EXTENSION OF THE ALGORITHMS WITH SATURATION FUNCTIONS FOR A NONLINEAR H_∞/PID CONTROLLER

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Abstract

In this paper an extension of the algorithms with saturation functions to the nonlinear H_∞ control for robot manipulators introduced in [4] has been carried out. Based on the classical algorithm, this extension copes with a nonlinear equation of the closed-loop error. The resulting Lyapunov's equation in the linear case is substituted for a Hamilton-Jacobi-Bellman-Isaacs equation. A modified expression for the control signal increment is supplied, and the local closed-loop stability of this approach is discussed. Finally, simulation results for an industrial robot –the RM-10– have been presented. Results obtained with this method have been compared with the ones attained with the original controllers to show the improvements supplied by this method.

Keywords: Robot manipulators, nonlinear H_∞ control, robust control, Lyapunov-based algorithms.

1 Introduction

During last years many publications have appeared related to the nonlinear H_∞ control theory (see, for example, [2],[8]), whose solution to the continuous nonlinear system was given by *van der Schaft* in his famous article [7]. In this paper, applications related with robotics are of special interest, where this theory has been used successfully in ([1],[3], [5]), among others.

In [1], a vector of disturbance signals acting upon the input channels is used to represent the combined effect of modelling errors and external disturbances. Thus, this analytical solution to the nonlinear H_∞ control problem provides for the control system the ability to reject these disturbances (maintaining small tracking error) without excessive control effort. However, this issue (successfully applied in [5]) assumes null-average disturbances and a perfect robot model. In this way, uncertainties were considered like a vector of disturbance signals acting on input channels (torques).

The first restriction was solved in [4], where an additional integral term was included to cope with persistent disturbances, such as constant weights at the end-effector. This controller was interpreted like a computed torque control with and external PID, whose gain matrices vary with the position and velocity of the robot joints. A particular case of a cost variable weighting matrix was also presented in which the resulting external nonlinear PID does not depend on the attenuation level γ of the H_∞ formulation. However, a perfect model was still supposed, with the same interpretation of the uncertainty.

This paper deals with this last restriction, assuming bounded uncertainties in the robot model. The proposed solution is based on the *classical algorithms with saturation functions* (see, for example, [6]), where a linear equation of the closed-loop error is handled. This classical method can not be applied to the controller proposed in [4] since the gain matrices of its external PID vary along the time. Thus, in this paper the classical algorithm has been extended to be able to cope with the resulting nonlinear equations of the error.

The remainder of the paper is organized as follows: An approach upon the concepts of L_2 gain and H_∞ optimization in the context of nonlinear systems are introduced in Section 2. In Section 3 the main results presented in [4] are summarized. Next, a brief exposition of a classical algorithm with saturation function is carried out it Section 4. In Section 5, the classical algorithm is extended in order to be able to cope with the error nonlinear equation proposed in the previous controller. Simulation results for the RM-10 robot manipulator are shown in Section 6 as example to show the performance of these controllers. Finally, the major conclusions to be drawn are given in Section 7.

2 Nonlinear H_∞ control approach

The dynamical equation of a nth order smooth nonlinear system which is affected by an unknown disturbance can be expressed as follows:

$$\dot{x} = f(x,t) + g(x,t)\,u + k(x,t)\,d \tag{1}$$

where $u \in \Re^p$ is the vector of control inputs, $d \in \Re^q$ is the vector of external disturbances and $x \in \Re^n$ is the vector of states. Performance can be defined using the cost variable $z \in \Re^{(m+p)}$ given by the expression:

$$z = W \begin{bmatrix} h(x) \\ u \end{bmatrix}$$

where $h(x) \in \Re^m$ is the error vector to be controlled and $W \in \Re^{(m+p)\times(m+p)}$ is a weighting matrix. If we assume that the states x are available for measurement then the optimal H_∞ problem can be posed as follows [7]:

Find the smallest value $\gamma^* \geq 0$ such that for any $\gamma \geq \gamma^*$ there exists a state feedback $u = u(x,t)$ such that the L_2 gain from d to z is less than or equal to γ, that is,

$$\int_0^T \|z\|_2^2 \, dt \leq \gamma^2 \int_0^T \|d\|_2^2 \, dt \tag{2}$$

The integral expression on the left-hand side of expression (2) can be written as:

$$\|z\|_2^2 = z^T z = \left[\begin{array}{cc} h^T(x) & u^T \end{array} \right] W^T W \left[\begin{array}{c} h(x) \\ u \end{array} \right]$$

and the symmetric positive definite matrix $W^T W$ can be partitioned as follows:

$$W^T W = \left[\begin{array}{cc} Q & \bar{C} \\ \bar{C}^T & R \end{array} \right] > 0$$

where:

$$Q = \left[\begin{array}{ccc} Q_1 & Q_{12} & Q_{13} \\ Q_{12} & Q_2 & Q_{23} \\ Q_{13} & Q_{23} & Q_3 \end{array} \right] \qquad \bar{C} = \left[\begin{array}{c} \bar{C}_1 \\ \bar{C}_2 \\ \bar{C}_3 \end{array} \right]$$

The matrices Q and R are symmetric positive definite and the fact that $W^T W > 0$ guarantees that $Q - \bar{C} R^{-1} \bar{C}^T > 0$.

An optimal control signal u^* may be computed for system (1) if there exists a smooth solution $V(x,t)$, with $V(x_0,t) \equiv 0$ for $t \geq 0$, to the following Hamilton-Jacobi equation:

$$\begin{aligned} &\frac{\partial V}{\partial t} + \frac{\partial^T V}{\partial x} f(x,t) + \frac{1}{2} \frac{\partial^T V}{\partial x} \left[\frac{1}{\gamma^2} k(x,t) k^T(x,t) \right. \\ &\left. - g(x,t) R^{-1} g^T(x,t) \right] \frac{\partial V}{\partial x} - \frac{\partial^T V}{\partial x} G(x,t) R^{-1} C^T h(x) \\ &+ \frac{1}{2} h^T(x) \left(Q - \bar{C} R^{-1} \bar{C}^T \right) h(x) = 0 \end{aligned} \tag{3}$$

for each $\gamma > \sqrt{\sigma_{\max}(R)} \geq 0$. In such case, the optimal state feedback control law –see [1]– is derived as:

$$u^* = -R^{-1} \left(\bar{C}^T h(x) + g^T(x,t) \frac{\partial V}{\partial x} \right) \tag{4}$$

3 Nonlinear H_∞ optimization in manipulator motion control

The following Euler-Lagrange equations of motion are used to describe the behavior of a n degree-of-freedom (DOF) robot manipulator:

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau + d_\tau \tag{5}$$

where q is the vector of joint variables (joint positions) and $\dot{q}$ is its temporal derivative (joint speeds). It is supposed that these two vectors are available for measurements. The vector τ (torques applied on the axis of the joints) is the signal input of the system and d_τ represents the total effect of system modelling errors and the external disturbances. The inertia matrix $M(q)$ is symmetric and positive definite, $V(q,\dot{q})$ is the vector of centripetal and Coriolis terms and $G(q)$ is a vector which consists of the gravitational terms.

Denoting by q_r, $\dot{q}_r$ and $\ddot{q}_r$ the desired position, speed and acceleration of the joints, respectively, the tracking error vector x and its derivative $\dot{x}$ are defined as follows:

$$x = \left[\begin{array}{c} \dot{e} \\ e \\ \int e dt \end{array} \right] \quad \text{and} \quad \dot{x} = \left[\begin{array}{c} \ddot{e} \\ \dot{e} \\ e \end{array} \right]$$

where:

$$\ddot{e} = \ddot{q} - \ddot{q}_r, \quad \dot{e} = \dot{q} - \dot{q}_r,$$

$$e = q - q_r, \quad \int e dt = \int_o^t (q - q_r) \, dt.$$

For system (5) the following optimal control law is proposed in [4]:

$$\begin{aligned} \tau^* = M(q)\ddot{q}_r + V(q,\dot{q}) + G(q) - \\ - \left(K_D \dot{e} + K_P e + K_I \int e dt \right) \end{aligned} \tag{6}$$

In the particular case (see [4]) where the weighting matrix $W^T W$ satisfies $Q_1 = w_1^2 I$, $Q_2 = w_2^2 I$, $Q_3 = w_3^2 I$, $R = w_u^2 I$, $Q_{12} = Q_{13} = Q_{23} = O$, and $\bar{C}_1 = \bar{C}_2 = \bar{C}_3 = O$, the gain matrices take the form:

$$\begin{aligned} K_D &= \frac{\sqrt{w_2^2 + 2w_1 w_3}}{w_1} M + \frac{1}{2}\dot{M} + N + \frac{1}{w_u^2} I \\ K_P &= \frac{w_3}{w_1} M + \frac{\sqrt{w_2^2 + 2w_1 w_3}}{w_1} \left(\frac{1}{2}\dot{M} + N + \frac{1}{w_u^2} I \right) \\ K_I &= \frac{w_3}{w_1} \left(\frac{1}{2}\dot{M} + N + \frac{1}{w_u^2} I \right) \end{aligned}$$

being $N(q,\dot{q}) = V(q,\dot{q}) + G(q)$. These expressions have an important property: *they do not depend on the parameter* γ. Thereby we have algebraic expressions to compute the general optimal solution for this particular case.

4 The classical algorithm with saturation functions

In the following section a brief summary of an algorithm with saturation functions is exposed. A more detailed exposition can be found in [6].

Let $M(q)$, $V(q,\dot{q})$ and $G(q)$ be the dynamic matrices of the Euler-Lagrange equations (5) and $\hat{M}(q)$, $\hat{V}(q,\dot{q})$ and $\hat{G}(q)$ their respective estimations. The following hypotheses are assumed:

1. $\sup_{t \geq 0} \|\ddot{q}_r\| < Q_1 < \infty$.
2. $\|E(q)\| \equiv \|M(q)^{-1} \hat{M}(q) - I\| \leq \alpha \leq 1$ for some value of α.
3. $\|\Delta N(q,\dot{q})\| \equiv \|\Delta V(q,\dot{q}) + \Delta G(q)\| \leq \phi(x,t)$ for some time-bounded function $\phi(x,t)$, where in this case $x(t) = [\dot{e}(t) \; e(t)]^T$ and

   $$\Delta V(q,\dot{q}) = V(q,\dot{q}) - \hat{V}(q,\dot{q}) \qquad \Delta G(q) = G(q) - \hat{G}(q)$$

Once the values of the bounds have been calculated, this method proposes a computed torque structure where the robot dynamic is linearized with the estimated matrices . If there were no uncertainty and the external controller were a linear PD control law (with *constant* gain matrices K_P and K_D respectively), the closed-loop dynamic of the error would satisfy the following *linear* differential equation:

$$\dot{x}(t) = \bar{A}x(t) = (A - BK)x(t) \tag{7}$$

being the *constant* matrices A, B and K as follows:

$$A = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix} \qquad B = \begin{bmatrix} I \\ 0 \end{bmatrix} \qquad K = \begin{bmatrix} K_D & K_P \end{bmatrix}$$

Since estimations can not be perfect, the following algorithm is proposed to achieve an appropriate control signal $v(t)$:

1. Design a control law $v(t)$ as follows:

   $$v(t) = \ddot{q}_r(t) - Kx(t) + \Delta v(t)$$

   where $K = [K_D\ K_P]$ are the gain matrices of the external PD controller. The next dynamic equation of the closed-loop error is obtained with this linearization:

   $$\dot{x} = \bar{A}x + B(\Delta v + \eta)$$

   where K should be designed such that $\bar{A} = A - BK$ is Hurwitz, and η has the following expression:

   $$\eta = E\Delta v + E(\ddot{q}_r - Kx) + M^{-1}\Delta N$$

2. Compute the function $\rho(x, t)$ satisfying:

   $$\|\eta\| < \rho(x, t) \qquad\qquad \|\Delta v\| < \rho(x, t) \tag{8}$$

   as follows:

   $$\rho(x, t) = \frac{1}{1 - \alpha}(\alpha Q_1 + \|K\|\|x\| + \overline{M}\phi(x, t)) \tag{9}$$

   where $\|M(q)^{-1}\| \leq \overline{M}\ \forall q \in \Re^n$

3. Since $\bar{A}$ is Hurwitz, select a symmetric positive definite matrix X and find the unique symmetric positive definite solution P of the following Lyapunov's equation:

   $$\bar{A}^T P + P\bar{A} + X = 0 \tag{10}$$

4. Finally, compute the term $\Delta v(t)$ by means of the following expression:

   $$\Delta v(t) = \begin{cases} -\rho(x, t)\,\dfrac{B^T Px}{\|B^T Px\|} & if \quad \|B^T Px\| \geq \epsilon \\ \\ \dfrac{-\rho(x, t)}{\epsilon}\,B^T Px & if \quad \|B^T Px\| < \epsilon \end{cases} \tag{11}$$

5 Extension of the algorithm with saturation function

The main idea is to include an algorithm similar to the above exposed into the control law stated in Section 3. However, the above classical method can not be used since the external controller consists of nonlinear gain matrices, which implies a nonlinear closed-loop error equation.

This handicap can be solved taking into account the following points:

- The dynamic of the closed-loop error attained with the controller of Section 3 is stable for the nominal case (null uncertainty).

- The Lyapunov's equation 10 in the linear case is obtained from the assumption of a quadratic function $\Psi(x) = x^T Px$. The constant symmetric positive definite matrix P is the one used for the calculus of $\Delta v(t)$ in Equation 11 through the term $B^T Px$.

Therefore, an analogy between linear and nonlinear optimal control (see [2], for example) can be carried out, comparing the control laws obtained for linear case ($-B^T P^T x$) and for the nonlinear one ($-g^T \dfrac{\partial \Psi(x,t)}{\partial x}$).

Bearing in mind this analogy, the classical algorithm may be modified as follows:

Let $M(q)$, $V(q, \dot{q})$ and $G(q)$ be the dynamic matrices of the Euler-Lagrange equations (5) and $\hat{M}(q)$, $\hat{V}(q, \dot{q})$ and $\hat{G}(q)$ their respective estimations. Assuming the same hypothesis than the ones of the classical method, a control signal $v(t)$ can be attained by means of the following algorithm:

1. *Design an external control law v as follows:*

   $$v(t) = \ddot{q}_r(t) - \hat{M}^{-1}Kx(t) + \Delta v(t) \tag{12}$$

   where x is the error vector defined in Section 3, $K = [K_D\ K_P\ K_I]$ is the matrix of the external nonlinear PID controller of Equation 6 and $\Delta v(t)$ has the same meaning than the one of the classical method. Following the methodology exposed in Section 4 for the linear PD external controller, an expression for the dynamic of the closed-loop error can be obtained as follows:

   $$\dot{x} = \bar{f}(x, t) + B(\Delta v + \eta) = \bar{A}(q, \dot{q})x + B(\Delta v + \eta) \tag{13}$$

   where $\bar{f}(x, t) \equiv \bar{A}(q, \dot{q})x(t)$. In this case, the dynamic matrix of the error has the following expression:

   $$\bar{A}(q, \dot{q}) = A - B\hat{M}^{-1}K(q, \dot{q}) \tag{14}$$

   being A and B the same matrices of the classical algorithm. Notice the time dependency of the vector $\bar{f}(x, t)$ through the temporal variation of the joint positions and velocities.

2. *Compute a scalar function $\rho(e,t)$ by means of Equation (9), where Inequalities (8) are supposed to be satisfied. Notice that only a bound of the uncertainties is computed, which is independent of the method applied.*

3. *Find a scalar function $\Psi(x,t) \geq 0$ such that satisfies the following inequality:*

$$\frac{\partial \Psi(x,t)}{\partial t} + \frac{\partial^T \Psi(x,t)}{\partial x} \bar{f}(x,t) < 0 \quad \forall\, x \neq 0 \qquad (15)$$

where $\bar{f}(x,t) = \bar{A}(q,\dot{q})x(t)$. It must be remembered that the matrix K has been designed such that the dynamic vector of the error $\bar{f}$ is stable in the nominal case (null uncertainties).

4. *Finally, to complete the control law of Equation (12), design the term $\Delta v(t)$ by the following expression:*

$$\Delta v(t) = \begin{cases} -\rho(x,t) \dfrac{B^T \dfrac{\partial \Psi(x,t)}{\partial x}}{\left\| B^T \dfrac{\partial \Psi(x,t)}{\partial x} \right\|} & if \quad \left\| B^T \dfrac{\partial \Psi(x,t)}{\partial x} \right\| \geq \epsilon \\ \dfrac{-\rho(x,t)}{\epsilon}\, B^T \dfrac{\partial \Psi(x,t)}{\partial x} & if \quad \left\| B^T \dfrac{\partial \Psi(x,t)}{\partial x} \right\| < \epsilon \end{cases} \qquad (16)$$

Notice that $\Delta v(t)$ is again linearized for a bound value of $\| B^T \frac{\partial \Psi(x,t)}{\partial x} \|$ less than ϵ.

The proof of the closed-loop stability when (16) is applied may be carried out by means of the Lyapunov's Second Method. Thus, let $\Psi(x,t) \geq 0$ be a scalar function satisfying (15). The total temporal derivative of $\Psi(x,t)$ follows:

$$\frac{d\Psi(x,t)}{dt} = \frac{\partial \Psi(x,t)}{\partial t} + \frac{\partial^T \Psi(x,t)}{\partial x}\, \dot{x}(t)$$

Taking into account Equation (13), it follows that:

$$\begin{aligned} \frac{d\Psi(x,t)}{dt} &= \frac{\partial \Psi(x,t)}{\partial t} + \frac{\partial^T \Psi(x,t)}{\partial x} \left(\bar{f}(x,t) + B\ (\Delta v + \eta)\right) \\ &< \frac{\partial^T \Psi(x,t)}{\partial x}\, B\ (\Delta v + \eta) \end{aligned}$$

where it has been used that $\Psi(x,t)$ satisfies Equation (15). The temporal derivative of $\Psi(x,t)$ is negative if the following inequality is satisfied:

$$\frac{\partial^T \Psi(x,t)}{\partial x}\, B\ (\Delta v + \eta)\ \leq\ 0$$

Therefore, the term Δv must be chosen such that:

$$\frac{\partial^T \Psi(x,t)}{\partial x} B \Delta v \leq -\frac{\partial^T \Psi(x,t)}{\partial x} B \eta$$

Assume the worst case, that is, vectors $B^T \frac{\partial \Psi(x,t)}{\partial x}$ and η with the same direction. Then, Equation (8) enables us to conclude that Δv must be design with opposite direction respect to $B^T \frac{\partial \Psi(x,t)}{\partial x}$ (in order to counteract the effect of η) and with module equal to $\rho(x,t)$, that is:

$$\Delta v = -\rho(x,t) \frac{B^T \dfrac{\partial \Psi(x,t)}{\partial x}}{\left\| B^T \dfrac{\partial \Psi(x,t)}{\partial x} \right\|}$$

$\diamond$

Due to the complexity of the resulting HJBI equation, no analytical expression is available yet for the scalar function $\Psi(x,t)$ in Equation 15. In this application, we have used the expression $x^T P(t) x$ as an approximation of $\Psi(x,t)$. Matrix $P(t)$ is obtained as the solution of the following Lyapunov's equation:

$$\bar{A}_o^T P + P \bar{A}_o + X = 0$$

where X is again a symmetric positive definite matrix and A_o is the Jacobian of the vector $\bar{f}(x,t)$ evaluated at (t_o, x_o), that is:

$$\bar{A}_o = \left. \frac{\partial \bar{f}(x,t)}{\partial x} \right|_{t=t_o, x=x_o}$$

It must be noticed that $\bar{A}_o$ is Hurwitz (see Eq. 14) and therefore $P(t)$ is well defined. The Aizerman's conjecture (see [9]) is satisfied when the temporal derivative is assumed to be small enough with respect to the spatial one. Hence, under this assumption, the stability of this design can be guaranteed.

6 Simulation results for the RM-10 industrial robot

The RM-10 robot manipulator, shown in Figure 1, is a six-degree-of-freedom revolute joint manipulator arm. All the six

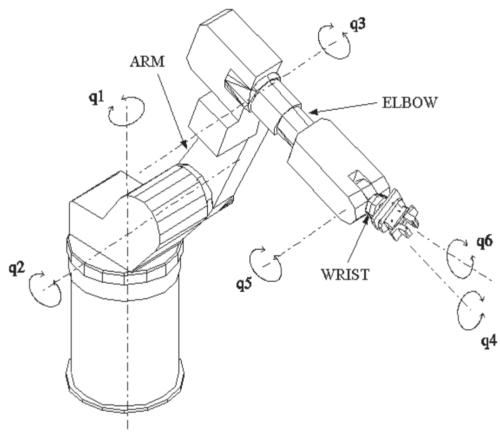

Figure 1: The RM-10 Robot Manipulator

joints are driven by DC-brushless low-inertia electric motors which provide a uniform torque for all joint positions, and enables high control torque peaks. Torque is delivered to the joint axis through gear reductions, thus RM-10 is an indirect-drive manipulator.

Before accomplishing the design of a controller it is necessary to obtain a dynamic model of the robot manipulator. According to the Euler-Lagrange formulation [6], the dynamic model of a general n-link rigid-body robot is a second order nonlinear equation, as shown in Equation 5. In this case, the motion equation is complex and contains a number of hard-to-handle nonlinear terms. In order to simplify the controller design, some terms (friction, nondiagonal terms of the inertia matrix, ...) in equation (5) have been neglected, and included as model uncertainty. This yields a very simplified model that only takes into account diagonal terms.

A number of additional parameters are required to characterize the dynamic model of the robot manipulator, such as link masses and inertias. These parameters have been estimated by geometric measurements and dynamical experiments of the robot arm. In Table 1, the estimated masses of the different links of the robot are shown. These values may help to make an idea of the characteristics of the robot. A diagonal $W^T W$

link	mass (Kg)
1	38.65
2	51.80
3	84.10
4	33.89
5	7.36
6	5.00

Table 1: Estimated masses of the links

weighting matrix has been considered to design the controller exposed in Section 3. Table 2 shows the values for the diagonal weighting submatrices used for the RM-10 control synthesis.Only the three first joints (the most important in mass)

Signal to minimize	Weighting matrix
Velocity error $\dot{e}$	$Q_1 = \left(\frac{1}{3}\right)^2 I$
Position error e	$Q_2 = I$
Integral error $\int edt$	$Q_3 = 3^2 I$
Additional control effort u	$R = 0.005^2 I$

Table 2: Weights for the controller

have been controlled in the tests presented in this paper. Simulations have been performed using a complete accurate model of the robot. The position references (which are computed by a trajectory generator) are fifth degree polynomials between the initial position $\left[\begin{array}{ccc} q_1 & q_2 & q_3 \end{array}\right] = \left[\begin{array}{ccc} 0 & -\frac{\pi}{2} & 0 \end{array}\right]$ rad to final position equal to $\left[\begin{array}{ccc} q_1 & q_2 & q_3 \end{array}\right] = \left[\begin{array}{ccc} 1.5 & \frac{\pi}{2} & \frac{\pi}{6} \end{array}\right]$ rad with initial and final speeds and accelerations equal to zero. The transition time is 3 seconds. In addition, in order to check the performance supplied by the controllers, an external persistent torque has been applied onto joints 1, 2 and 3 at times $t = 0.5$ s, $t = 1$ s and $t = 1.5$ s respectively.

In order to show the improvements achieved with the proposed method, results obtained with three different controllers will be exposed: the nonlinear PD controller (*NL PD*) proposed in [1], the nonlinear PID controller (*NL PID*) proposed in [4] and the nonlinear PID controlled with the modified algorithm (*NLS PID*) proposed in this paper.

The position and velocity errors corresponding to the *NL PD* controller are shown in Figure 2. In this case, the magnitude of the position and velocity errors (maximum peak) are about 0.2 degrees and 0.08 rad/s respectively. Notice that there is a non-null steady state error of the position since this controller has no integral effect. The same variables in case of the *NL*

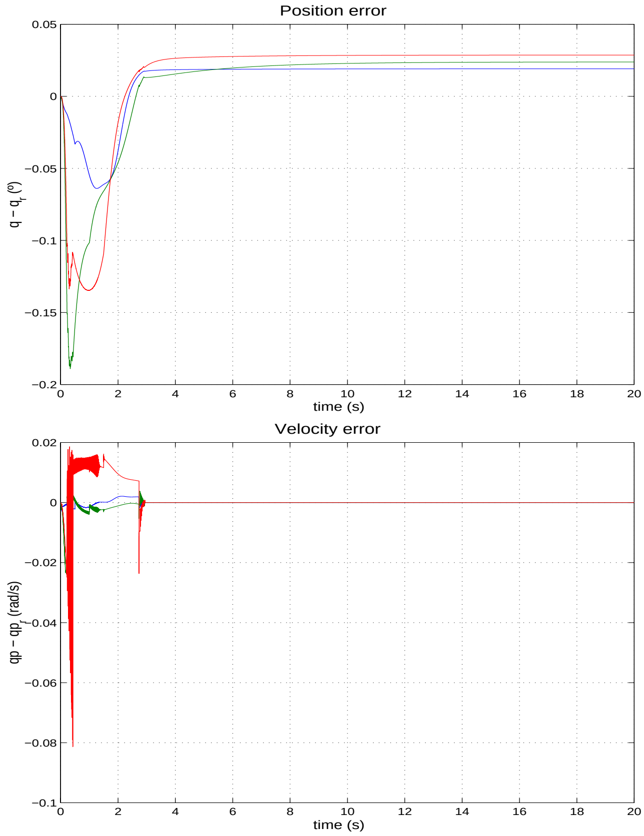

Figure 2: Results with the *NL PD* controller.

PID controller are shown in Figure 3. It can be seen how both maximum errors of position and velocity (about 0.13 degrees and 0.06 rad/s respectively) are lightly attenuated respect to the ones of the previous controller. However, it is important to notice that a null steady state position error is achieved with this controller.

Finally, Figure 4 shows the results obtained with the *NLS PID* controller proposed in this paper. The parameters used for the calculus of the saturation function have been calculated computationally. A value equal to 0.1 has been selected for ϵ in this application. It can be observed that, besides achieving a null steady state position error, the magnitude of both errors have been drastically decreased. The maximum values are about 0.025 degrees for the position error and about 0.003 rad/s in case of the velocity error.

7 Summary

An extension of the algorithms with saturation functions to the nonlinear H_∞ control for robot manipulators introduced in [4] has been carried out. Based on the classical method, where

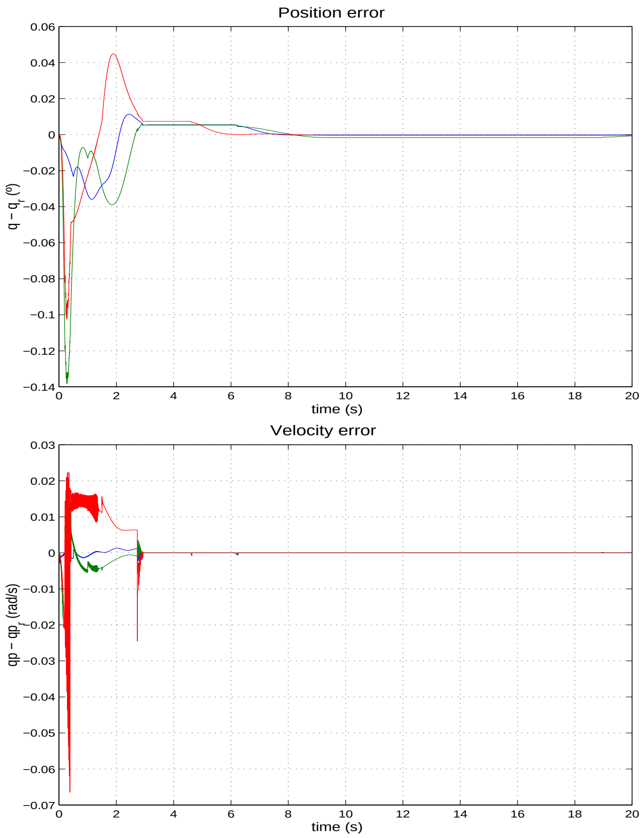

Figure 3: Results with the *NL PID* controller.

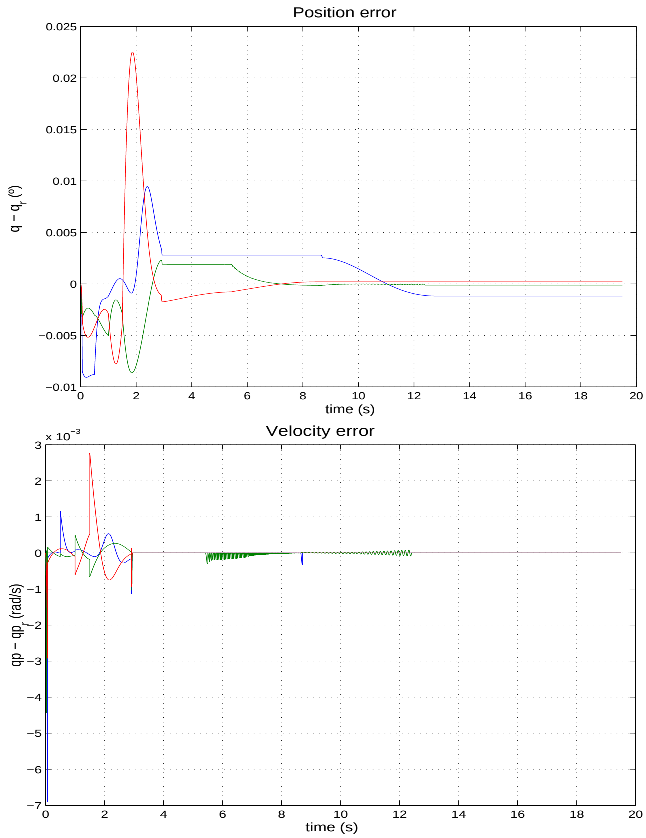

Figure 4: Results with the *NLS PD* controller.

a linear equation is used, this extension copes with a nonlinear equation of the error. The resulting Lyapunov's equation has been substituted for a Hamilton-Jacobi-Bellman-Isaacs inequality, which was solved by linearization at each operating point. A modified expression for the control signal increment has been supplied, showing that closed-loop stability has been achieved. Simulation results for an industrial robot have been presented. Several tests were carried out taking into account the differences between the model used for the controllers synthesis and the one implemented on the simulator. Finally, improvements obtained with the proposed algorithm have0 been shown by comparison with the results attained with the original controllers.

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