OPTIMAL TUNING OF PI CONTROLLERS FOR FIRST ORDER PLUS DEAD TIME/LONG DEAD TIME MODELS USING DIMENSIONAL ANALYSIS

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Abstract

For first order plus dead time models, an optimal method for tuning PI controllers is presented using dimensional analysis and numerical optimisation techniques. Considering a step change in setpoint, optimal equations for determining PI parameters are obtained through minimising the integral of absolute error (IAE). The optimisation process is constrained to guarantee a minimum Gain margin (G.M.) of 2 and a minimum Phase Margin (P.M.) of 60°. The proposed formulas can also be used for first order systems with long dead time. Simulation results show that the proposed method has a considerable superiority over conventional techniques. In addition, the closed loop system shows a robust performance in the face of model parameters uncertainties.

1 Introduction

In the earlier paper [16], the use of dimensional analysis in tuning of PID controllers for first order plus dead time (FOPDT) models was proposed. As an extension of this study, the current paper considers tuning of PI controllers for first order systems with dead time/long dead time. In addition, because gain and phase margins are often used as a measure of robustness [10], the optimal formulas are determined so that the predefined amounts for G.M. and P.M. are guaranteed.

Despite the continual advances in control theory, the PI controller is still the most commonly used controller in the process control industry [2]. This is mainly due to its noticeable effectiveness and its simple structure which is conceptually easy to understand. According to the reports, more han 90% of the industrial controllers are PID, mostly PI, controllers [5,11,12]. In [3] a typical paper mill was reported with more than 2000 control loops while 97% of these loops used PI controllers. As a result, any improvement in the PI controller tuning methods is priceless because of its broad range of applications. A number of analytical and numerical methods have been proposed for tuning this controller since the 1940s, many of them reported in [1]. These methods are usually different in complexity, flexibility and in the amount of process knowledge used. Nevertheless, there is no generally accepted design method for this controller [2]. Therefore, the design of PI controllers still remains a challenge before researchers and engineers.

Traditionally, PI controllers have been tuned empirically, e.g., by the first method of Ziegler and Nichols described in [18]. This method, called the "*continuous cycling method*" has been widely known as a fairly accurate heuristic method to determine good settings of PI and PID controllers for a wide range of common industrial processes [9]. It also has the advantage of requiring very little information about the process, however, it requires knowledge about the ultimate data which are obtained by destabilising the system under proportional feedback. Moreover, the method inherently leads to an oscillatory response in the face of a change in the setpoint [7, 14].

A large number of industrial plants can approximately be modelled by a first order plus dead time transfer function as follows:

$$G(s) = \frac{Ke^{-t_d s}}{Ts+1}. \tag{1}$$

In order to design PI controllers for this important category of industrial plants, various methods have been suggested during the past sixty years. The second method of Ziegler and Nichols known as the "*process reaction curve*" *method* [19] and that of Cohen and Coon [4] are the most prominent methods mentioned in most control textbooks. Similar to the Ziegler and Nichols methods, Cohen and Coon technique sometimes brings about oscillatory responses, because it was designed to provide closed loop responses with a damping ratio of 25% [16]. However, Ziegler and Nichols methods are still widely used, either in their original form or with some modifications [1].

2 Proposed method

An efficient design method should cope with a wide range of systems. It should satisfy the design specifications and be robust in the face of model uncertainties. The aim of this paper is to propose a set of formulas for tuning PI controllers for FOPDT models. Therefore, as shown in equation (2), the PI parameters should be defined based on the model parameters:

$$K_c = f_1(K, t_d, T).$$
$$T_i = f_2(K, t_d, T). \quad (2)$$

The problem is that it is very difficult to determine these functions, because each parameter of the controller is a function of three parameters of the model. Therefore, we propose to use *dimensional analysis* to reduce the number of parameters involved. Dimensional analysis is a mathematical tool often applied in physics and engineering to simplify a problem by reducing the number of variables to the smallest number of essential ones [20]. In other words, dimensional analysis is a process for eliminating extraneous information from a relation between quantities [17].

Definition 1:
A dimensionless number is a pure number without any physical unit. Such a number is typically defined as a product or ratio of quantities that have units, in such a way that all units can be cancelled.

Buckingham's pi-theorem:
Any equation such as

$$f(x_1, x_2, ..., x_n) = 0. \quad (3)$$

with nonzero $x_1, x_2, ..., x_n$, is equivalent to an equation of the form

$$g(p_1, p_2, ..., p_k) = 0. \quad (4)$$

where $p_1, p_2, ..., p_k$ are independent dimensionless numbers. Here $k = n - m$ where m is the number of fundamental units used. In other words, m is the minimum number of $x_1, x_2, ..., x_n$, which include all the units in equation (3).
In equation (1), the unit of t_d and T is time. The unit of K is equal to $\frac{\text{unit of output}}{\text{unit of input}}$ which is different from a plant to another one. K and t_d (or T) are fundamental units and include all the units in equation (1). As a result, there is only one dimensionless number in the model, namely $\frac{t_d}{T}$. Other dimensionless numbers for the model and the controller are:

KK_c and either of $\frac{T_i}{t_d}$ or $\frac{T_i}{T}$.

Based on Buckingham's pi-theorem, the PI parameters are obtained from the parameters of the model through determining the second and third dimensionless numbers from the first one, as shown below:

$$KK_c = g_1\left(\frac{t_d}{T}\right).$$
$$\frac{T_i}{t_d} = g_2\left(\frac{t_d}{T}\right). \quad (5)$$

Obviously, the functions in equation (5) can be determined much more easily than those in equation (2).
Considering a step change in the setpoint, this paper aims to determine g_1 and g_2 so that the IAE is minimised. In order to ensure that the resulting system has enough robustness, two constraints are used to guarantee a minimum G.M. of 2 and a minimum P.M. of 60°. In addition, as the phase margin is known to be related to the damping of the system, it also serves as a measure of performance [10].
First, the best values of controller parameters are determined for any given $\frac{t_d}{T}$ using genetic algorithms [6]. In order that the resulting formulas can also be applied to first order plus long dead time systems, $\frac{t_d}{T}$ is changed from 0.1 to 10. Then the optimal values of KK_c versus $\frac{t_d}{T}$ are drawn. Finally, g_1 is determined using curve-fitting techniques. The situation is same for g_2 if KK_c is replaced with $\frac{T_i}{t_d}$ or $\frac{T_i}{T}$.
Equations (6,7) represent the proposed formulas for tuning PI controllers:

$$KK_c = 0.4849\frac{T}{t_d} + 0.3047. \quad (6)$$

$$\frac{T_i}{T} = 0.4262\frac{t_d}{T} + 0.9581. \quad (7)$$

3 Simulation results

In this section, the performance of the proposed method is compared with that of other techniques. For simplicity, the Cohen-Coon, first Ziegler-Nichols, second Ziegler-Nichols, refined Ziegler-Nichols, Lee-Edgar, Hagglund and proposed methods are abbreviated to C.C., Z.N.1, Z.N.2, R.Z.N., L.E., Hag. and Pro., respectively.
For the first example, the performance of the proposed method is compared with that of the C.C., Z.N.1, Z.N.2, R.Z.N. [9] and L.E. [13] techniques.

Example 1:

$$G_1(s) = \frac{e^{-0.5s}}{s+1}. \quad (8)$$

The results of the comparison are shown in Table 1. The worst response is given by the C.C. method. Since the proposed method has the least percentage of overshoot and the highest phase margin and acceptable values of settling time and gain margin, it gives the best performance.

	C.C.	Z.N.1	Z.N.2	R.Z.N.	L.E.	Pro.
K_c	1.88	1.8	1.71	0.73	1.45	1.27
T_i	0.83	1.5	1.45	0.69	1.16	1.17
P.O.	55.3	21.28	18.36	13.03	15.68	7.54
T_s	6.9	5.24	3.54	4.58	3.68	3.81
G.M.	1.58	1.91	2	3.73	2.27	2.58
P.M.	29.08	51.61	53.6	53.19	54.3	59.95

Table 1: Comparison of the performance of different methods in controlling $G_1(s)$.

Figure 1 shows the closed loop step responses resulted from the R.Z.N., L.E. and proposed methods.

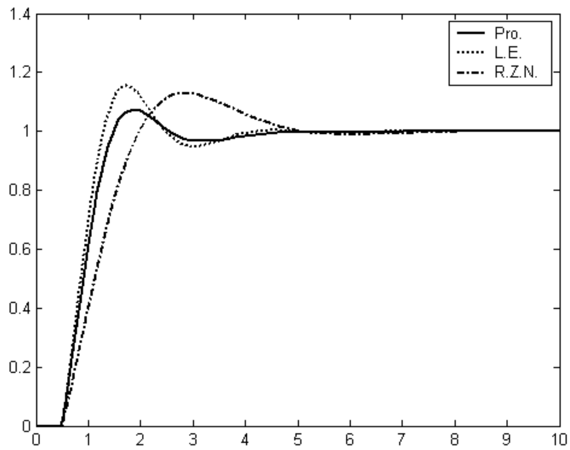

Figure 1. Closed loop step response resulted from applying the R.Z.N., L.E. and proposed methods to $G_1(s)$.

Example 2:

$$G_2(s) = \frac{(-s+1)e^{-s}}{(6s+1)(2s+1)^2}. \qquad (9)$$

In order to determine the PI parameters suggested by the Z.N.1 and R.Z.N. methods, first K_u and w are determined from the following equations:

$$K_u = \frac{\sqrt{36w^2+1}(4w^2+1)}{\sqrt{w^2+1}}. \qquad (10)$$

$$tg^{-1}(w) + w + tg^{-1}(6w) + 2tg^{-1}(2w) = p. \qquad (11)$$

In order to use the C.C., Z.N.2 and proposed techniques to obtain the PI parameters, $G_2(s)$ is approximately modelled with a FOPDT model, using half rule [15].

Half rule:
Let the original model be:

$$Q(s) = K\frac{\prod_{j=1}^{m}(-a_j s+1)}{\prod_{i=1}^{n}(b_i s+1)}e^{-qs}. \qquad (12)$$

where $b_1 \ge b_2 ... \ge b_n$ and $a_1, a_2, ..., a_m$ are positive values. Then the FOPDT is given by:

$$\hat{Q}(s) = \frac{Ke^{-t_d s}}{Ts+1}. \qquad (13)$$

where $t_d = q + \frac{b_2}{2} + \sum_{i=3}^{n} b_i + \sum_{j=1}^{m} a_j$ and $T = b_1 + \frac{b_2}{2}$.

Using half rule, the approximate model for $G_2(s)$ is given by:

$$\hat{G}_2(s) = \frac{e^{-5s}}{7s+1}. \qquad (14)$$

Table 2 shows the results of the comparison.

	C.C.	Z.N.1	Z.N.2	R.Z.N.	Pro.
K_c	1.34	1.59	1.26	0.71	0.98
T_i	6.9	14.47	15	6.6	8.84
P.O.	46.08	20.44	3.17	11.19	10.93
T_s	73.4	54.7	61.8	33.1	40.1
G.M.	1.72	1.89	2.4	3.18	2.65
P.M.	31.15	48.83	67.19	56.35	58.32

Table 2: Comparison of the performance of different methods in controlling $G_2(s)$.

The response of the C.C. method is again the worst, while the performance of the Z.N.2, R.Z.N. and proposed techniques are the best. Figure 2 shows the closed loop step responses resulted from the Z.N.2, R.Z.N and proposed methods. Although the response given by the Z.N.2 method shows the least overshoot, it has the biggest undershoot and longest settling time. The percentage of overshoot and phase margin of the proposed method are slightly better than those of the R.Z.N. From settling time and gain margin points of view, the response of the R.Z.N. is better than that of the proposed method, however, the proposed technique suggests a faster response with a smaller rise time.

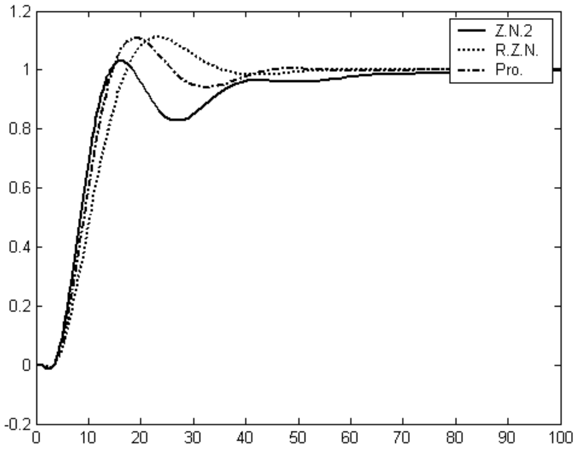

Figure 2. Closed loop step response resulted from applying the Z.N.2, R.Z.N. and proposed methods to $G_2(s)$.

Example 3:

$$G_3(s) = \frac{e^{-10S}}{s+1}. \qquad (15)$$

This example indicates a system with long dead time. Control of systems with long dead time is difficult [1]. For this example, the performance of the proposed method is compared with that of the C.C., Z.N.1, Z.N.2, R.Z.N. and Hagglund [8] techniques to show the effectiveness of the proposed method when facing with such systems. The results of the comparison are shown in Table 3.

	C.C.	Z.N.1	Z.N.2	R.Z.N.	Hag	Pro.
K_c	0.17	0.47	0.09	0.38	0.25	0.35
T_i	2.87	18.66	30	5.61	3.9	5.22
P.O.	3.38	0	0	1.99	1.34	1.23
T_s	45	182	1362	45.2	25.7	45.7
G.M.	2.83	2.18	11.5	2.13	2.58	2.22
P.M.	61.42	100.01	93.27	66.07	62.83	65.2

Table 3: Comparison of the performance of different methods to control $G_3(s)$.

The responses resulted from Z.N.1 and Z.N.2 methods are very slow, however, all of the remaining methods show good responses. The response of the Hag. method has the least settling time, while the least percentage of overshoot comes from the proposed method. From G.M. and P.M. points of view, the C.C. and R.Z.N. techniques are the best, respectively.

4 Robustness studies

In order to investigate the robustness of the mentioned methods in the face of model uncertainties, the model parameters in $G_1(s)$ and $G_3(s)$ are deviated as much as 20% of their nominal values. The worst case is related to an increase of 20% in K and t_d and a decrease of 20% in T [16]. Considering $G_1(s)$, the perturbed model is given by:

$$\hat{G}_1(s) = \frac{1.2\,e^{-0.6S}}{0.83s+1}. \qquad (16)$$

Table 4 shows the results of the comparison.

	C.C.	Z.N.1	Z.N.2	R.Z.N.	L.E.	Pro.
K_c	1.88	1.8	1.71	0.73	1.45	1.27
T_i	0.83	1.5	1.45	0.69	1.16	1.17
P.O.	Unstable closed loop system	65.79	60.55	27.07	51.83	37.68
T_s		17.44	13.28	5.05	9.25	6.03
G.M.		1.19	1.24	2.27	1.41	1.6
P.M.		18.71	22.9	44.22	31.27	41.13

Table 4: Comparison of the performance of different methods in controlling $\hat{G}_1(s)$.

Facing with model uncertainties, the closed loop system resulted from the C.C. controller parameters is unstable, while the Z.N.1 and Z.N.2 methods resulted in highly oscillatory responses. Among the remaining techniques the performance of the proposed method is acceptable, whilst the best and worst responses are related to the R.Z.N. and L.E. techniques, respectively.
Considering $G_3(s)$, the perturbed model is given by:

$$\hat{G}_3(s) = \frac{1.2\,e^{-12S}}{0.83s+1}. \qquad (17)$$

The results of comparison are shown in Table 5.

	C.C.	Z.N.1	Z.N.2	R.Z.N.	Hag	Pro.
K_c	0.17	0.47	0.09	0.38	0.25	0.35
T_i	2.87	18.66	30	5.61	3.9	5.22
P.O.	26.08	0	0	38.6	27.29	34.75
T_s	71.7	153	1139	85	77.4	84.2
G.M.	2	1.77	9.37	1.61	1.87	1.67
P.M.	47.68	97.42	93.54	49.88	48.34	49.42

Table 5: Comparison of the performance of different methods in controlling $\hat{G}_3(s)$.

While the C.C. method shows the highest degree of robustness and gives the best response, Z.N.1 and Z.N.2 methods bring about sluggish responses. The proposed, R.Z.N., and Hag. methods show relatively robust performances and give acceptable responses.

5 Conclusions

In this paper an optimal technique for tuning PI controller parameters for FOPDT models was proposed. Dimensional analysis and numerical optimisation methods were used to simplify the procedure of obtaining optimal formulas for control parameters. Simulation studies for three common examples showed that the proposed method could deal with the FOPDT models over a large range of $\frac{t_d}{T}$. In addition, for systems of higher orders which are capable of being reduced to FOPDT models, the performance of the method is quite satisfactory. Comparing the proposed method with well-known techniques, suggested that the proposed method was advantageous to most of them such as Ziegler-Nichols and Cohen-Coon methods. In addition, robustness studies proved that the PI controller given by the proposed formulas was satisfactorily robust against model uncertainties.

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