PARAMETER ESTIMATION OF RAILWAY VEHICLE DYNAMIC MODEL USING RAO-BLACKWELLISED PARTICLE FILTER

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Abstract

This paper presents the development of a new method for parameter estimation in linear state space model. The proposed method is based on a Rao-Blackwellised particle filter. The simulation results with a railway vehicle dynamic model are provided which demonstrate the effectiveness of the proposed method in comparison with the conventional EKF-based method.

1 Introduction

The faults in the suspension elements of a railway vehicle will leads to the changes in the parameter values of the vehicle dynamic model, which in turn may lead to the deterioration of ride quality and increase of wheel-rail wear; in some cases, it may even cause a safety problem. Thus, an appropriate method is required to perform parameter estimation so that the vehicle condition monitoring system is able to detect and isolate incipient faults. The problem of parameter estimation in dynamic systems has been intensively studied and the basic estimation techniques are well established, but, because of the complex nature of railway vehicle dynamics, the application of the techniques to this particular problem is not straightforward.

The dynamic behaviour of the railway vehicle is usually described by a linear stochastic state space model. Conventionally, parameter estimation in such a system is performed with the extended Kalman filter (EKF) based approach. Although the EKF approach to joint parameter and state estimation problem for linear systems with unknown parameters is well known and widely spread, it has been shown that in general the estimates may be biased or divergent (see *e.g.* [11] and references therein). In this paper, a new parameter estimation method for linear stochastic state space model is developed and applied for estimation of the parameters in the railway vehicle dynamic model. The new method is based on the newly-developed Rao-Blackwellised particle filter. The effectiveness of the new method is demonstrated through the simulation with the railway vehicle dynamic model and its performance is compared with the conventional EKF approach.

The rest of the paper is organised as follows. In Section 2, the modelling of plan view dynamics of a half railway vehicle is presented followed by a brief review of the EKF approach to parameter estimation for linear state space model in Section 3. Section 4 gives a brief introduction to the Rao-Blackwellised particle filter (RBPF) and presents the development of the new parameter estimation method based on RBPF. In Section 5, the new method is applied to the plan view dynamic model derived in Section 2 and the results of parameter estimation are compared with the EKF approach. Conclusions are drawn in Section 6.

2 Modelling of railway vehicle dynamics

Failure modes effects and criticality analysis (FMECA) from railway vehicle manufacturers has highlighted that the secondary lateral and anti-yaw dampers in a railway vehicle suspension system have a lower reliability record than most other suspension elements and are difficult to test in situ. The former causes degraded ride quality, whereas the latter affects running stability. The other key area is concerned with the wheel profile: progressive wear can profoundly influence the contact mechanics and may cause unstable running at speed.

In this section, a plan view dynamic model of a half body vehicle is developed with the intention to detect the faults in these two dampers. Figure 1 gives the plan view of half of a conventional bogie vehicle and the configuration of the sensors. The plan view equations of motion of the vehicle travelling on straight track are derived as follows:

$$\begin{aligned}
&m_{w1}\ddot{y}_{w1} + \tfrac{2f_{22}}{v}\dot{y}_{w1} + K_y(y_{w1}-y_b) - 2f_{22}\Psi_{w1} \\
&-K_y a\Psi_b = 0 \\
&I_{w1}\ddot{\Psi}_{w1} + \tfrac{2f_{11}l^2}{v}\dot{\Psi}_{w1} + K_x l^2(\Psi_{w1}-\Psi_b) \\
&+\tfrac{2f_{11}\lambda l}{r_0}y_{w1} = \tfrac{2f_{11}\lambda l}{r_0}y_{t1} \\
&m_{w2}\ddot{y}_{w2} + \tfrac{2f_{22}}{v}\dot{y}_{w2} + K_y(y_{w2}-y_b) - 2f_{22}\Psi_{w2} \\
&+K_y a\Psi_b = 0 \\
&I_{w2}\ddot{\Psi}_{w2} + \tfrac{2f_{11}l^2}{v}\dot{\Psi}_{w2} + K_x l^2(\Psi_{w2}-\Psi_b) \\
&+\tfrac{2f_{11}\lambda l}{r_0}y_{w2} = \tfrac{2f_{11}\lambda l}{r_0}y_{t2} \\
&m_b\ddot{y}_b + K_y(y_b-y_{w1}) + K_y(y_b-y_{w2}) + K_{sy}(y_b-y_{bd}) \\
&+K_{sry}(y_b-y_{sr}) = 0 \\
&I_b\ddot{\Psi}_b + C_{say}l_{bw}^2\dot{\Psi}_b + (2a^2K_y+2K_x l^2)\Psi_b - K_x l^2\Psi_{w1} \\
&-aK_y y_{w1} - K_x l^2\Psi_{w2} + aK_y y_{w2} = 0 \\
&C_{sy}(\dot{y}_{sr}-\dot{y}_{bd}) = K_{sry}(y_b-y_{sr}) \\
&m_{bd}\ddot{y}_{bd} - C_{sy}(\dot{y}_{sr}-\dot{y}_{bd}) + K_{sy}(y_{bd}-y_b) = 0
\end{aligned} \tag{1}$$

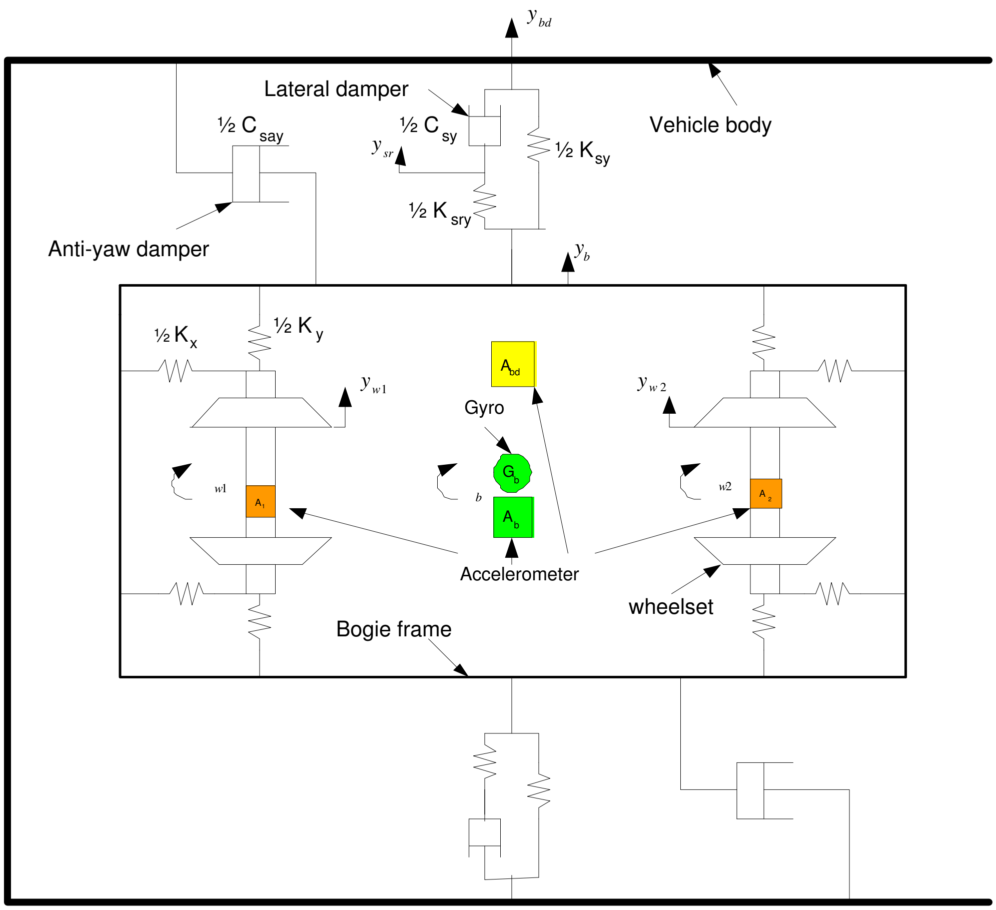

Figure 1: Plan view of half bogie vehicle and sensor configuration

The meaning of variables and parameter values in the equation are given in the Appendix. Defining the relative lateral def lections between wheel and track as $d_1 = y_{w1} - y_{t1}$ and $d_2 = y_{w2} - y_{t2}$, the above equations can be rearranged as follows:

$$
\begin{aligned}
&\ddot{y}_{w1} = -\frac{2f_{22}}{m_{w1}v}\dot{y}_{w1} - \frac{K_y}{m_{w1}}y_{w1} + \frac{2f_{22}}{m_{w1}}\Psi_{w1} + \frac{K_y}{m_{w1}}y_b \\
&+\frac{aK_y}{m_{w1}}\Psi_b \\
&\ddot{\Psi}_{w1} = -\frac{2f_{11}l^2}{I_{w1}v}\dot{\Psi}_{w1} - \frac{K_x l^2}{I_{w1}}\Psi_{w1} + \frac{K_x l^2}{I_{w1}}\Psi_b - \frac{2f_{11}\lambda l}{I_{w1}r_0}d_1 \\
&\ddot{y}_{w2} = -\frac{2f_{22}}{m_{w2}v}\dot{y}_{w2} - \frac{K_y}{m_{w2}}y_{w2} + \frac{2f_{22}}{m_{w2}}\Psi_{w2} + \frac{K_y}{m_{w2}}y_b \\
&-\frac{aK_y}{m_{w2}}\Psi_b \\
&\ddot{\Psi}_{w2} = -\frac{2f_{11}l^2}{I_{w2}v}\dot{\Psi}_{w2} - \frac{K_x l^2}{I_{w2}}\Psi_{w2} + \frac{K_x l^2}{I_{w2}}\Psi_b - \frac{2f_{11}\lambda l}{I_{w2}r_0}d_2 \\
&\ddot{y}_b = \frac{K_y}{m_b}y_{w1} + \frac{K_y}{m_b}y_{w2} - \frac{2K_y+K_{sy}+K_{sry}}{m_b}y_b + \frac{K_{sy}}{m_b}y_{bd} \\
&+\frac{K_{sry}}{m_b}y_{sr} \\
&\ddot{\Psi}_b = \frac{aK_y}{I_b}y_{w1} + \frac{K_x l^2}{I_b}\Psi_{w1} - \frac{aK_y}{I_b}y_{w2} + \frac{K_x l^2}{I_b}\Psi_{w2} \\
&-\frac{C_{say}l_{bw}^2}{I_b}\dot{\Psi}_b - \frac{2a^2K_y+2K_x l^2}{I_b}\Psi_b \\
&\ddot{y}_{bd} = \frac{K_{sy}+K_{sry}}{m_{bd}}y_b - \frac{K_{sy}}{m_{bd}}y_{bd} - \frac{K_{sry}}{m_{bd}}y_{sr} \\
&\dot{y}_{sr} = \dot{y}_{bd} + \frac{K_{sry}}{C_{sy}}y_b - \frac{K_{sry}}{C_{sy}}y_{sr} \\
&\dot{d}_1 = \dot{y}_{w1} - \dot{y}_{t1} \\
&\dot{d}_2 = \dot{y}_{w2} - \dot{y}_{t2}
\end{aligned}
\tag{2}
$$

It can been seen that the dynamic behaviour of a railway vehicle is very complex and highly interactive. For the development of a model-based filter, such as an EKF, a state-space form of system model can be derived from (2) as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{G}\boldsymbol{\beta} \tag{3}$$

where

$$
\begin{aligned}
\mathbf{x} \;&=\; \begin{bmatrix} \dot{y}_{w1} & y_{w1} & \dot{\Psi}_{w1} & \Psi_{w1} & \dot{y}_{w2} & y_{w2} & \dot{\Psi}_{w2} & \Psi_{w2} & \dot{y}_b & y_b \\ \dot{\Psi}_b & \Psi_b & \dot{y}_{bd} & y_{bd} & y_{sr} & d_1 & d_2 \end{bmatrix}^T \\
\boldsymbol{\beta} \;&=\; \begin{bmatrix} \dot{y}_{t1} & \dot{y}_{t2} \end{bmatrix}^T
\end{aligned}
$$

matrices $\mathbf{A}$ and $\mathbf{G}$ can be readily derived from the equations (2) and $\boldsymbol{\beta}$ can be approximated as white Gaussian noise vector [9].

The vehicle is equipped with five sensors (a gyro and four accelerometers) as shown in Figure 1 which can measure the lateral accelerations of the two wheelsets ($\ddot{y}_{w1}$ and $\ddot{y}_{w2}$), the lateral acceleration and yaw velocity of the bogie ($\ddot{y}_b$ and $\dot{\Psi}_b$) and the lateral acceleration of the vehicle body ($\ddot{y}_{bd}$). The measurement equation is given as follows:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{4}$$

where, $\mathbf{y} = \begin{bmatrix} \ddot{y}_{w1} & \ddot{y}_{w2} & \ddot{y}_b & \dot{\Psi}_b & \ddot{y}_{bd} \end{bmatrix}^T$, $\mathbf{v}$ represents the measurement noise vector and the measurement matrix $\mathbf{H}$ is obtained readily from the system matrix $\mathbf{A}$:

$$\mathbf{H} = \begin{bmatrix} \mathbf{A}(1,:) \\ \mathbf{A}(5,:) \\ \mathbf{A}(9,:) \\ \mathbf{A}(12,:) \\ \mathbf{A}(13,:) \end{bmatrix} \tag{5}$$

Faults in the secondary lateral and anti-yaw dampers are ref lected by the changes in the damping coefficients C_{sy} and C_{say} in equations (2), which in turn will change the matrix $\mathbf{A}$ of the above dynamic model (3). Therefore, the faults can be detected and isolated by monitoring the changes in these two parameters. To facilitate presentation, the above dynamic model (3) and (4) is rewritten as follows to include explicitly the parameters associated with the faults to be detected and isolated:

$$\dot{\mathbf{x}} = \mathbf{A}(\boldsymbol{\theta})\mathbf{x} + \mathbf{G}\boldsymbol{\beta} \tag{6}$$
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{7}$$

where, $\boldsymbol{\theta}$ collects all the parameters to be monitored which will determine matrices $\mathbf{A}$ (in the present case, $\boldsymbol{\theta} = \begin{bmatrix} C_{sy} & C_{say} \end{bmatrix}^T$, the extension to include the wheel profile-related parameters such as conicity λ will be discussed in another paper). Note that $\mathbf{H}$ is independent of $\boldsymbol{\theta}$.

3 Parameter estimation using EKF

One way to deal with the parameter estimation problem in the system described by (6) and (7) is to view the parameters as additional states, or more precisely, to augment the state vector $\mathbf{x}$ with the parameter vector $\boldsymbol{\theta}$ as $\boldsymbol{\xi} = [\mathbf{x}^T \ \ \boldsymbol{\theta}^T]^T$ and re-write the state space model in terms of $\boldsymbol{\xi}$, we then have the following set of equations:

$$\begin{aligned} \dot{\boldsymbol{\xi}} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\boldsymbol{\theta}} \end{bmatrix} &= \mathbf{f}(\boldsymbol{\xi}) + \mathbf{G}'\boldsymbol{\beta}' \\ &= \begin{bmatrix} \mathbf{A}(\boldsymbol{\theta})\mathbf{x} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{G} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta} \\ \mathbf{n} \end{bmatrix} \end{aligned} \tag{8}$$
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{9}$$

For most practical applications, the measurements are usually sampled-data (i.e. discrete) resulting from the digital implementation and the discrete version of above model is given as follows (see *e.g.* [11],[12]):

$$\begin{aligned} \boldsymbol{\xi}_{k+1} = \begin{bmatrix} \mathbf{x}_{k+1} \\ \boldsymbol{\theta}_{k+1} \end{bmatrix} &= \mathbf{g}(\boldsymbol{\xi}_k) + \begin{bmatrix} \boldsymbol{\Gamma}(\boldsymbol{\theta}_k) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{w}_k \\ \mathbf{n}_k \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{\Phi}(\boldsymbol{\theta}_k)\mathbf{x}_k \\ \boldsymbol{\theta}_k \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Gamma}(\boldsymbol{\theta}_k) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{w}_k \\ \mathbf{n}_k \end{bmatrix} \end{aligned} \tag{10}$$
$$\mathbf{y}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \tag{11}$$

where, $\boldsymbol{\Phi}(\boldsymbol{\theta}_k) = e^{\mathbf{A}(\boldsymbol{\theta}_k)T}$, $\boldsymbol{\Gamma}(\boldsymbol{\theta}_k) = \int_0^T e^{\mathbf{A}(\boldsymbol{\theta}_k)\tau}\mathbf{G}d\tau$ and $\mathbf{w}_k$, $\mathbf{n}_k$ are white Gaussian noises of appropriate strength. To obtain the parameter estimation recursively, we shall consequently faced with a general nonlinear filtering problem.

If the sampling time T is short compared with the system time constants, the system can be treated as linear time-invariant over the sampling interval T. Therefore the above nonlinear filtering problem for estimating augmented state $\boldsymbol{\xi}$ so as to determine the parameters $\boldsymbol{\theta}$ is attacked by the sampled-data EKF as follows [11],[12],[2]:

- Measurement Update at the sampling time instant k

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\boldsymbol{\Psi}^T\mathbf{R}_k^{-1} \tag{12}$$
$$\mathbf{R}_k = \boldsymbol{\Psi}\mathbf{P}_{k|k-1}\boldsymbol{\Psi}^T + \mathbf{Q}_v \tag{13}$$
$$\hat{\boldsymbol{\xi}}_k = \hat{\boldsymbol{\xi}}_{k|k-1} + \mathbf{K}_k(\mathbf{y}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1}) \tag{14}$$
$$\mathbf{P}_k = \mathbf{P}_{k|k-1} - \mathbf{K}_k\boldsymbol{\Psi}\mathbf{P}_{k|k-1} \tag{15}$$

where,

$$\boldsymbol{\Psi} = \left[\ \frac{\partial}{\partial \mathbf{x}}(\mathbf{H}\mathbf{x}) \ \middle| \ \frac{\partial}{\partial \boldsymbol{\theta}}(\mathbf{H}\mathbf{x}) \ \right] = \left[\ \mathbf{H} \ \middle| \ \mathbf{0} \ \right]$$

- Time Propagation over the sampling interval (i.e between measurements)

$$\hat{\boldsymbol{\xi}}_{k+1|k} = \begin{bmatrix} \boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_k) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \hat{\boldsymbol{\xi}}_k \tag{16}$$

$$\mathbf{P}_{k+1|k} = \mathbf{F}_k\mathbf{P}_k\mathbf{F}_k^T + \begin{bmatrix} \boldsymbol{\Gamma} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{Q}_\mathbf{w} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_\mathbf{n} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Gamma}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{17}$$

where, $\boldsymbol{\Phi}(\hat{\boldsymbol{\theta}}_k) \approx \mathbf{I} + \mathbf{A}(\hat{\boldsymbol{\theta}}_k)T$, $\boldsymbol{\Gamma} \approx \mathbf{I}\mathbf{G}T$ and

$$\begin{aligned} \mathbf{F}_k &= \frac{\partial}{\partial \boldsymbol{\xi}}\mathbf{g}(\boldsymbol{\xi}) \Big|_{\boldsymbol{\xi}=\hat{\boldsymbol{\xi}}_k} \\ &= \left[\begin{array}{c|c} \frac{\partial}{\partial \mathbf{x}}(\boldsymbol{\Phi}(\boldsymbol{\theta})\mathbf{x}) & \frac{\partial}{\partial \boldsymbol{\theta}}(\boldsymbol{\Phi}(\boldsymbol{\theta})\mathbf{x}) \\ \hline \mathbf{0} & \mathbf{I} \end{array} \right]_{\substack{\mathbf{x}=\hat{\mathbf{x}}_k \\ \boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_k}} \\ &= \left[\begin{array}{c|c} \boldsymbol{\Phi}(\boldsymbol{\theta}) & T\frac{\partial}{\partial \boldsymbol{\theta}}(\mathbf{A}(\boldsymbol{\theta})\mathbf{x}) \\ \hline \mathbf{0} & \mathbf{I} \end{array} \right]_{\substack{\mathbf{x}=\hat{\mathbf{x}}_k \\ \boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_k}} \end{aligned} \tag{18}$$

As indicated in [1],"Although this extended Kalman filter approach appears perfectly straightforward, experience has shown that with the usual state-space model, it does not work well in practice".

4 Parameter estimation using RBPF

4.1 Background

Starting from the seminal paper of Gordon, Salmond and Smith [7], the particle filter (PF) (also known as Monte Carlo filter), a simulation-based method for nonlinear non-Gaussian state estimation, has gone through a dynamic and rapid development. In the mid 1990s, several particle filter algorithms were proposed independently under the different names, see [4] for a summary of the state-of-the-art.

The PF is developed in the framework of recursive Bayesian estimation which attempt to approximate the complete probability density function (pdf) of the state to be estimated as opposed to just estimating the first few central moments of it such as in EKF. The major innovation of the PF is to approximate the required, usually complicated, pdf by a swarm of interacting points called "particles" which can be considered as the realizations or samples from the required pdf, rather than by a function over the state space. As such, the method is not restricted by considerations of analytic tractability. The PF will propagate and update these particles and their mean and covariance matrix are approximations to the Bayesian estimates.

The use of PF for simultaneously estimating the states and parameters has been proposed in [8] which extends the idea used in Section 3 to the general nonlinear non-Gaussian state space model, where a random walk model, $\boldsymbol{\theta}_k = \boldsymbol{\theta}_{k-1} + \mathbf{w}'_k$, with $\mathbf{w}'_k$ a zero mean Gaussian white noise is used for parameter evolution to allow the exploration of the parameter space and reduce sample degeneracy in particle filtering. A similar idea was used in [7], where an additional independent random disturbance or "roughening jitter" was introduced to the sampled state particles in an attempt to deal with sample degeneracy. Extending this idea to the present case for parameter estimation, the standard deviation σ of the Gaussian roughening jitter corresponding to a particular component of the parameter vector $\boldsymbol{\theta}$ is given by $\sigma = KEN^{-\frac{1}{d}}$ as suggested in [7], where E is the interval between the maximum and the minimum samples of this component, d is the dimension of the parameter vector $\boldsymbol{\theta}$ (i.e. the number of the parameters to be estimated), K is a constant tuning parameter and N is the number of particles used in PF. More recently, Liu and West [10] suggested an approach to improve the precision of the parameter estimation by using kernel smoothing with shrinkage for parameter evolution. In their approach, the unwanted information loss effect or over-dispersion of the samples for the fixed parameter caused by the independent random shock $\mathbf{w}'_k$ is corrected by the introduction of negative correlations between $\boldsymbol{\theta}_{k-1}$ and the random shock $\mathbf{w}'_k$.

One of the major drawbacks of the particle filter for parameter estimation as described above is that sampling in high-dimensional space can be inefficient because a large number of samples is needed to represent the required pdf. A standard technique to increase the efficiency of sampling techniques is to reduce the size of the augmented state space by marginalizing out some of the variables analytically; this is an example of the techniques called Rao-Blackwellisation. Combining this technique with the above particle filter results in Rao-Blackwellised particle filter (RBPF) (see *e.g.* [3] [5]). RBPF has been applied for state estimation of the jump Markov linear systems in [6] and a hybrid filter is obtained where a part of the calculations is realized analytically and the other part using Monte Carlo methods. In the following, we extend this idea to solve the problem of estimating the unknown parameters in linear state space models.

4.2 System description

Suppose the observations are generated by the model specified as follows:

$$\begin{aligned} \boldsymbol{\theta}_k &\sim p(\boldsymbol{\theta}_k|\boldsymbol{\theta}_{k-1}) && (19)\\ \mathbf{x}_k &= \boldsymbol{\Phi}(\boldsymbol{\theta}_k)\mathbf{x}_{k-1} + \boldsymbol{\Gamma}(\boldsymbol{\theta}_k)\mathbf{w}_k && (20)\\ \mathbf{y}_k &= \mathbf{H}\mathbf{x}_k + \mathbf{v}_k && (21) \end{aligned}$$

where $p(\cdot|\boldsymbol{\theta}_{k-1})$ denotes the pdf conditional on $\boldsymbol{\theta}_{k-1}$, the equations (20) and (21) are the discrete version of equations (6) and (7) with $\boldsymbol{\Phi}(\boldsymbol{\theta}_k) = e^{\mathbf{A}(\boldsymbol{\theta})T}|_{\boldsymbol{\theta}=\boldsymbol{\theta}_k}$, $\boldsymbol{\Gamma}(\boldsymbol{\theta}_k) = \int_0^T e^{\mathbf{A}(\boldsymbol{\theta}_k)\tau}\mathbf{G}d\tau$, $\mathbf{w}_k$ and $\mathbf{v}_k$ are zero mean white Gaussian with diagonal covariance matrices $\mathbf{Q}_w$ and $\mathbf{Q}_v$. A PF can be designed based on the above model with the aim to determine the unknown parameters $\boldsymbol{\theta}$ by estimating the augmented states $\boldsymbol{\xi}_k = [\mathbf{x}_k^T, \boldsymbol{\theta}_k^T]^T$.

4.3 Algorithm for parameter estimation

As mentioned previously, the use of Rao-Blackwellisation techniques can increase the efficiency of sampling in PF by reducing the size of the state space to be sampled through marginalization, which results in RBPF. In our present application, the dimension of the augmented state vector $\boldsymbol{\xi}_k$ is $17 + 2 = 19$, but the model (19) and (20) has a tractable substructure which can be analytically marginalized out. In fact, for each realization (or sample) of $\boldsymbol{\theta}_k$, we have a single linear Gaussian state space model. As such, the marginalization can be carried out exactly using the Kalman filter algorithm. The resulting RBPF is similar to PF but we only need to sample the parameter vector $\boldsymbol{\theta}_k$ (in present case, the dimension of $\boldsymbol{\theta}_k$ is 2, thus the size of the space to be sampled is drastically reduced). Then for each sample of $\boldsymbol{\theta}_k$, the mean and covariance of state $\mathbf{x}_k$ are updated using Kalman filter.

Let $\mathcal{Z}_k$ denote the set of measurements up to time k, i.e. $\mathcal{Z}_k = \{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_k\}$. The proposed RBPF-based parameter estimation algorithm is summarized as follows.

Initialization: for $i = 1, \cdots, N$, draw samples $\boldsymbol{\theta}_0(i)$ from the initial pdf $p(\boldsymbol{\theta}_0) = p(\boldsymbol{\theta}|\mathcal{Z}_0)$ and set $\hat{\mathbf{x}}_0(i) = \hat{\mathbf{x}}_0$, $\mathbf{P}_0(i) = \mathbf{P}_0$, where, $\hat{\mathbf{x}}_0$ is the initial state estimate and $\mathbf{P}_0$ is the initial state estimation covariance matrix.

For $k = 1, 2, \cdots$, repeat the following steps:

- For $i = 1, 2, \cdots, N$, draw sample $\tilde{\boldsymbol{\theta}}_k(i)$ from $p(\boldsymbol{\theta}_k|\boldsymbol{\theta}_{k-1}(i))$.
- For $i = 1, 2, \cdots, N$, propagate the mean $\hat{\mathbf{x}}_{k-1}(i)$ and covariance $\mathbf{P}_{k-1}(i)$ of the state $\mathbf{x}_{k-1}$ as follows:

$$\begin{aligned} \tilde{\mathbf{x}}_{k|k-1}(i) &= \boldsymbol{\Phi}(\tilde{\boldsymbol{\theta}}_k(i))\hat{\mathbf{x}}_{k-1}(i) \\ \tilde{\mathbf{P}}_{k|k-1}(i) &= \boldsymbol{\Phi}(\tilde{\boldsymbol{\theta}}_k(i))\mathbf{P}_{k-1}(i)\boldsymbol{\Phi}^T(\tilde{\boldsymbol{\theta}}_k(i)) + \\ &\quad \boldsymbol{\Gamma}(\tilde{\boldsymbol{\theta}}_k(i))\mathbf{Q}_w\boldsymbol{\Gamma}^T(\tilde{\boldsymbol{\theta}}_k(i)) \end{aligned}$$

- For $i = 1, 2, \cdots, N$, evaluate and normalize the importance weights:

$$\tilde{\alpha}_k(i) = p(\mathbf{y}_k|\mathcal{Z}_{k-1}, \tilde{\boldsymbol{\theta}}_k(i)) \sim \mathcal{N}(\tilde{\mathbf{y}}_{k|k-1}(i), \tilde{\mathbf{R}}_k(i))$$

$$\alpha_k(i) = \frac{\tilde{\alpha}_k(i)}{\sum_{j=1}^{N} \tilde{\alpha}_k(j)} \quad (22)$$

where,

$$\begin{aligned} \tilde{\mathbf{y}}_{k|k-1}(i) &= \mathbf{H}\tilde{\mathbf{x}}_{k|k-1}(i) \\ \tilde{\mathbf{R}}_k(i) &= \mathbf{H}\tilde{\mathbf{P}}_{k|k-1}(i)\mathbf{H}^T + \mathbf{Q}_v \end{aligned}$$

- Parameter estimate is calculated as follows:

$$\hat{\boldsymbol{\theta}}_k = \sum_{i=1}^{N} \alpha_k(i)\tilde{\boldsymbol{\theta}}_k(i) \quad (23)$$

- Resampling particles $\{\tilde{\mathbf{x}}_{k|k-1}(i), \tilde{\mathbf{P}}_{k|k-1}(i), \tilde{\boldsymbol{\theta}}_k(i) : i = 1, 2, \cdots, N\}$ with sampling probabilities proportional to $\alpha_k(i)$ to obtain N particles $\{\hat{\mathbf{x}}_{k|k-1}(i), \mathbf{P}_{k|k-1}(i), \boldsymbol{\theta}_k(i) : i = 1, 2, \cdots, N\}$.

- For $i = 1, 2, \cdots, N$, performing measurement update for state vector $\mathbf{x}$ using Kalman recursion to obtain particle $\{\hat{\mathbf{x}}_k(i), \mathbf{P}_k(i), \boldsymbol{\theta}_k(i)\}$ given $\{\hat{\mathbf{x}}_{k|k-1}(i), \mathbf{P}_{k|k-1}(i), \boldsymbol{\theta}_k(i)\}$, where

$$\begin{aligned} \hat{\mathbf{x}}_k(i) &= \hat{\mathbf{x}}_{k|k-1}(i) + \mathbf{K}_k(i)(\mathbf{y}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1}(i)) \\ \mathbf{P}_k(i) &= (\mathbf{I} - \mathbf{K}_k(i)\mathbf{H})\mathbf{P}_{k|k-1}(i) \\ \mathbf{K}_k(i) &= \mathbf{P}_{k|k-1}(i)\mathbf{H}^T\mathbf{R}_k^{-1}(i) \\ \mathbf{R}_k(i) &= \mathbf{H}\mathbf{P}_{k|k-1}(i)\mathbf{H}^T + \mathbf{Q}_v \end{aligned}$$

5 Simulation experiments

In this section, the RBPF-based parameter estimation method proposed in this paper is applied for estimating the parameters $\boldsymbol{\theta} = \begin{bmatrix} C_{sy} & C_{say} \end{bmatrix}^T$ in the plan view dynamic model (6) derived in Section 2. The parameter evolution density takes the following form as suggested in [10]:

$$p(\boldsymbol{\theta}_k|\boldsymbol{\theta}_{k-1}) \sim \mathcal{N}(\boldsymbol{\theta}_k|a\boldsymbol{\theta}_{k-1} + (1-a)\bar{\boldsymbol{\theta}}_{k-1}, h^2\mathbf{V}_{k-1}) \quad (24)$$

where $h^2 = 1 - a^2$ and $a = (3\delta - 1)/2\delta$, δ is a discount factor typically around $0.95 \sim 0.99$. $\bar{\boldsymbol{\theta}}_{k-1}$ and $\mathbf{V}_{k-1}$ are the mean and variance of the Monte Carlo approximation $\{\tilde{\boldsymbol{\theta}}_{k-1}(i), \alpha_{k-1}(i)\}$ to $p(\boldsymbol{\theta}|\mathcal{Z}_{k-1})$ respectively. $\delta = 0.99$ and particle number $N = 1000$ are chosen for the following simulation. The measurement sampling frequency is $1kHz$, *i.e.* $T = 1mS$. The results of parameter estimation are shown in Figure 2 and Figure 3. For comparison, the EKF approach described in Section 3 is also applied and the results are shown as in Figure 4 and Figure 5. It can been seen, from these figures, the parameter estimates from RBPF-based method converge to the true values, whereas the EKF approach is not able to estimate the parameters correctly. In Figure 4, the EKF estimate of C_{sy} rises and falls severely in the initial period of estimation, and then tends to settle down at a negative value which is physically not meaningful. The EKF estimate of C_{say} does not converge and drifts away from its true value as shown in Figure 5 (in all the figures, the dashed line represents the true value of the parameter and solid line represents the estimated value of the parameter).

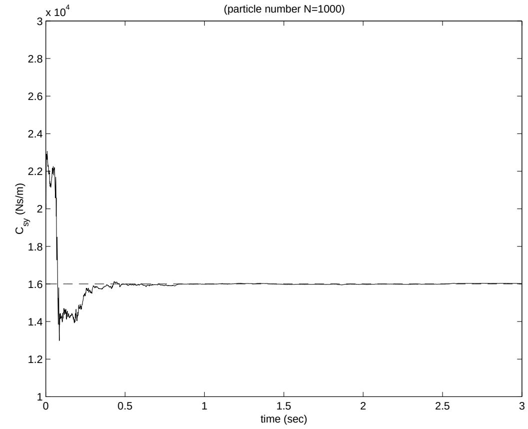

Figure 2: Estimation of C_{sy} using RBPF with 1000 particles

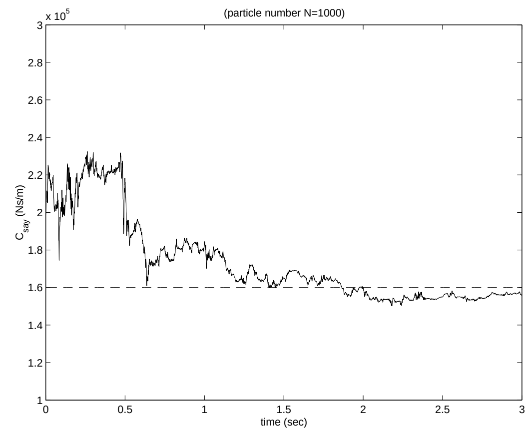

Figure 3: Estimation of C_{say} using RBPF with 1000 particles

6 Conclusions

A RBPF-based parameter estimation method is proposed in this paper. The method is used to solve the parameter estimation problem of the railway vehicle dynamic model where the conventional EKF approach fails. This example clearly demonstrates the inadequacy of the EKF-based approach and the RBPF-based method proposed in this paper offers much promise. Further work is being carried out to address the robustness issue for the proposed method.

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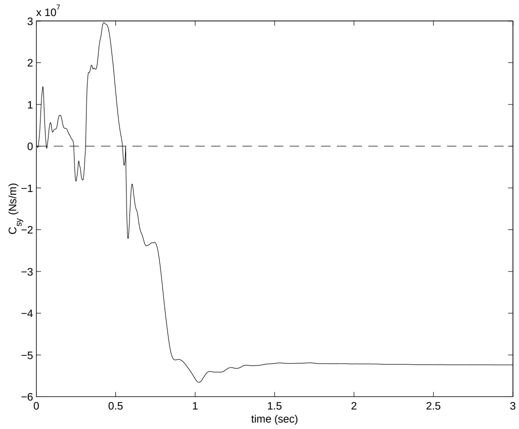

Figure 4: Estimation of C_{sy} using EKF approach

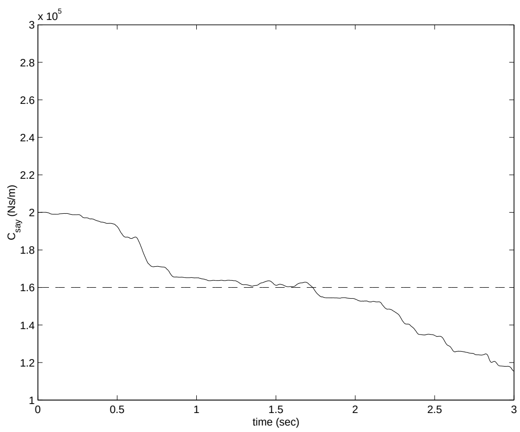

Figure 5: Estimation of C_{say} using EKF approach

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Appendix. Symbols and Parameters

y_{w1}	lateral displacement of the leading wheelset
y_{w2}	lateral displacement of the trailing wheelset
y_b	lateral displacement of the bogie
y_{bd}	lateral displacement of vehicle body
Ψ_{w1}	yaw angle of the leading wheelset
Ψ_{w2}	yaw angle of the trailing wheelset
Ψ_b	yaw angle of the bogie
y_{t1}	lateral track displacement at the leading wheels
y_{t2}	lateral track displacement at the trailing wheels
m_w	Wheelset mass(1250kg)
I_w	Wheelset yaw inertia(700kgm^2)
m_b	Bogie mass(2580kg)
I_b	Bogie yaw inertia(2160kgm^2)
m_{bd}	Half of vehicle body mass(13000kg)
K_y	Primary lateral stiffness per wheelset(5000kN/m)
K_x	Primary longitudinal stiffness per wheelset (9726kN/m)
K_{sy}	Secondary lateral stiffness per bogie(242kN/m)
K_{sry}	Secondary lateral end stiffness per bogie (2420kN/m)
C_{sy}	Secondary lateral damping per bogie(16kNs/m)
C_{say}	Secondary anti-yaw damping per bogie (160kNs/m)
f_{11}	longitudinal creep coefficient(10MN)
f_{22}	lateral creep coefficient(10MN)
a	Semi wheel-wheel spacing(1.3m)
l	Half gauge(0.717m)
λ	Conicity
v	Vehicle forward velocity(45.3m/s)
r_0	Wheelset radius(0.455m)
l_{bw}	Semi bogie width(1.3m)