# ROBUST OUTPUT FEEDBACK CONTROL FOR THE LATERAL DYNAMICS OF A RAILWAY CAR

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## Abstract

In this paper the problem of lateral dynamics control of a railway car is addressed. A full dynamic model, including car, bogies and wheelsets dynamics is considered, leading to a 34-th order linear system. Two different sensor/actuator configurations are considered and corresponding performances are critically compared. The control strategy is based on a MIMO high order sliding manifold approach, which provides high robustness with respect to system uncertainties and exogenous disturbances. The design procedure yields a MIMO controller in terms of LMFD description. Simulations are used to show the effectiveness of the proposed approach and to carry out the comparisons among the two considered solutions.

## 1 Introduction

Control of various kinds of vibrations in railway vehicles is a problem that has attracted the interest of many researchers in the last decade. While traditionally vibration alleviation control strategies were used only to increase riding comfort, the use of high speed trains with lighter cars has dramatically increased the importance of vibration reduction controllers [1], in order to compensate not only for increased noise levels, but also for possible unstable behaviors exhibited above the "critical speed". For instance, it is well known that the coupled lateral and yaw vibration of the bogie, caused by the interaction between wheel and rail, can result in the so-called "hunting instability", which degrades both the wheel and the line and, in some cases, increases the risk of derailment.

In order to reduce vibrations, both primary and secondary suspension systems are used in railway cars. However, the effectiveness of traditional passive suspension systems is limited in frequency, because their major contribution is around a fixed nominal resonance frequency. To overcome this limitation and increase the damping in a broad frequency band, active suspension systems have to be designed and controlled.

Active suspensions control systems have been approached from many points of view. Optimal LQ-based control strategies are widely discussed in [2] and the same paper collects a large number of references approaching the problem with classical techniques, nonlinear programming and fuzzy logic approaches. $\mathcal{H}_\infty$ approaches have also been applied, the reader can refer for instance to [3] in which lateral, yaw and roll motions are reduced by using a frequency shaping approach. Moreover, also $\mu$-synthesis approaches [4] have shown their effectiveness in the control of longitudinal dynamics of a half-car model.

Very recent papers attack the problem from different a point of view: in [5] a double nested control loop is considered, where the inner loop controls the ride and the outer the attitude. The two loops are decoupled by means of an input decoupling transformation, and the resulting controller reduces heave, pitch and roll motions. Also controller parameterizations based on LFT approach have been used [6] and tested on quarter-car, half-car and full car models, considering also the nonlinear dynamic model of the car. Hunting instability is dealt with in [7], where active electro-mechanic yaw dampers and experimental results are presented.

Finally, mechatronic approaches have been proposed, using a new concept for the control of car vibrations. Specifically, active steering [8] with different control strategies and active tilt [9] with experimental tests have been considered.

In this paper we propose a robust controller design based on a high order sliding manifold approach. Specifically, the order of the sliding strategy is defined by the actuators/sensors location. A 34-th order linear model is derived and two cases are considered. In the first case, four linear actuators on the wheelsets are considered, and colocated sensors are used. In the second case, the sensors are moved to the bogie, in order to operate them in a less harsh environment. This control strategy has been chosen for its robustness properties, since some parameters of the vehicle are very uncertain and extyernal disturbances have to be considered.

In [10], [11] it has been shown that, by using the mathematical tools of the Singular Perturbation Theory [12], it is possible to design a state feedback controller that has robustness properties similar to these of high-gain control systems. However, the controller does not exhibit any "peaking" phenomenon that affects the latter. In fact, it is well known [14] that using high gain systems the system state is decomposed into a "fast" and "slow" part, while in the proposed control strategy the whole state is "slow", while the control is the "fast" variable.

In [13], the above procedure has been extended to the case of MIMO output feedback. In detail, the controller order is based on the (multivariable) pole-zero excess, that can be exactly defined based on the plant Markov's parameters. A number of transfer zeros is added by the controller so as to fill up the pole-zero excess, and, to make the controller proper, suitable faraway poles are introduced and justified by using the singular perturbation theory. The zeros are located by defining a suitable time-varying sliding manifold so that the system output and its successive time derivatives up to the order of sliding are assigned with a desired behavior. The resulting controller is again very robust against external disturbances, like a high-gain controller.

In this paper, based on the methodology presented in [13], we propose two control strategies for robustly stabilizing the lateral dynamics of a railway car. Simulation results address the case of the lateral control of a railway vehicle, which is subject to lateral disturbances on the wheels. The disturbances are such as to yield flange contacts of the wheels on the rail in the non controlled car, and are strongly rejected by the controller.

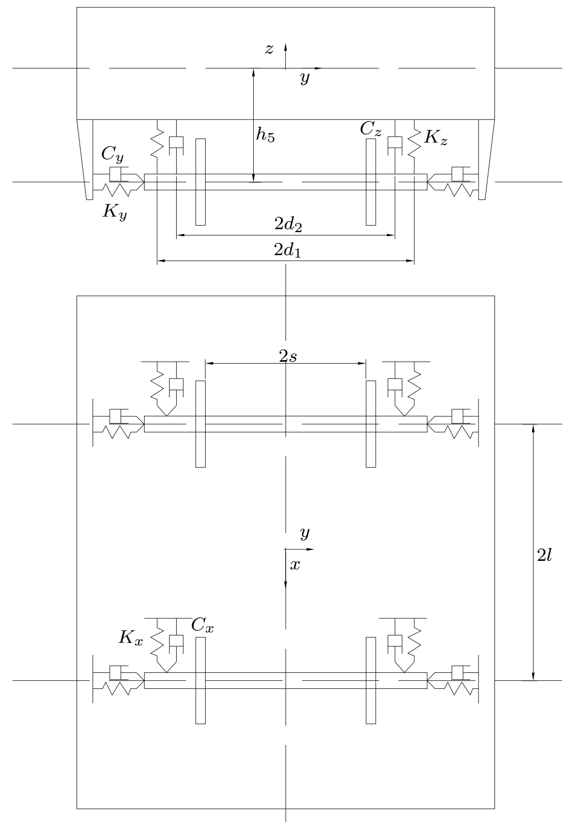


Figure 1: Bogie and wheelset.

## 2 System modeling

In Fig. 1 the bogie is reported, while in Fig. 2 a scheme of half a car is depicted.

The model of lateral dynamics of the railway vehicle comprises 17 dof's, whose meaning is summarized in Table 1.

| Car | | Front bogie | | Rear bogie | |
|---|---|---|---|---|---|
| $y_c$ | lat. displ. | $y_{b1}$ | lat. displ. | $y_{b2}$ | lat. displ. |
| $\alpha_c$ | yaw | $\alpha_{b1}$ | yaw | $\alpha_{b2}$ | yaw |
| $\theta_c$ | roll | $\theta_{b1}$ | roll | $\theta_{b2}$ | roll |

| Wheelsets (front bogie) | | Wheelsets (rear bogie) | |
|---|---|---|---|
| $y_1$ | lat. displ. (front) | $y_3$ | lat. displ. (front) |
| $\alpha_1$ | yaw (front) | $\alpha_3$ | yaw (front) |
| $y_2$ | lat. disp.(rear) | $y_4$ | lat. disp. (rear) |
| $\alpha_2$ | yaw (rear) | $\alpha_4$ | yaw (rear) |

Table 1: Degrees-of-freedom meaning

Based on the derivation of the dynamic models of railway vehicle systems in [15], the equations of motion can be written as

$$M_c\ddot{y}_c + 4K_{by}\;\; 2y_c + 2\theta_c h_1 - y_{b1} - y_{b2} + (\theta_{b1} + \theta_{b2})h_2 \;\; + 4C_{by}\;\; 2\dot{y}_c + 2\dot{\theta}_c h_3 - \dot{y}_{b1} - \dot{y}_{b2} + (\dot{\theta}_{b1} + \dot{\theta}_{b2})h_4 \;\; = 0$$

$$J_{zc}\ddot{\alpha}_c + 2K_{by}\;\; 2\alpha_c(L + a) - (\alpha_{b1} + \alpha_{b2})a + y_{b1} - y_{b2} + (\theta_{b2} - \theta_{b1})h_2 \;\; (L + a) + 2K_{by}\;\; 2\alpha_c(L - a) + (\alpha_{b1} + \alpha_{b2})a + y_{b1} - y_{b2} + (\theta_{b2} - \theta_{b1})h_2 \;\; (L - a) + 2C_{by}\;\; 2\dot{\alpha}_c(L + a) - (\dot{\alpha}_{b1} + \dot{\alpha}_{b2})a + \dot{y}_{b1} - \dot{y}_{b2} + (\dot{\theta}_{b2} - \dot{\theta}_{b1})h_4 \;\; (L + a) + 2C_{by}\;\; 2\dot{\alpha}_c(L - a) + (\dot{\alpha}_{b1} + \dot{\alpha}_{b2})a + \dot{y}_{b1} - \dot{y}_{b2} + (\dot{\theta}_{b2} - \dot{\theta}_{b1})h_4 \;\; (L - a) + 2K_{bx}(2\alpha_c - \alpha_{b1} - \alpha_{b2})d_5^2 + 2C_{bx}(2\dot{\alpha}_c - \dot{\alpha}_{b1} - \dot{\alpha}_{b2})d_5^2 = 0$$

$$J_{xc}\ddot{\theta}_c + 4K_{by}\;\; 2y_c + 2\theta_c h_1 + (\theta_{b1} - \theta_{b2})h_2 - y_{b1} - y_{b2} \;\; h_1 + 4C_{by}\;\; 2\dot{y}_c + 2\dot{\theta}_c h_3 + (\dot{\theta}_{b1} - \dot{\theta}_{b2})h_4 - \dot{y}_{b1} - \dot{y}_{b2} \;\; h_3 + 4K_{bz}(2\theta_c - \theta_{b1} - \theta_{b2})d_3^2 + 4C_{bz}(2\dot{\theta}_c - \dot{\theta}_{b1} - \dot{\theta}_{b2})d_4^2 + M_c g/2(2y_c - y_{b1} - y_{b2}) = 0$$

$$M\ddot{y}_{b1} + 2K_y(2\theta_{b1}h_5 + 2y_{b1} - y_1 - y_2) + 2C_y(2\dot{\theta}_{b1}h_5 + 2\dot{y}_{b1} - \dot{y}_1 - \dot{y}_2) - 2k_{by}(y_c + \theta_c h_1 - y_{b1} + \theta_{b1}h_2 - \alpha_c L) - 2C_{by}(\dot{y}_c + \dot{\theta}_c h_3 - \dot{y}_{b1} + \dot{\theta}_{b1}h_4 - \dot{\alpha}_c L) = 0$$

$$J_z\ddot{\alpha}_{b1} + 2K_y(y_1 - y_2 + 2\alpha_{b1}l)l + 2K_x(2\alpha_{b1} - \alpha_1 - \alpha_2)d_1^2 + 2C_y (\dot{y}_1 - \dot{y}_2 + 2\dot{\alpha}_{b1}l)l + 2C_x(2\dot{\alpha}_{b1} - \dot{\alpha}_1 - \dot{\alpha}_2)d_2^2 + 2K_{bx}(\alpha_{b1} - \alpha_c)d_5^2 + 2C_{bx}(\dot{\alpha}_{b1} - \dot{\alpha}_c)d_5^2 + 4K_{by}(\alpha_{b1} - \alpha_c)a^2 + 4C_{by}(\dot{\alpha}_{b1} - \dot{\alpha}_c)a^2 = 0$$

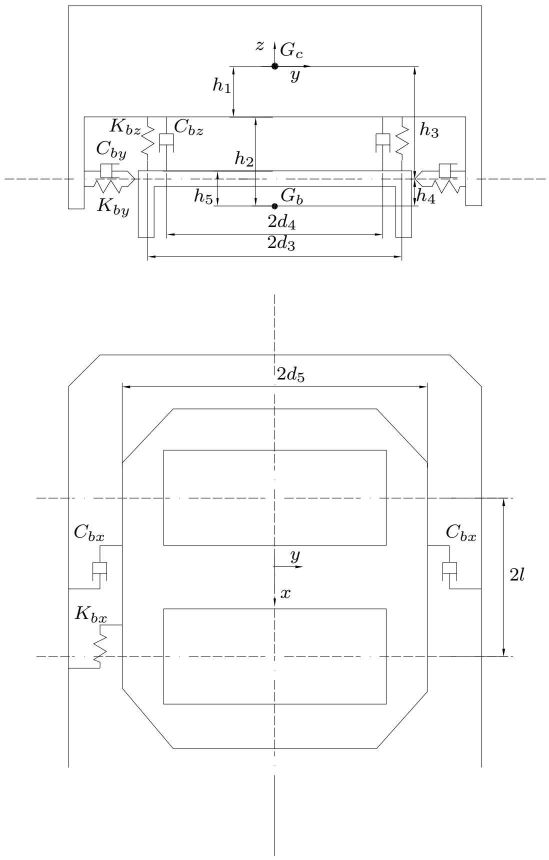


Figure 2: Half a car with bogie.

$$J_x\ddot{\theta}_{b1} + 2K_y(2y_{b1} + 2\theta_{b1}h_5 - y_1 - y_2)h_5 + 2C_y(2\dot{y}_{b1} + 2\dot{\theta}_{b1}h_5 - \dot{y}_1 - \dot{y}_2)h_5 + 2K_z\theta_{b1}d_1^2 + 2C_z\dot{\theta}_{b1}d_2^2 + 4K_{bz}(\theta_{b1} - \theta_c)d_3^2 + 4C_{bz}(\dot{\theta}_{b1} - \dot{\theta}_c)d_4^2 + 4K_{by}(y_c + \theta_c h_1 + \theta_{b1}h_2 - y_{b1} - \alpha_c a)h_2 + 4C_{by}(\dot{y}_c + \dot{\theta}_c h_1 + \dot{\theta}_{b1}h_2 - \dot{y}_{b1} - \dot{\alpha}_c a)h_4 + M_c g/2(y_c - y_{b1}) + M_b g/2(2y_{b1} - y_1 - y_2) = 0$$

$$M\ddot{y}_{b2} + 2K_y(2\theta_{b2}h_5 + 2y_{b2} - y_3 - y_4) + 2C_y(2\dot{\theta}_{b2}h_5 + 2\dot{y}_{b2} - \dot{y}_3 - \dot{y}_4) - 2K_{by}(y_c + \theta_c h_1 - y_{b2} + \theta_{b2}h_2 - \alpha_c L) - 2C_{by}(\dot{y}_c + \dot{\theta}_c h_3 - \dot{y}_{b2} + \dot{\theta}_{b2}h_4 - \dot{\alpha}_c L) = 0$$

$$J_z\ddot{\alpha}_{b2} + 2k_y(y_3 - y_4 + 2\alpha_{b2}l)l + 2k_x(2\alpha_{b2} - \alpha_3 - \alpha_4)d_1^2 + 2C_y(\dot{y}_3 - \dot{y}_4 + 2\dot{\alpha}_{b2}l)l + 2C_x(2\dot{\alpha}_{b2} - \dot{\alpha}_3 - \dot{\alpha}_4)d_2^2 + 2K_{bx}(\alpha_{b2} - \alpha_c)d_5^2 + 2C_{bx}(\dot{\alpha}_{b2} - \dot{\alpha}_c)d_5^2 + 4K_{by}(\alpha_{b2} - \alpha_c)a^2 + 4C_{by}(\dot{\alpha}_{b2} - \dot{\alpha}_c)a^2 = 0$$

$$J_x\ddot{\theta}_{b2} + 2K_y(2y_{b2} + 2\theta_{b2}h_5 - y_3 - y_4)h_5 + 2C_y(2\dot{y}_{b2} + 2\dot{\theta}_{b2}h_5 - \dot{y}_3 - \dot{y}_4)h_5 + 2K_z\theta_{b2}d_1^2 + 2C_z\dot{\theta}_{b2}d_2^2 + 4K_{bz}(\theta_{b2} - \theta_c)d_3^2 + 4C_{bz}(\dot{\theta}_{b2} - \dot{\theta}_c)d_4^2 + 4K_{by}(y_c + \theta_c h_1 + \theta_{b2}h_2 - y_{b2} - \alpha_c a)h_2 + 4C_{by}(\dot{y}_c + \dot{\theta}_c h_1 + \dot{\theta}_{b2}h_2 - \dot{y}_{b2} - \dot{\alpha}_c a)h_4 + M_c g/2(y_c - y_{b2}) + M_b g/2(2y_{b2} - y_3 - y_4) = 0$$

$$M_a\ddot{y}_1 + 2K_y(y_1 + \alpha_{b1}l - y_{b1} - \theta_{b1}h_5) + 2C_y(\dot{y}_1 + \dot{\alpha}_{b1}l - \dot{y}_{b1} - \dot{\theta}_{b1}h_5) + 2F_0(\dot{y}_1/v - \alpha_1) + F_u y_1 = u_1$$

$$J_s\ddot{\alpha}_1 + 2K_x(\alpha_1 - \alpha_{b1})d_1^2 + 2C_x(\dot{\alpha}_1 - \dot{\alpha}_{b1})d_1^2 + 2F_0 s(s\dot{\alpha}_1/v + y_1\gamma/r) = 0$$

$$M_a\ddot{y}_2 + 2K_y(y_2 - \alpha_{b1}l - y_{b1} - \theta_{b1}h_5) + 2C_y(\dot{y}_2 - \dot{\alpha}_{b1}l - \dot{y}_{b1} - \dot{\theta}_{b1}h_5) + 2F_0(\dot{y}_2/v - \alpha_2) + F_u y_2 = u_2$$

$$J_s\ddot{\alpha}_2 + 2K_x(\alpha_2 - \alpha_{b1})d_1^2 + 2C_x(\dot{\alpha}_2 - \dot{\alpha}_{b1})d_1^2 + 2F_0 s(s\dot{\alpha}_2/v + y_2\gamma/r) = 0$$

$$M_a\ddot{y}_3 + 2K_y(y_3 + \alpha_{b2}l - y_{b2} - \theta_{b2}h_5) + 2C_y(\dot{y}_3 + \dot{\alpha}_{b2}l - \dot{y}_{b2} - \dot{\theta}_{b2}h_5) + 2F_0(\dot{y}_3/v - \alpha_3) + F_u y_3 = u_3$$

$$J_s\ddot{\alpha}_3 + 2K_x(\alpha_3 - \alpha_{b2})d_1^2 + 2C_x(\dot{\alpha}_3 - \dot{\alpha}_{b2})d_1^2 + 2F_0 s(s\dot{\alpha}_3/v + y_3\gamma/r) = 0$$

$$M_a\ddot{y}_4 + 2K_y(y_4 - \alpha_{b2}l - y_{b2} - \theta_{b2}h_5) + 2C_y(\dot{y}_4 - \dot{\alpha}_{b2}l - \dot{y}_{b2} - \dot{\theta}_{b2}h_5) + 2F_0(\dot{y}_4/v - \alpha_4) + F_u y_4 = u_4$$

$$J_s\ddot{\alpha}_4 + 2K_x(\alpha_4 - \alpha_{b2})d_1^2 + 2C_x(\dot{\alpha}_4 - \dot{\alpha}_{b2})d_1^2 + 2F_0 s(s\dot{\alpha}_4/v + y_4\gamma/r) = 0$$

where $u_i$, $i = 1, \ldots, 4$ are the control inputs acting on the lateral dynamics of the wheelsets. The dynamic parameters of the model are explained in Tab. 2. In the following, the subscript $x$ denotes the longitudinal direction, the subscript $y$ denotes the lateral direction and the subscript $z$ denotes the vertical direction.

| Parameter | Definition |
|---|---|
| $M_c, M, M_a$ | Car, bogie and wheelset mass |
| $g$ | gravity acceleration magnitude |
| $J_{xc}, J_x$ | Car and bogie roll inertia |
| $J_{zc}, J_z, J_s$ | Car, bogie and wheelset yaw inertia |
| $K_x, K_y, K_z$ | Primary suspensions stiffness |
| $K_{bx}$,$K_{by}$,$K_{bz}$ | Secondary suspensions stiffness |
| $C_x, C_y, C_z$ | Primary suspensions damping |
| $C_{bx}, C_{by}, C_{bz}$ | Secondary suspensions damping |
| $r, \gamma$ | Wheel radius and conicity |
| $v$ | Vehicle travel speed |
| $F_0, F_u$ | Creep coefficients |
| $h_1$ | Vertical distance car barycenter-secondary suspensions |
| $h_2$ | Vert. dist. bogie barycenter-secondary suspensions |
| $h_3$ | Vert. dist. car barycenter-secondary lateral dampers |
| $h_4$ | Vert. dist. bogie barycenter-secondary lateral dampers |
| $h_5$ | Vert. dist. bogie barycenter-primary suspensions |
| $2d_1$ | Lateral distance primary vertical suspensions |
| $2d_2$ | Lateral distance primary vertical dampers |
| $2d_3$ | Lateral distance secondary vertical suspensions |
| $2d_4$ | Lateral distance secondary vertical dampers |
| $2d_5$ | Lateral distance secondary longitudinal suspensions |
| $2a$ | Longitudinal distance secondary suspensions |
| $2s$ | Wheelset gauge |
| $2l$ | Spacing between the two wheelsets in the same bogie |
| $2L$ | Spacing between the two bogies |

Table 2: Dynamic parameters

The above model can be written in state space form

$$\dot{x} = Ax + Bu + \chi \quad (1)$$

$$y = Cx \quad (2)$$

where $x \in \mathbb{R}^{34}$ is the state vector, collecting the variables in Table 1 and their time derivatives, $u \in \mathbb{R}^4$ is the input vector,

$d \in \mathbb{R}^4$ is a state disturbance vector and $y \in \mathbb{R}^4$ is the system output vector.

## 3 Control strategy

In this Section, we present the control system design. The objective of the controller is to reject the disturbances while guaranteeing closed-loop stability. Specifically, assuming that some bounded disturbances enter the system we want the system output and its first $\rho$ time derivatives to asymptotically remain in a bounded neighborhood of zero, i.e.

$$\lim_{t\to\infty} \|y^{(k)}(t)\| < \delta_k, \quad k = 0,\dots\rho-1 \tag{3}$$

for given small real numbers $\delta_k > 0, k = 0,\dots\rho-1$, where $\rho$ is a given integer to be defined next, based on the MIMO pole-zero excess.

In order to fulfill the above requirement, we define a time-varying sliding manifold $\mathcal{S}$ as

$$\mathcal{S} = \left\{(x,t) \in \mathbb{R}^{34} \times \mathbb{R}_+ \;:\; \sigma^{(k)}(y,t) = 0,\; k = 0,\dots,\rho-1\right\} \tag{4}$$

where $\sigma : \mathbb{R}^4 \times \mathbb{R}_+ \mapsto \mathbb{R}^4$ is given by

$$\sigma(y,t) = -y + \eta(t), \quad \eta(t) = e^{Wt}\sum_{i=0}^{\rho-1} c_i \frac{t^i}{i!} \tag{5}$$

and $W$ is a Hurwitz $4\times4$ real matrix to be suitably selected, while $c_i, i = 0,\dots,\rho-1$ are real vectors given by the recursive equation

$$\begin{aligned} c_k &= y^{(k)}(0) - \sum_{i=0}^{k-1}\begin{pmatrix} k \\ i \end{pmatrix} W^{k-i} c_i, \\ &\quad k = 1,\dots,\rho-1,\; c_0 = y(0), \end{aligned} \tag{6}$$

being $y^{(k)}(0), k = 0,\dots,\rho-1$ the initial conditions of the system output and its time derivatives. The function $\eta(t)$ takes into account initial conditions on the output and its time derivatives so as to have $\sigma^{(k)}(y,t)|_{t=0} = 0,\; k = 0,\dots,\rho-1$.

The design of the robust controller is addressed by the following Theorem, that is a slightly modified version of Theorem 1 in [13]:

**Theorem 1** *Consider the completely controllable and observable MIMO plant*

$$\dot{x} = Ax + Bu + \chi \tag{7}$$

$$y = Cx \tag{8}$$

*where* $x \in \mathbb{R}^n$, $u \in \mathbb{R}^r$, $y \in \mathbb{R}^r$.

*Let the control law be defined by the differential equation*

$$\begin{aligned} &\epsilon^\nu D_\nu u^{(\nu)} + \epsilon^{\nu-1} D_{\nu-1} u^{(\nu-1)} + \cdots + \epsilon D_1 \dot{u} \\ &= N_\rho \sigma^{(\rho)} + N_{\rho-1}\sigma^{(\rho-1)} + \cdots + N_1\dot{\sigma} + N_0\sigma, \end{aligned} \tag{9}$$

*where* $\epsilon > 0$ *is a "small" real constant, and* $D_i, i = 1,\dots,\nu$, $N_i, i = 0,\dots,\rho$ *are real constant* $r\times r$ *matrices to be selected, with* $\nu$ *and* $\rho$ *integers such that* $\nu \geq \rho$.

*Assume that:*

(i) *the plant is minimum phase;*

(ii) *the disturbances are "matched", i.e. there exists* $g \in \mathbb{R}^r$ *such that* $\chi = Bg$;

(iii) *the integer* $\rho$ *and the matrices* $N_k$, $k = 1,\dots,\rho$ *are such that the* $\rho$*-th Markov parameter* $H_\rho = CA^{\rho-1}B$ *is invertible and let* $N_\rho H_\rho = D_0$, *with* $D_0$ *invertible* $r\times r$ *real matrix, while the previous Markov parameters are zero,* $H_i = 0$, $i = 0,\dots,\rho-1$;

(iv) *the polynomial*

$$\det\left(D_\nu s^\nu + D_{\nu-1}s^{\nu-1} + \cdots + D_1 s + D_0\right) \tag{10}$$

*is strictly Hurwitz;*

(v) *the polynomial*

$$\det\left(N_\rho s^\rho + N_{\rho-1}s^{\rho-1} + \cdots + N_0\right) \tag{11}$$

*is strictly Hurwitz;*

(vi) *there exists a real* $\gamma < 0$ *such that*

$$Re\lambda_{\max}(W) < \gamma < 0 \tag{12}$$

*where* $Re\lambda_{\max}(X)$ *denotes the largest real part of the eigenvalues of the matrix* $X$.

*Then, there exist* $\epsilon_0 > 0$, $\delta > 0$, $\lambda < 0$, *with* $\lambda > \gamma$, *such that for any* $\epsilon \in (0,\epsilon_0]$, *the solution* $(x(t,\epsilon), u(t,\epsilon))$ *of (7), (9), is such that*

$$\sum_{k=0}^{\rho-1} \|y^{(k)}(t,\epsilon)\| \leq \delta + ae^{\lambda t} \quad \text{for any} \quad t \in [0,+\infty), \tag{13}$$

*where* $a$ *is a positive constant depending on the plant initial conditions.*

The proof of this Theorem can easily be deduced form the one in [13].

The above Theorem suggests us a procedure for designing the controller:

1. Select the boundary layer dynamics by assigning $\nu+1$ matrices $D_k$, $k = 0,\dots\nu$ such that eqn. (10) holds;
2. select $\rho$ matrices $\bar{N}_k$, $k = 0,\dots,\rho-1$ such that the polynomial

$$\det\left(s^\rho + \bar{N}_{\rho-1}s^{\rho-1} + \cdots + \bar{N}_0\right) \tag{14}$$

is Hurwitz;

3. let $N_\rho = D_0 H_\rho^{-1}$, $N_k = N_\rho \bar{N}_k$, $k = 0, \ldots, \rho - 1$.
4. select a "small" $\epsilon$ and compute the controller.

The MFD of the controller is obviously computed as

$$\begin{aligned} C(s,\epsilon) \quad = \quad & \left(D_\nu \epsilon^\nu s^\nu + D_{\nu-1}\epsilon^{\nu-1}s^{\nu-1} + \cdots + D_1 \epsilon s\right)^{-1} \\ & \left(N_\rho s^\rho + N_{\rho-1}s^{\rho-1} + \cdots + N_0\right). \end{aligned} \quad (15)$$

and the control signal is $u(s) = C(s,\epsilon)\sigma(s)$. As far as the selection of $W$, it defines the way the system output recovers possible initial offsets, hence it can be chosen as a Hurwitz matrix with eigenvalues related to output recovery time.

## 4 Simulation results

The procedure presented in the previous section is applied in two cases. Four linear actuators have been considered on the lateral displacement of the four wheelsets. Moreover, a band-limited random disturbance has been considered the front wheelset of the front bogie, and the same disturbance has been applied to the other wheelsets considering a time delay corresponding to the train speed ($300$ km/h) and the wheelset distances. The disturbance is such that the open-loop lateral displacement of the wheelsets is $5$mm. In the first simulation set we have considered sensors on the four wheelset displacement. Hence we have a colocated actuators/sensors configuration, which results in $\rho = 2$ and $H_2$ diagonal. Using the procedure in the previous Section, we have selected $D_k$'s, $\bar{N}_k$'s and $W$ as diagonal matrices, hence the resulting controller is decentralized. The results of the simulation show very good robustness properties, with high disturbance rejection. However, placing sensors on the wheelsets can be not realistic, since the wheelset vibrations can reduce sensors life time. Hence, a second sensor configuration has been considered, moving the sensors on the bogies in order to put them in a less harsh environment. Specifically, lateral displacement and yaw have been measured for each bogie, and in this case $\rho = 4$. Using again the procedure given in the previous Section, in order to simplify computation we have still chosen $D_k$'s, $\bar{N}_k$'s and $W$ diagonal, but, since the Markov parameter $H_4$ is no longer diagonal, the resulting controller is fully coupled. Simulation results are shown and compared in Figures 3, 4, 5. In Figure 3 the wheelset lateral displacement is depicted, and it is apparent that the first strategy, colocated feedback, exhibits better performance. Nevertheless, in both cases significant displacement reduction have been achieved with respect to open-loop ($5$ mm). Moreover, also body acceleration have been considered as a measure of ride comfort. In Figure 4 the closed-loop behavior is reported in the two cases. Again, the colocated strategy works better, and again closed-loop systems outperform the open-loop system, whose maximum acceleration is about $0.3\,\mathrm{m/s^2}$. The same performances can be shown to hold on car yaw and roll accelerations. Finally, in Figure 5 the controller output are shown, whose shapes are such as to compensate for applied disturbances.

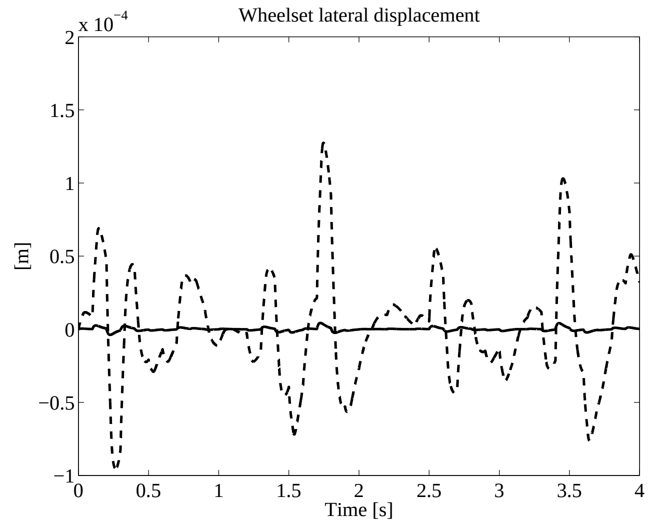


Figure 3: Closed-loop front wheelset lateral displacement (front bogie). Solid line: colocated feedback; dashed line: noncolocated feedback.

## 5 Conclusions

In this paper the model of the lateral dynamics of a railway car has been presented, and a robust MIMO control strategy has been presented and applied to control the car. The car has two bogies, each bearing two wheelsets, hence a 17 dofs' model results. The control strategy is based on the Theory of Singular Perturbations, and exhibits very good robustness properties with respect to external disturbances and uncertain model parameters. Simulations are performed in order to test the proposed strategy in two cases: a colocated feedback policy, that is shown to give the best results in terms of disturbance rejection, and a noncolocated centralized controller, that, although slightly degrading the closed-loop performances, allows better sensor placement. Performances are also evaluated in terms of passengers comfort.

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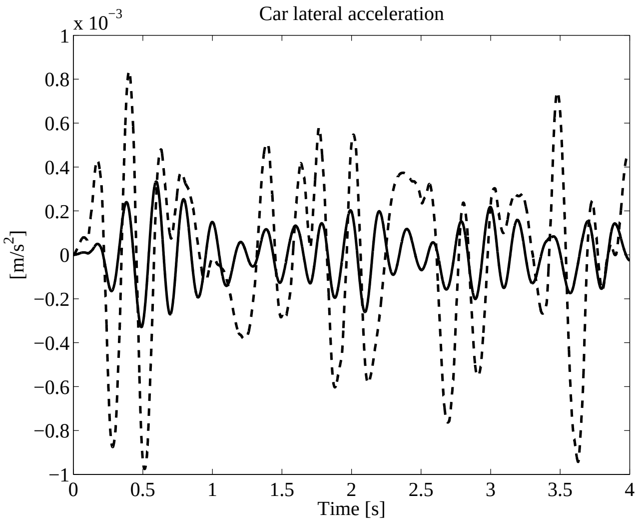


Figure 4: Closed-loop car lateral acceleration. Solid line: colocated feedback; dashed line: noncolocated feedback.

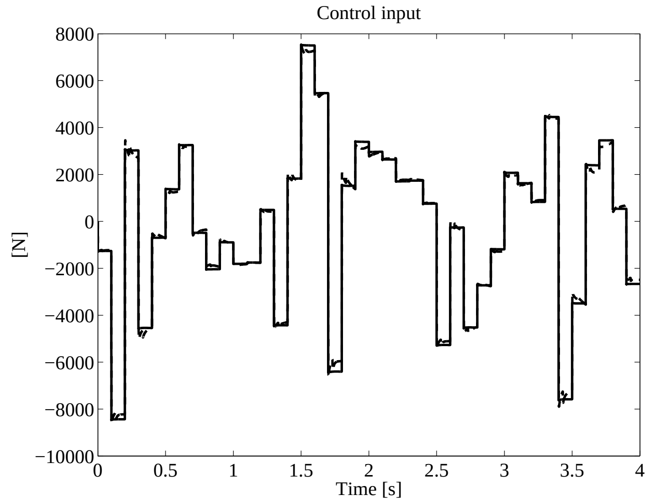


Figure 5: Control input: front wheelset, front bogie. Solid line: colocated feedback; dashed line: noncolocated feedback.

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