In the controller, the states (masses) and expressions for the external flows are needed. For a real well, these are not easily obtained by measurements. However, there is a development towards more advanced instrumentation systems, which may be used to design a nonlinear state observer based on the simple nonlinear well model, for example an extended Kalman filter. Initial results in this regard can be found in [7].

Much work remains on the connection between the mass setpoints (M_g^* and M_o^*) and the performance of the well. In the simulations shown, an unsatisfactorily “trial-and-error”-method was used to find these values.

An approach where the oil and gas are treated as a single phase can also be developed by the theory in this paper. In this case, the production choke must be used to control the (total) outflow of mass. Simulations (not included herein) show that this control strategy also stabilizes the well. Such a control strategy can be advantageous, since there can be situations where the gas-lift choke is not available for control, for example if the amount of available lift gas topside is given by production constraints. Other advantages are that the controller is independent (and hence robust) to mass transfer between the oil and gas phase in the tubing, and that tuning (in terms of total mass setpoint) is significantly easier. The expected disadvantages are a smaller region of attraction, and that the achievable performance of the well (the oil production) is lower.

5 Concluding remarks

A controller for a class of positive systems is suggested, leading to closed loop stability of a set. The main restriction of the system class is the assumptions (Assumption A3) that ensure that the “Lyapunov function” used in the proof of the main result is decreasing when the input saturates.

The controller is successfully implemented on an oil well simulated in the multiphase flow simulator OLGA 2000.

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References

[1] G. Bastin. Issues in modelling and control of mass balance systems. In D. Aeyels, F. Lamnabi-Lagarrigue, and A. van der Schaft, editors, *Stability and stabilization of Nonlinear Systems*. Springer, 1999.

[2] G. Bastin and L. Praly. Feedback stabilisation with positive control of a class of dissipative mass-balance systems. In *Proc. 14th IFAC World Congress, Beijing, China*, 1999.

[3] K. H. Bendiksen, D. Malnes, R. Moe, and S. Nuland. The dynamic two-fluid model OLGA: Theory and application. *SPE Production Engineering*, pages 171–180, 1991.

[4] R. Chabour and B. Kalitine. Semi-definite Lyapunov functions - stability and stabilizability. *IEEE Trans. Aut. Control*, 2002. To appear.

[5] P. De Leenheer and D. Aeyels. Stabilization of positive systems with first integrals. *Automatica*, 38(9):1583–1589, 2002.

[6] G. O. Eikrem, B. A. Foss, L. Imsland, H. Bin, and M. Golan. Stabilization of gaslifted wells. In *Proc. 15th IFAC World Congress, Barcelona, Spain*, 2002.

[7] G. O. Eikrem, L. Imsland, and B. A. Foss. Stabilization of gas lifted wells based on state estimation, 2003. Proc. Adchem 2003.

[8] L. Farina. Positive systems in the state space approach: Main issues and recent results. In *Proceedings of MTNS*, 2002.

[9] K. Godfrey. *Compartmental models and their application*. Academic Press Inc. [Harcourt Brace Jovanovich Publishers], London, 1983.

[10] M. Golan and C. H. Whitson. *Well Performance*. Prentice-Hall, 2nd edition, 1991.

[11] L. S. Imsland. *Topics in Nonlinear Control - Output Feedback Stabilization and Control of Positive Systems*. PhD thesis, Dept. of Eng. Cyb., NTNU, 2002.

[12] J. A. Jacquez and C. P. Simon. Qualitative theory of compartmental systems. *SIAM Rev.*, 35(1):43–79, 1993.

[13] B. Jansen, M. Dalsmo, L. Nøkleberg, K. Havre, V. Kristiansen, and P. Lemetayer. Automatic control of unstable gas lifted wells. In *SPE Annual Technical Conference and Exhibition*, October 1999. Paper no.: SPE 56832.

[14] D. Luenberger. *Introduction to dynamic systems*. John Wiley & Sons, 1979.

[15] Scandpower. *OLGA 2000 user's manual*, 2000.