MIN-MAX MODEL PREDICTIVE CONTROL OF A LABORATORY PLANT

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Abstract

This work describes the application of a Min-Max predictive controller to a control laboratory plant. Min-max formulations of Model Based Predictive Control (MBPC) are one of the possible approaches in the literature to deal with the control of plants subject to bounded uncertainties. One of the drawbacks of Min-Max MBPC is the amount of calculation required to find a control sequence. The controller used in this paper is relatively efficient numerically: the control sequence is calculated solving a linear programming problem with a reasonable number of constraints. This allows the calculation time to be small enough to apply the controller to reasonably fast real systems. The Min-Max MBPC was tested on-line in a two-tank laboratory plant under different working situations to check the robustness, stability and general performance of the designed controller. The results are quite encouraging.

1 Introduction

MBPC is an advanced control strategy that has reached great acceptance both in industry and in academia. There are several reasons for this popularity: the basic ideas are easy to follow and these techniques have been successfully applied to rather different processes. Moreover, MIMO formulations and the inclusion of constraints on the variables of interest are straightforward.

However, there is a drawback that appears many times in the literature: the need of stability and robustness results. Of course, there are interesting results that have appear during the last years giving an answer to this requirement ([4], [6], [7], [8], [12]).

One of the possible approaches followed to obtain the desired stability and robustness in predictive control is the Min-Max approach. A Min-Max formulation of MBPC involves the minimization of the maximum of the cost function that defines the error between the predicted output sequence and the reference trajectory with respect to the control sequence (or the rate of variation of the control sequence). Depending on how this is done, different Min-Max predictive controllers have been proposed.

Min-Max MBPC is a robust predictive control strategy, the main drawback that can be pointed out is the computational burden and the difficulty of using closed-loop information to reduce conservativeness. Different authors ([1], [2], [3], [6], among others) have proposed ways to reduce to the Min-Max computational effort.

Campo and Morari [6] pioneered a Min-Max predictive controller with an ∞-norm based criterion. They transformed the min-max problem into a linear program (LP). Unfortunately the number of constraints could be large. Allwright and Papavasillou [2] developed a different LP, with a smaller number of constraints, for solving the same min-max problem. Kim and Kwon [9] gave an interesting solution to the Min-Max predictive control problem: the control law is explicitly obtained. From a computational point of view this is very fast, but there are no constraints on the input or output variables.

The Min-Max Model Predictive Controller by Álamo *et al.* [1] calculates rather fast the control to be applied, but, again, no constraints are considered. The work by Ramírez and Camacho [13] is interesting in two ways: it is one of the few applications to a real process and the numerical approach is rather innovative. But as the authors remarked in the conclusions, further work should be done to include constraints.

Megías *et al.* [12] implemented a quasi-infinite horizon 1-norm GPC combined with a global uncertainty description and an uncertainty band-updating procedure such that only a LP problem is solved to compute the control law.

The approach followed in this paper is an extension of the one presented by Allwright in [3]. The reasons to select this approach were: constraints are considered, a simple prediction model is used and the Min-Max problem is solved as a LP problem. Taking [3] as a starting point, some features have been made to obtain better results for a real plant.

It is clear conceptually that this type of min-max approach leads to very conservative control policies. Nevertheless we thought it important to see what actually happens when such policies are applied to laboratory equipment, as a starting point for robustness studies.

The paper is structured as follows: section 2 gives an introduction to the Min-Max predictive control technique used, section 3 describes the two-tanks plant where the method has been applied, section 4 presents results from the implementation, and finally some conclusions are given and future work is discussed.

2 Min-Max Model Predictive Control

Model Based Predictive Control [5][11] is a control strategy based on the explicit use of a model to predict the process output over a period of time. At each sampling time the future control signals are calculated by minimization of a cost function, which is usually defined as a weighted combination of tracking errors and control variations. A receding control horizon technique is normally applied: the calculations are repeated every sampling time, to take into account the difference between the predicted value and the measured value.

Many of the formulations of MBPC cannot guarantee a robust behaviour when there mismatches between the prediction model and the plant or under the presence of disturbances. Min-Max MBPC can deal with these problems. As has been pointed out above, the presented in this paper is based on the Min-Max MBPC of [3] with some changes to improve the performance and reduce the computational time when working on a real system.

The prediction model from [3] has output at time $k+l$, where k denotes the present time, given by

$$y^{k+l}(\xi,u,v)=\sum_{i=1}^{N}H_{\xi}^{i}u^{k+l-i}$$

where $u=\left[\left(u^{k}\right)' \quad \left(u^{k+1}\right)' \quad \dots \quad \left(u^{k+M-1}\right)'\right]'$ is the vector of 'future' controls and $v=\left[\left(u^{k-1}\right)' \quad \left(u^{k-2}\right)' \quad \dots \quad \left(u^{k-N}\right)'\right]'$ is the 'initial condition' at time k.

Here $\xi\in\Xi:=\{\xi\in R^{q}:\underline{\xi_j}\le\xi_j\le\overline{\xi_j},1\le j\le q\}$ parametrises the uncertainty in the impulse response terms H_{ξ}^{i} according to

$$H_{\xi}^{i}=\sum_{j=1}^{q}H^{ij}\xi_j\ .$$

The vector of future outputs at time k is

$$y(\rho,u,v)=\begin{bmatrix} y^{k+1}\left(\xi^{k+1},u,v\right)' \\ y^{k+2}\left(\xi^{k+2},u,v\right)' \\ \vdots \\ y^{k+M}\left(\xi^{k+M},u,v\right)' \end{bmatrix}'$$

and the desired future output at time $k+l$ is s^{k+l}.

The min max problem solved at each 'present time' k in order to implement the receding horizon control law is:

$$\min_{\substack{\underline{u}\le u\le\overline{u} \\ \underline{y}\le y(\rho,u,v)\le\overline{y},\forall\rho\in\Xi}} \quad \max_{\rho\in\Xi}\sum_{i=1}^{N}\left\|y^{k+i}(\rho,\mathrm{u},\mathrm{v})-\mathrm{s}^{k+i}\right\|_{\infty}$$

For the application described here, the above formulation has been modified in the following ways.

1 The prediction and control horizons have been allowed to be different, instead of identical. This enables a reduction in the number of variables and the computation time and also allows less constrained responses to be accepted.

2 The cost interval has changed from {1:M}:={1,2, …, M} to $\{N_1,N_2\}$ so that it is possible to ignore the system response for times k+1, …, k+ N_1-1 which is very useful when, for example, the system has an initial inverse response.

3 In the algoritm of [3] for solving the min max problem, an important step involves writing H_{ξ}^{i} as $H_{\eta}^{i}+\sum_{j=1}^{q}H^{ij}\delta_j$ where η is the center of interval Ξ, where δ=ξ-η. We have found that numerical precision is improved if η is normalized to $\left(\sum_{j=1}^{q}\eta_j\right)^{-1}\eta$.

4 Constraints on the rate of change $\Delta u^{k+l}=u^{k+l}-u^{k+l-1}$ and $\Delta^2u^{k+l}=\Delta u^{k+l}-\Delta u^{k+l-1}=u^{k+l}-2u^{k+l-1}+u^{k+l-2}$ of the input have been introduced. These two types of constraints allow to take into account the variations of the control signal.

5 The desired future output s will be referred as reference trajectory and it can be calculated in different ways. The approach followed in this work is to determine it along the prediction horizon as a first order filter: $s^{k+l}=\alpha\, s^{k+l-1}+(1-\alpha)r^{k+l}$ and $s^k=y^k$. Where r is the desired future output or set-point, this value can be constant or variable along the prediction horizon. The range of α is 0<α≤1. The closer to 1, the faster the output response. The closer to 0, the slower the output response.

6 Plant description

The plant is located at the Department of Systems Engineering and Automatic Control of the University of Valladolid, Spain. As depicted in Figure 1, the process is composed of two tanks (height: 800 millimeters; internal diameter: 94 millimeters). The water is pumped independently to both of them. The control signals are the pumps flow rates Q_1 and Q_2. The liquid leaves the tanks by gravity from outlets near the bottom of the tanks with flow rates q_1 and q_2. There are additional outlets that connect the tanks via a short pipe. The liquid levels in the tanks are measured by capacitive sensors ABB KENT Taylor, model II80LS. The pumps are FLOJET 4405-343, 24V/2-4Amp. The data acquisition card is a Measurement Computing CIODAS16. Figure 2 is a photo of the real plant.

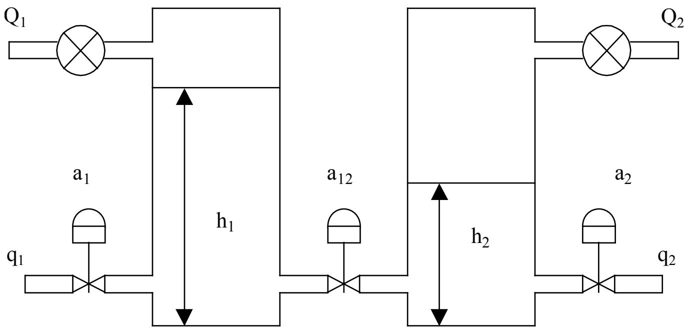

Figure 1: Plant scheme

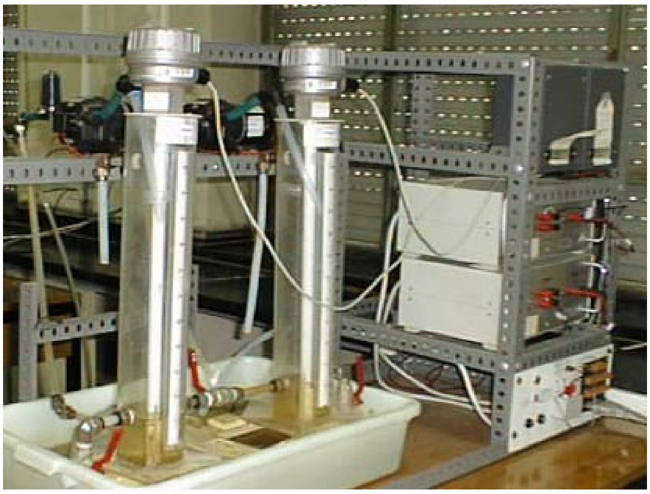

Figure 2: Two-tanks laboratory plant

Different experiments were carried out to identify linear models at different operating points. The identification of the system gave very good results as it can be seen in Figure 3. This graph shows both the real and the model data corresponding to the level of the first tank.

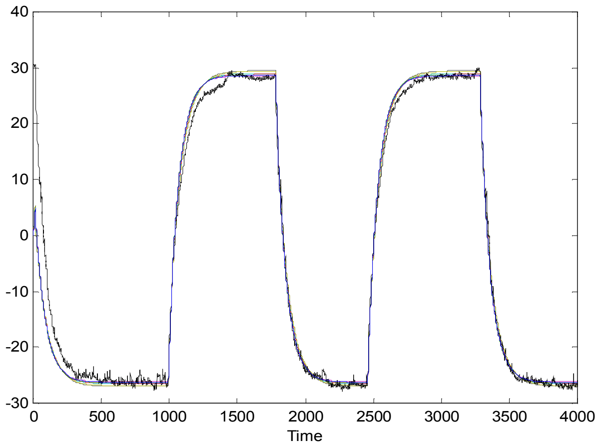

Figure 3: Measured and simulated model output

The variations in the steady state gain in the models over the range of operating points are:

$$\text{from } DC_{\text{lower\_oper\_po}} = \begin{pmatrix} 1.6747 & 1.4349 \\ 1.7317 & 1.5637 \end{pmatrix} \text{ to}$$

$$DC_{\text{upper\_oper\_po}} = \begin{pmatrix} 1.9012 & 1.5247 \\ 1.5599 & 1.5222 \end{pmatrix}.$$

The variations in the settling time (in seconds) are:

$$\text{from } ST_{\text{lower\_oper\_po}} = \begin{pmatrix} 146 & 166 \\ 168 & 142 \end{pmatrix} \text{ to}$$

$$ST_{\text{upper\_oper\_po}} = \begin{pmatrix} 134 & 210 \\ 192 & 124 \end{pmatrix}.$$

7 Experimental results

The closed-loop experiments were carried out using a PC running at 600MHz and with 64 Mb of memory. Although the machine is not state-of-the-art, the speed of the calculations was acceptable.

The system has a time constant of 30 seconds, approximately and the sampling time considered was 2.5 seconds, i.e., every 2.5 seconds a new control signal was sent to the plant.

Different experiments were carried out to check the robustness and conservativeness of the Min-Max controller at different operating points, with varying uncertainty sizes and tuning parameters. The causes of the uncertainty were the variations in the dynamic behaviour with the operating point and changes in the inlet flow of pH.

To compare the effect of these variations the following set-points changes were applied. The tank 1 set-point was set to 55 centimeters and when t=400 seconds it was changed to 40 cm. The set-point of the second tank remained constant for the duration of the experiment and equal to 60 cm. In both cases, the set-points were fixed far from the operating points.Figure 4, Figure 5 and Figure 6 show the experimental results.

The order in which variables are shown is always the same: the first plot is the level in the first tank and its set-point, the second one represents the second tank and the last one is the evolution of the two input variables.

The main tuning parameters are:

- N_1=2
- N_2=12
- N_u=5
- α=0.5

The input constraints are selected to be:

- u_{max} = [100 100]
- u_{min} = [0 0]
- Δu_{max} = [10 10]
- Δu_{min} = [-10 -10]
- $\Delta^2 u_{max}$ = [5 5]
- $\Delta^2 u_{min}$ = [-5 -5].

The output constraints are set between 10% and 90% of the maximum level.

The aim of the experiment shown in Figure 5 was to illustrate that when working far from the nominal point, the control results are not good if only one model and no uncertainty were considered. The system is working fully coupled. The results show overshoots and great disturbances between loops. We can observe that the system response is rather fast and accurate regarding the steady state. Nevertheless, due to the coupling there is a rather strong disturbance in the second output every time there is a change in the set-point of the first tank.

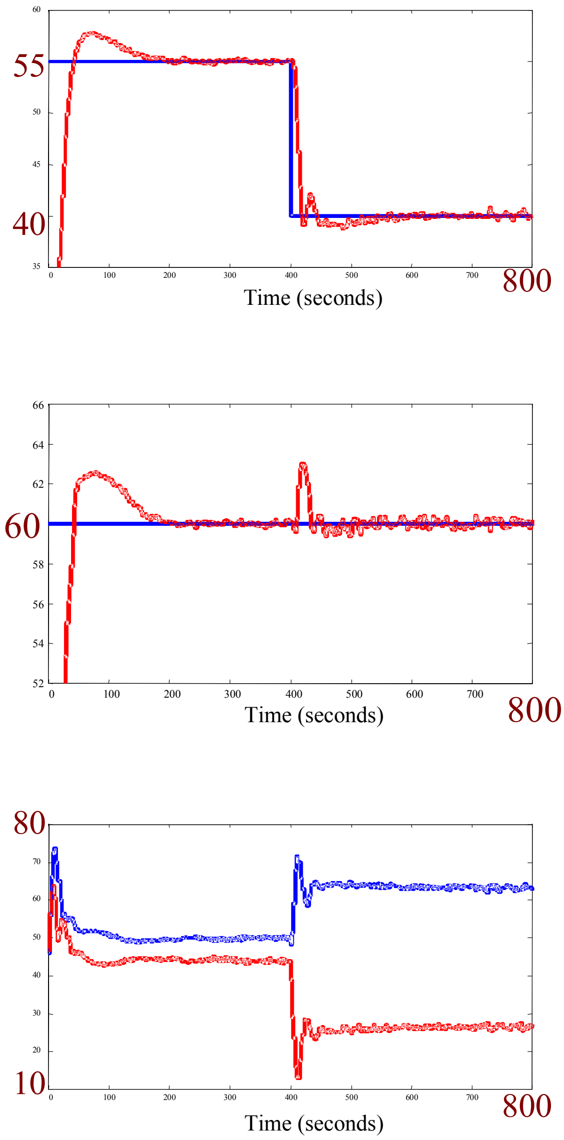

Figure 4: One model, without uncertainty

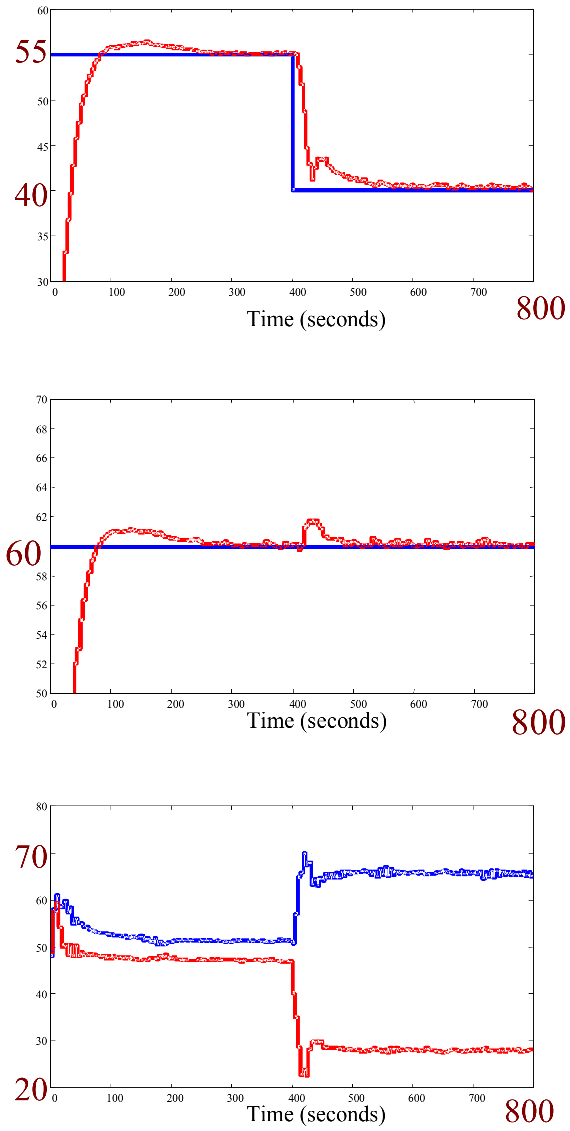

Figure 5: One model with uncertainty

The last experiment (Figure 6) shown consisted in considering two models and uncertainty in both of them. Maintaining the previous settings, the uncertainty of the second system is $\xi_{\min} = 0.8, \xi_{\max} = 1.2$. The response is slower than in the second experiment, the changes of the control variables are even softener and the second level deviations from the set-point are very small. This is the most robust controller of the three presented in the paper.

8 Conclusions

This paper has presented the application of a Min-Max MBPC that is related to the ground-breaking work of Campo and Morari [6] to a laboratory plant. Taking [3] as starting point, some modifications have been made in order to make the controller suitable for an on-line implementation.

The experiments carried out showed that the controller improves the robustness of the controlled system and that is possible to apply this technique to real processes.

Future work includes the use of different prediction models: step response or transfer function and comparisons with other robust controller for constrained plant, with particular emphasis on the conservativeness of the resulting control policies.

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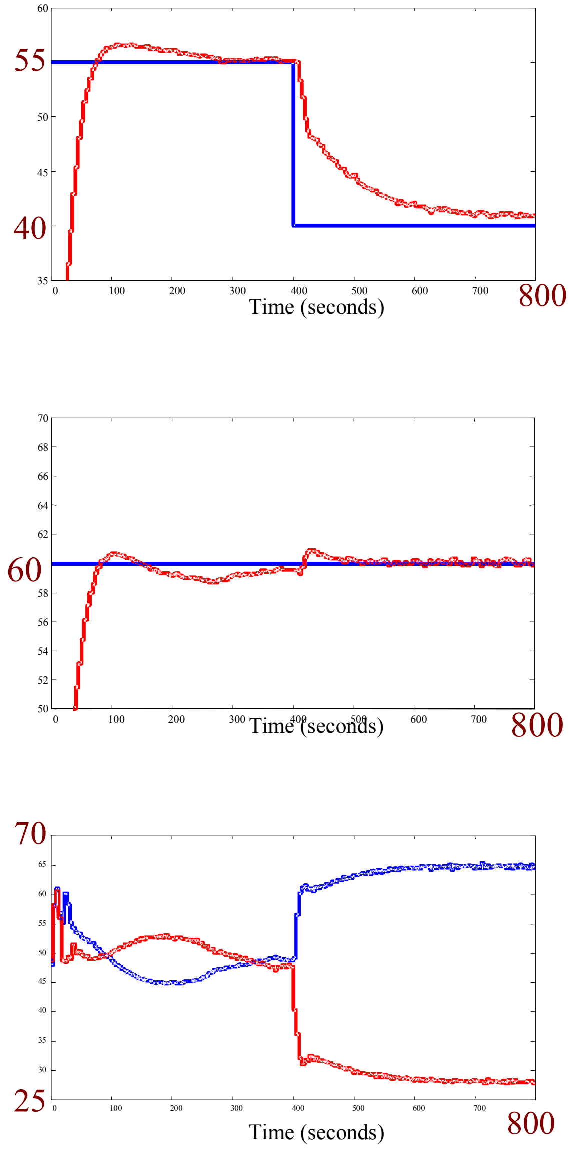

Figure 6: Two models with uncertainty

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