ROBUST TRANSIENT STABILIZATION OF A SYNCHRONOUS GENERATOR WITH PARAMETER UNCERTAINTY

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Abstract

A robust nonlinear state feedback control achieving transient stabilization is designed on the basis of the standard third order model of a synchronous generator connected to an infinite bus. Sudden mechanical power failures, short circuits, infinite bus perturbations may drive the generator out of step. The proposed robust nonlinear excitation control prevents the machine from going out of step in the presence of any mechanical or electrical parameter perturbation. L_∞ and L_2 disturbance attenuation are guaranteed from the power angle and relative speed regulation errors with respect to time-varying parameter perturbations from nominal values. The operating condition is the only equilibrium point of the closed loop system with an explicitly computable stability region, when all parameters are equal to their nominal values.

1 Introduction

Transient stability and voltage regulation for power systems are classical control problems. Dynamical models of increasing complexity have been developed both for a single machine connected to an infinite bus and for multimachine networks ([1], [3]). They all show an intrinsic nonlinear nature since the electric power provided by each generator is a nonlinear function of the generators state variables. As a consequence there are several stable and unstable equilibrium points. Early studies aimed at determining the stability regions of desired operating conditions by means of Lyapunov functions in order to study the effect of perturbations ([1], [9]). In fact sudden mechanical (load shedding and generation tripping) and electrical (short circuits with changes in the power network structure) perturbations may destabilize the operating conditions and force one or more generators to go out of step and to be disconnected from the network. The transient stabilization problem consists in the design of an excitation feedback control for each generator which keeps each generator at synchronous speed when perturbations occur. Voltage regulation is not considered at this stage. Linear controllers are actually employed which are designed on the basis of linear approximations around operating conditions: only small perturbations and small deviation from operating conditions can be handled. It is clear that nonlinear controllers [7] are required to handle the large perturbations that typically occur in power systems. As a first step in this direction it was shown that several power systems models are linearizable by state feedback ([2], [4], [14]) so that the operating condition is the only equilibrium point for the closed loop system and very large stability regions can be assured by nonlinear feedback linearizing controllers provided that state variable measurements and physical parameters are available to the controller: transient stabilization along with voltage regulation can be achieved in this case. Adaptive versions of feedback linearizing controls were then developed in [12], [15] and [8] so that the knowledge of some critical parameters which may change during operations is no longer needed to guarantee speed regulation. Robust nonlinear state feedback controls have also been investigated in [13] while in [10] robust adaptive nonlinear controllers are developed assuming additive disturbances and unknown electrical parameters.

In this paper we address the problem of designing a nonlinear state feedback control for a third order model of a synchronous generator connected to an infinite bus relying only on the nonlinearities but not on the parameters which are not assumed to be known by the controller and in fact are also allowed to be time-varying to account for unmodelled dynamics. There are ten physical parameters (both mechanical and electrical) in the third order model and they can be time varying and undergo sudden variations due to short circuits, turbine failures, load shedding, infinite bus voltage and frequency perturbations. Following the theoretical developments in [5], [6], [11] (even though they do not apply to the model considered here in which uncertain parameters multiply the control input), a robust nonlinear state feedback control is designed which makes the operating condition the only equilibrium point for the closed loop system when the parameters assume their nominal value: it is exponentially stable with an explicitly computed very large stability region. The robustness with respect to time varying variations from nominal values is achieved guaranteeing boundedness and arbitrary L_∞ and L_2 disturbance attenuation from the regulation errors with respect to the variations of all parameters from their nominal values.

1 Problem Formulation

The well-known classical third order dynamic model of a synchronous generator connected to an infinite bus is given as follows (see [1]):

$$\begin{aligned}
\dot{\delta} &= \omega \\
\dot{\omega} &= -\frac{D}{H}\omega - \frac{\omega_s}{H}P_e + \frac{\omega_s}{H}P_m \qquad (1)\\
\dot{P}_e &= -\frac{1}{T'_d}P_e + \frac{X_d - X'_d}{X_{d_s}X'_{d_s}}V_s^2\omega\sin^2\delta \\
&\quad + P_e\omega\mathrm{cotg}\delta + \frac{V_sK_c}{X_{d_s}T'_d}(\sin\delta)u_f
\end{aligned}$$

where δ(rad) is the power angle of the generator relative to the angle of the infinite bus rotating at synchronous speed ω_s; ω(rad/s) is the angular speed of the generator relative to ω_s, i.e., $\omega = \omega_g - \omega_s$ and ω_g is the generator angular speed; $P_e(p.u.)$ denotes the active electrical power delivered by the generator to the infinite bus; (δ, ω, P_e) constitute the state variables; $H(s)$ is the inertia constant, $D(p.u.)$ is damping constant and $P_m(p.u.)$ is the mechanical input power; $V_s(p.u.)$ is the voltage at the infinite bus; $X_{d_s} = X_s + X_d = X_T + \frac{1}{2}X_L + X_d(p.u.)$ is the total reactance which takes into account $X_d(p.u.)$, the generator direct axis reactance, $X_L(p.u.)$, the transmission line reactance, and $X_T(p.u.)$, the reactance of the transformer; $X'_{d_s} = X_s + X'_d = X_T + \frac{1}{2}X_L + X'_d(p.u.)$ with X'_d denoting the generator direct axis transient reactance; $u_f(p.u.)$, which constitutes the control signal, is the input to the SCR amplifier of the generator; K_c is the gain of the excitation amplifier and $T'_d = T_dX'_{d_s}/X_{d_s}$ with $T_d(s)$ being the direct axis short circuit time constant.
The generator terminal voltage is given by

$$V_t = \left[\frac{X_s^2P_e^2}{V_s^2\sin^2\delta} + \frac{X_d^2V_s^2}{X_{d_s}^2} + \frac{2X_sX_d}{X_{d_s}}P_e\mathrm{cotg}\delta\right]^{\frac{1}{2}}$$

which is to be regulated to its reference value $V_{t_r} = 1(p.u.)$, while the relative speed ω is to be regulated to zero.

In practice, the exact values of the machine physical parameters (H,X_d,X'_d,X_T,X_L,T_d,K_c) in model (1) are hard to obtain, and (P_m,D,ω_s,V_s) are lumped parameters which account for unmodelled dynamics such as turbine dynamics, load dynamics, damper windings and multimachine dynamics. Those parameters may undergo sudden on line variations due to mechanical and electrical perturbations and faults. For instance, ω_s and P_m will change considerably when the mechanical power is perturbed by load shedding or turbine failures, and their variation is not measurable. The infinite bus voltage V_s may change as a consequence of perturbations occurred in the network. Furthermore, when a fault occurs in the bus or causes a change in the structure of the electrical network, the reactance of the transmission line will be changed and as a result X_{d_s} and X'_{d_s} will change considerably. Thus, there is a need for the excitation control law which not only stabilize the nominal system at a desired equilibrium with a large stability region, but also guarantees robustness with respect to unknown parameter perturbations. In the next section, we will present an approach to design such a feedback control law so that the closed loop system is exponentially stable with arbitrary small L_∞ and L_2 gains from the parameter perturbations to the regulation error. To this end, we reduce the eleven physical parameters in the model (1) as follows

$$\begin{aligned}
\dot{\delta} &= \omega \\
\dot{\omega} &= -\theta_1\omega - \theta_2P_e + \theta_3 \qquad (2)\\
\dot{P}_e &= -\theta_4P_e + \theta_5\omega\sin^2\delta + P_e\omega\mathrm{cotg}\delta + \frac{1}{\theta_6}(\sin\delta)u_f
\end{aligned}$$

where the new parametrization requires only six positive parameters $\theta_i > 0$, $(i = 1, 2, \cdots, 6)$ defined as $\theta_1 = \frac{D}{H}$, $\theta_2 = \frac{\omega_s}{H}$, $\theta_3 = \frac{\omega_s}{H}P_m$, $\theta_4 = \frac{1}{T'_d}$, $\theta_5 = \frac{X_d - X'_d}{X_{d_s}X'_{d_s}}V_s^2$, $\theta_6 = \frac{X_{d_s}T'_d}{V_sK_c}$. While the variables (ω, P_e, V_t) can be measured, δ measurements are not available, even though δ can be obtained by time integration of ω measurements. In the following, we will denote by c_i $(i = 1, 2, \cdots, 6)$ the known constant nominal values of the parameters θ_i and by $\tilde{\theta}_i = \theta_i - c_i$ $(i = 1, 2, \cdots, 6)$ the parameter variations from nominal values. Due to physical considerations we restrict the operation of the system to the open set

$$- = \left\{(\delta, \omega, P_e) : (\delta, \omega, P_e) \in (0, \pi) \times \Re \times \Re^+\right\}. \qquad (3)$$

The asymptotically stable nominal operating condition is given by

$$\delta = \delta_s, \quad \omega = 0, \quad P_e = P_{es} = \frac{c_3}{c_2}, \quad u_{fs} = \frac{c_4c_3c_6}{c_2\sin\delta_s} \qquad (4)$$

where $\delta_s \in (0, \frac{\pi}{2})$, guarantees terminal voltage regulation $[V_t = 1(p.u.)]$. Note that there is another unstable equilibrium point $(\delta_u, 0, P_m)$ with $\sin\delta_u = \sin\delta_s$ for equation (2) which may be very close to the stable one making the stability region very small. The control problem can be formulated as follows: find a state feedback control law [k is the arbitrary positive real scalar to be chosen]

$$u_f = \varphi(\delta, \omega, P_e, \delta_s, k) \tag{5}$$

for the system (2) such that the closed loop system has the asymptotically stable operating condition (4) as an exponentially stable equilibrium point when the parameters assume their nominal values and in addition satisfies in terms of $z(t) = [(\delta(t)-\delta_s), \omega(t)]^T$, $x(t) = [z(t), (P_e(t)-P_{es})]^T$ and $w = [\tilde{\theta}_1, \tilde{\theta}_2, \cdots, \tilde{\theta}_6]^T$ the following properties:

(S1) *exponential stability and L_∞ disturbance attenuation.*

$$\|z(t)\|^2 \le g(x(0))e^{-ct} + \frac{1}{k}\gamma_1(\|w\|), \quad \forall t \ge 0 \tag{6}$$

holds, where $g(x(0)) \ge 0$, $c > 0$, $\gamma_1(r)$ is a class $\mathcal{K}$ function and k is an arbitrary positive real scalar;

(S2) *L_2 disturbance attenuation.*

$$\int_0^T \|z(\tau)\|^2 d\tau \le h(x(0)) + \frac{1}{k}\int_0^T \gamma_2(\|w\|)d\tau \tag{7}$$

holds for any given $T > 0$, where $h(x(0)) \ge 0$, $\gamma_2(r)$ is a class $\mathcal{K}$ function and k is an arbitrary positive real scalar.

The effect of physical parameters variations from their nominal values is quantified by (6) and (7): (6) gives an upper bound on L_∞ regulation errors while (7) gives a bound on L_2 regulation errors. This formulation allows us to solve the transient stability problem for the desired operating condition and to characterize the robustness with respect to parameter variations from their nominal values.

2 Robust Control Design

Defining $\tilde{\delta}(t) = \delta(t) - \delta_s$ and $\tilde{\omega}(t) = \omega(t) - \omega_r(t) = \omega(t) + k_\delta\tilde{\delta}(t)$ with $k_\delta > 0$, from (2) we have

$$\begin{aligned} \dot{\tilde{\delta}} &= -k_\delta\tilde{\delta} + \tilde{\omega} && (8)\\ \dot{\tilde{\omega}} &= -\theta_1\omega - \theta_2 P_e + \theta_3 + k_\delta\omega. && (9)\end{aligned}$$

Recall that $\theta_i > 0$ $(i = 1, 2, \cdots, 6)$. Let $\theta_7 = \frac{1}{\theta_2}$, $c_7 = \frac{1}{c_2}$ $\left(\tilde{\theta}_7 = \theta_7 - c_7 = -\frac{1}{\theta_2 c_2}\tilde{\theta}_2\right)$ and define

$$\tilde{P}_e(\delta, \omega, P_e, \delta_s, k) = P_e - P_{er}(\delta, \omega, \delta_s, k) \tag{10}$$

with

$$P_{er}\Big(\tilde{\omega}(\delta, \omega, \delta_s), v_1(\delta, \omega, \delta_s, k)\Big) = c_7 v_1 + \frac{k}{4}\tilde{\omega}v_1^2 \tag{11}$$

where v_1 is to be designed and $k > 0$ is the constant which appears in the disturbance attenuation inequalities (6) and (7). Then, the equation (9) can be rewritten as

$$\dot{\tilde{\omega}} = -v_1 - \theta_1\omega - \theta_2\tilde{P}_e + \theta_3 + k_\delta\omega + \frac{\tilde{\theta}_7}{\theta_7}v_1 - \frac{k}{4\theta_7}\tilde{\omega}v_1^2. \tag{12}$$

Hence, by choosing v_1 as $(k_\omega > 0)$

$$v_1 = -c_1\omega + c_3 + k_\delta\omega + \frac{k}{4}\tilde{\omega}\omega^2 + \frac{k}{4}\tilde{\omega} + k_\omega\tilde{\omega} + \tilde{\delta} \tag{13}$$

the equation (12) becomes

$$\begin{aligned} \dot{\tilde{\omega}} &= -k_\omega\tilde{\omega} - \tilde{\delta} - \theta_2\tilde{P}_e + [\phi_{21} \;\; \phi_{23}]\begin{bmatrix}\tilde{\theta}_1\\ \tilde{\theta}_3\end{bmatrix} \\ &\quad -\frac{k}{4}\tilde{\omega}(\phi_{21}^2 + \phi_{23}^2) + \frac{\tilde{\theta}_7}{\theta_7}v_1 - \frac{k}{4\theta_7}\tilde{\omega}v_1^2 \qquad (14)\end{aligned}$$

with $\phi_{21} = -\omega$, $\phi_{23} = 1$. Note that $\dot{\tilde{\omega}} = \dot{\omega} - \dot{\omega}_r = \dot{\omega} + k_\delta\omega$. Differentiating (11) and (13) with respect to time yields

$$\begin{aligned} \dot{P}_{er} &= \Big[(\frac{k}{4}\omega^2 + \frac{k}{4} + k_\omega + k_\delta - c_1 + \frac{k}{2}\tilde{\omega}\omega)\cdot \\ &\quad \cdot (c_7 + \frac{k}{2}\tilde{\omega}v_1) + \frac{k}{4}v_1^2\Big](-\theta_1\omega - \theta_2 P_e + \theta_3) \\ &\quad + k_\delta\Big[(\frac{k}{4}\omega^2 + \frac{k}{4} + k_\omega + \frac{1}{k_\delta})(c_7 + \frac{k}{2}\tilde{\omega}v_1) \\ &\quad + \frac{k}{4}v_1^2\Big]\omega. \qquad (15)\end{aligned}$$

Therefore, from the model (2) and the last expression (15), we get

$$\dot{\tilde{P}}_e = \dot{P}_e - \dot{P}_{er} = \frac{1}{\theta_6}(\sin\delta)u_f + \phi_{30} + \sum_{i=1}^{5}\phi_{3i}\theta_i \tag{16}$$

where

$$\begin{aligned} \phi_{30} &= -k_\delta\Big[(\frac{k}{4}\omega^2 + \frac{k}{4} + k_\omega + \frac{1}{k_\delta})(c_7 + \frac{k}{2}\tilde{\omega}v_1) \\ &\quad + \frac{k}{4}v_1^2\Big]\omega + P_e\omega\,\mathrm{cotg}\delta, \\ \phi_{31} &= \omega\phi_s, \quad \phi_{32} = P_e\phi_s, \quad \phi_{33} = -\phi_s, \quad \phi_{34} = -P_e, \\ \phi_{35} &= \omega\sin^2\delta, \\ \phi_s &= \Big[(\frac{k}{4}\omega^2 + \frac{k}{4} + k_\omega + k_\delta - c_1 + \frac{k}{2}\tilde{\omega}\omega)\cdot \\ &\quad \cdot (c_7 + \frac{k}{2}\tilde{\omega}v_1) + \frac{k}{4}v_1^2\Big].\end{aligned}$$

Using the same strategy employed in choosing v_1, we define the robust control law as follows

$$u_f(\delta, \omega, P_e, \delta_s, k) = \frac{c_6}{\sin\delta}v_2 - \frac{k}{4\sin\delta}\tilde{P}_e v_2^2 \tag{17}$$

with v_2 yet to be designed, which substituted in (16) gives

$$\dot{\tilde{P}}_e = v_2 - \frac{\tilde{\theta}_6}{\theta_6}v_2 - \frac{k}{4\theta_6}\tilde{P}_e v_2^2 + \phi_{30} + \sum_{i=1}^{5}\phi_{3i}\theta_i. \tag{18}$$

Choosing v_2 as $(k_p > 0)$

$$\begin{aligned} v_2 &= -k_p\tilde{P}_e + c_2\tilde{\omega} - \frac{k}{4}\tilde{\omega}^2\tilde{P}_e - \phi_{30} - \sum_{i=1}^{5}\phi_{3i}c_i \\ &\quad -\frac{k}{4}\tilde{P}_e\sum_{i=1}^{5}\phi_{3i}^2 \qquad (19)\end{aligned}$$

and substituting it in (18), we obtain

$$\begin{aligned}\dot{\tilde{P}}_e &= -k_p\tilde{P}_e + c_2\tilde{\omega} + \sum_{i=1}^{5}\phi_{3i}\tilde{\theta}_i - \frac{k}{4}\tilde{P}_e\sum_{i=1}^{5}\phi_{3i}^2 \\ &\quad -\frac{\tilde{\theta}_6}{\theta_6}v_2 - \frac{k}{4\theta_6}\tilde{P}_e v_2^2 - \frac{k}{4}\tilde{\omega}^2\tilde{P}_e. \qquad (20)\end{aligned}$$

Thus, the closed loop system dynamics are given from (8), (14) and (20) as

$$\begin{aligned}\dot{\tilde{\delta}} &= -k_\delta\tilde{\delta} + \tilde{\omega} \\ \dot{\tilde{\omega}} &= -k_\omega\tilde{\omega} - \tilde{\delta} - \theta_2\tilde{P}_e + \Phi_2\tilde{\theta} - \frac{k}{4}\tilde{\omega}\Phi_2\Phi_2^T \\ &\quad +\frac{\tilde{\theta}_7}{\theta_7}v_1 - \frac{k}{4\theta_7}\tilde{\omega}v_1^2 \qquad (21) \\ \dot{\tilde{P}}_e &= -k_p\tilde{P}_e + c_2\tilde{\omega} + \Phi_3\tilde{\theta} - \frac{k}{4}\tilde{P}_e\Phi_3\Phi_3^T \\ &\quad -\frac{\tilde{\theta}_6}{\theta_6}v_2 - \frac{k}{4\theta_6}\tilde{P}_e v_2^2 - \frac{k}{4}\tilde{\omega}^2\tilde{P}_e\end{aligned}$$

where $\tilde{\theta} = [\tilde{\theta}_1,\ \tilde{\theta}_2, \cdots, \tilde{\theta}_5]^T$, $\Phi_2 = [\phi_{21},\ \ 0,\ \ \phi_{23},\ \ 0,\ \ 0\ ]$ and $\Phi_3 = [\phi_{31},\ \ \phi_{32},\ \ \phi_{33},\ \ \phi_{34},\ \ \phi_{35}\ ]$.

Consider the quadratic positive definite function

$$V(\tilde{\delta}, \tilde{\omega}, \tilde{P}_e) = \frac{1}{2}(\tilde{\delta}^2 + \tilde{\omega}^2 + \tilde{P}_e^2). \qquad (22)$$

Computing the derivative of function V along the trajectories of the closed loop system, we obtain

$$\begin{aligned}\dot{V} &\leq -(k_\delta\tilde{\delta}^2 + k_\omega\tilde{\omega}^2 + k_p\tilde{P}_e^2) + |\tilde{\omega}\Phi_2\tilde{\theta}| \\ &\quad -\frac{k}{4}\tilde{\omega}^2\Phi_2\Phi_2^T + \left|\frac{\tilde{\theta}_7}{\theta_7}\tilde{\omega}v_1\right| - \frac{k}{4\theta_7}\tilde{\omega}^2 v_1^2 \\ &\quad +|\tilde{\theta}_2\tilde{\omega}\tilde{P}_e| - \frac{k}{4}\tilde{\omega}^2\tilde{P}_e^2 + |\tilde{P}_e\Phi_3\tilde{\theta}| - \frac{k}{4}\tilde{P}_e^2\Phi_3\Phi_3^T \\ &\quad +\left|\frac{\tilde{\theta}_6}{\theta_6}\tilde{P}_e v_2\right| - \frac{k}{4\theta_6}\tilde{P}_e^2 v_2^2. \qquad (23)\end{aligned}$$

Using repeatedly the inequality

$$\left|\frac{\zeta^T\xi}{a}\right| \quad \leq \quad \frac{k}{4}\frac{\zeta^T\zeta}{a} + \frac{1}{k}\frac{\xi^T\xi}{a}\ (\zeta, \xi \in \Re^p, p \in \mathcal{N}^+, a, k \in \Re^+)$$

and recalling that $\theta_7 = \frac{1}{\theta_2}$ and $\tilde{\theta}_7 = -\frac{1}{\theta_2 c_2}\tilde{\theta}_2$, we obtain the dissipation inequality

$$\dot{V} \leq -(k_\delta\tilde{\delta}^2 + k_\omega\tilde{\omega}^2 + k_p\tilde{P}_e^2) + \frac{1}{k}\sum_{i=1}^{6}\epsilon_i\tilde{\theta}_i^2 \qquad (24)$$

with $\epsilon_1 = \epsilon_3 = \epsilon_4 = \epsilon_5 = 2$, $\epsilon_2 = 3 + \frac{1}{\theta_2 c_2^2}$ and $\epsilon_6 = \frac{1}{\theta_6}$.
Let $\gamma(\|w\|) = \sum_{i=1}^{6}\epsilon_i\tilde{\theta}_i^2$. Then, (24) becomes

$$\dot{V} \leq -(k_\delta\tilde{\delta}^2 + k_\omega\tilde{\omega}^2 + k_p\tilde{P}_e^2) + \frac{1}{k}\gamma(\|w\|) \qquad (25)$$

which implies, according to (22)

$$\dot{V} \leq -\min\Big\{2k_\delta, 2k_\omega, 2k_p\Big\}V + \frac{1}{k}\gamma(\|w\|). \qquad (26)$$

Let $U(t) = V(\tilde{\delta}(t), \tilde{\omega}(t), \tilde{P}_e(t)) = \frac{1}{2}\Big[\tilde{\delta}^2(t) + \tilde{\omega}^2(t) + \tilde{P}_e^2(t)\Big]$ and $c = \min\big\{2k_\delta, 2k_\omega, 2k_p\big\}$.
From (26) it follows

$$U(t) \leq U(0)e^{-ct} + \frac{1}{k}\int_0^t \gamma(\|w\|)e^{c(\tau-t)}d\tau. \qquad (27)$$

Since $\|z(t)\|^2 \leq \max\Big\{4, 2 + 4{k_\delta}^2\Big\}U(t)$, from (27) we obtain

$$\begin{aligned}\|z(t)\|^2 &\leq \max\Big\{4, 2 + 4{k_\delta}^2\Big\}\Big[U(0)e^{-ct} \\ &\quad +\frac{1}{k}\int_0^t \gamma(\|w\|)e^{c(\tau-t)}d\tau\Big]\end{aligned}$$

which implies (S1) with

$$g(x(0)) = \max\Big\{2, 1 + 2{k_\delta}^2\Big\}\Big[\tilde{\delta}^2(0) + \tilde{\omega}^2(0) + \tilde{P}_e^2(0)\Big].$$

On the other hand, integrating (25) over $[0, T)$, we obtain

$$\int_0^T \|z(\tau)\|^2 d\tau \leq \frac{\max\Big\{2, 1 + 2{k_\delta}^2\Big\}}{\min\Big\{k_\delta, k_\omega\Big\}}\Big[U(0) + \frac{1}{k}\int_0^T \gamma(\|w\|)d\tau\Big]$$

which implies (S2) with

$$h(x(0)) = \frac{\max\Big\{1, \frac{1}{2} + {k_\delta}^2\Big\}}{\min\Big\{k_\delta, k_\omega\Big\}}\Big[\tilde{\delta}^2(0) + \tilde{\omega}^2(0) + \tilde{P}_e^2(0)\Big].$$

At the equilibrium $\Big(\tilde{\delta} = 0, \tilde{\omega} = 0, \tilde{P}_e = 0, \tilde{\theta}_i = 0\ (i = 1, 2, \cdots, 6)\Big)$ for the closed loop system (21) we obtain from (17), (19)

$$\delta = \delta_s, \quad \omega = 0, \quad P_e = c_7 c_3 = \frac{c_3}{c_2}, \quad u_f = \frac{c_3 c_4 c_6}{c_2\sin\delta_s}$$

in accordance to (4). When $\tilde{\theta}_i = 0$ $(i = 1, 2, \cdots, 6)$, that is all parameters are equal to their nominal values, the function V in (22) is a Lyapunov function according to (24) and the stability region of the exponentially stable operating condition $(\delta_s, 0, P_m)$ is given by

$$\mathcal{S} = \left\{(\delta, \omega, P_e) : V(\tilde{\delta}, \tilde{\omega}, \tilde{P}_e) < \frac{\delta_s^2}{2}\right\}$$

so that the singularities at $\delta = 0$ and $\delta = \pi$ for the control (17) are avoided. In the presence of parameters variations from their nominal values, i.e., $\tilde{\theta}_i \neq 0$ $(i = 1, 2, \cdots, 6)$, the control parameter k directly affects the influence of each parameter variation on the regulation errors both in L_∞ and L_2 so that both transients and peaks can be controlled to enhance the robustness and prevent the generator from going out of step.

Remark: Following the design of the controller and the analysis of its properties, it is obvious that (25) is still valid even if the parameters θ_i $(i = 1, 2, \cdots, 6)$ are time varying: in the case, the inequality (25) and consequently the inequalities (7) and (8) hold with $\gamma(\|w(t)\|) = \sum_{i=1}^{6}\epsilon_i(t)\tilde{\theta}_i^2(t)$.

3 Simulation Results

We tested by simulation the proposed controller (17), (19) with control parameters: $k_\delta = k_\omega = k_p = 60$, $k = 0.1$ with reference to a synchronous generator characterized by the following nominal values of the parameters:

$\omega_s = 314.159$rad/s	$D = 5$p.u.	$H = 8$s
$T_d = 6.9$s	$K_c = 1$	$X_d = 1.863$p.u.
$X'_d = 0.257$p.u.	$X_T = 0.127$p.u.	$X_L = 0.4853$p.u.

The operating point δ_s=1.256 rad, P_m=0.9 p.u., ω=0 guarantees V_t=1 p.u., with V_s=1 p.u.. The goal of the simulation is to verify the effect of a severe fault on the turbine, a change in the structure of the electrical network and a variation of the gain of the excitation amplifier. It was considered a fast reduction (50% of the initial value) of the mechanical input power and a sudden increase (20% of the nominal value) of the transmission line reactance X_L and of the excitation amplifier gain K_c. The simulation was done according to the following time sequence:

1. The system is in pre-faulted state.
2. At $t = 0.6$ s the mechanical input power begins to decrease.
3. At $t = 1.9$ s the mechanical input power is 50% of the initial value.
4. At $t = 2$ s the transmission line reactance begins to increase.
5. At $t = 2.2$ s the transmission line reactance is 120% of the nominal value.
6. At $t = 2.4$ s the gain of the excitation amplifier begins to increase.
7. At $t = 2.6$ s the gain of the excitation amplifier is 120% of the nominal value.

Figs. 2.a)-2.c) show that the power angle δ, the relative angular speed ω of the generator and the electrical power P_e go smoothly to a stable equilibrium point for the perturbed system, according to (21).
Fig. 3.a) shows how the output voltage drops during the mechanical and the electrical perturbations, while Fig. 3.b) shows that the control signal is smooth and kept inside the prescribed physical bounds.
Transient and steady state errors are small and can be made even smaller by increasing the parameter k. The choice of the control parameters is mainly constrained by the limitation of the control signal.

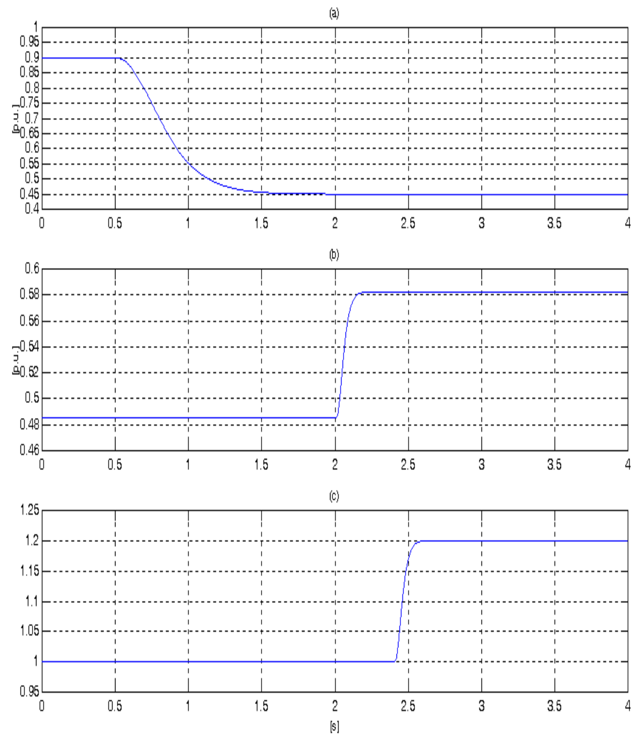

Figure 1: a) Mechanical input power P_m b) Transmission line reactance X_L c) Excitation amplifier gain K_c

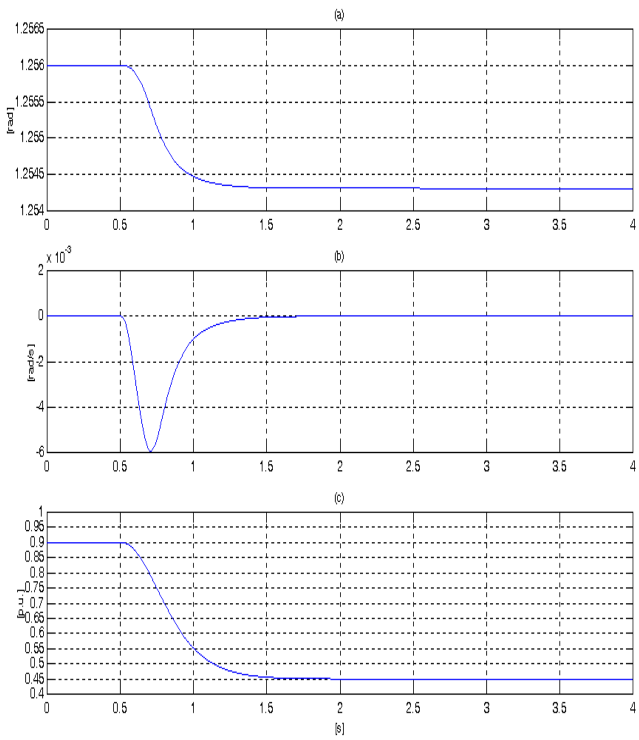

Figure 2: a) Power angle δ b) Relative angular speed of the generator ω c) Active electrical power P_e

4 Conclusions

For the well-known third order model (1) of a synchronous generator connected to an infinite bus involving significant nonlinearities and eleven physical parameters which are difficult to measure and may undergo large and sudden on line variations, the design of a state feedback control which achieves transient stabilization and is robust with

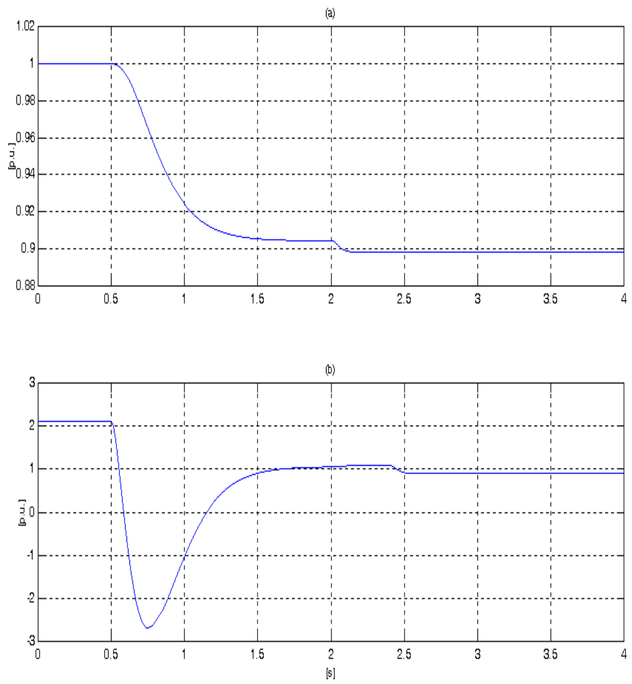

Figure 3: a) Generator terminal voltage V_t b) Control signal u_f

respect to all parameter perturbations both in L_∞ and L_2 sense is addressed and solved. The proposed nonlinear controller guarantees: i) exponential stability of the operating condition with an explicitly computed stability region, when all parameters are equal to their nominal values; ii) an explicitly quantified robustness with respect to parameter variations from nominal values.

The control performance was tested by simulating severe perturbations on three physical parameters: excellent transient and steady state behaviours are observed.

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