INSTRUMENTAL VARIABLES APPROACH TO IDENTIFICATION OF POLYNOMIAL WIENER SYSTEMS

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Abstract

A new approach to identification of Wiener systems by using the instrumental variables method is presented. In this approach, an inverse characteristic of the nonlinear element is represented by a polynomial of a known order. It is shown that parameters of a modified series-parallel Wiener model estimated with the least squares method are non-consistent. To obtain consistent parameter estimates, the instrumental variables method is used. The instrumental variables are generated by filtering the system input with the linear dynamic model obtained with the least squares method. In this paper, the known least squares identification method, based on the modified series-parallel model, is also extended to Wiener systems with inverse nonlinear characteristics, which polynomial representation does not contain the first order term. Two simulation examples are also included to show the effectiveness and practical feasibility of the presented approach.

1 Introduction

Various approaches to the Wiener system identification, which have been proposed in a few last decades, are based on correlation analysis [2, 3], non-parametric regression [5, 6], linear regression [9, 10, 11, 12, 13, 14], and nonlinear optimization [1, 7, 8, 15, 16]. Wigren proposed two recursive identification algorithms based on the prediction error method in which the characteristic of the nonlinear element is piecewise linearized [15] or assumed known [16]. In these algorithms, parameter estimation is formulated as a nonlinear optimization problem and solved by the Gauss-Newton method. An alternative to the above approaches is identification of the inverse Wiener system [14]. The inverse Wiener model is the Hammerstein model, which is more convenient for identification, but not all Wiener systems are invertible. Moreover, if an inverse parallel model is employed, the linear dynamic system should be minimum phase. Kalafatis *et al.* [11] considered the least squares identification of Wiener systems using a frequency sampling filter model of the linear dynamic system and a power series approximation of the inverse nonlinear element. The identification of the linear dynamic system with the steady-state gain of the linear dynamic model constrained to one, was considered by Pearson *et al.* [14]. Assuming that the inverse nonlinear steady-state characteristic is known, they proposed the weighted least squares approach to identification of the linear dynamic system. With the least squares algorithm of Janczak [9, 10], a polynomial model of the inverse nonlinear element and a non-inverted model of the linear dynamic system can be obtained. In this approach, the class of identified systems is restricted by an assumption that the inverse nonlinear element contains the first order term. The same assumption is also necessary in the adaptive least squares parameter estimation method proposed by Marciak *et al.* [13]. Their approach is based on replacing the ARX model used in [9, 10] with an orthonormal basis functions-based one.

The motivation of this work is to extend the class of identified Wiener systems by replacing this assumption with another one of a non-zero second order term. The main contribution of the paper is a new identification algorithm for Wiener systems that extends the least squares based approach [9, 10] to systems with inverse nonlinear characteristics, which polynomial representations do not contain the first order term. To solve the problem of parameter estimates non-consistency, an instrumental variables (IV) method, with instrumental variables generated via filtering the system input through a linear dynamic model obtained with the least squares method, is used.

The paper is organized as follows. The identification problem is formulated in Section 2. Then a modified definition of identification error is introduced in Section 3. Section 3 contains also the details of the least squares and instrumental variables parameter estimation. The effectiveness of the approach is illustrated with two simulation examples in Section 4. Finally, a few conclusions are presented in Section 5.

2 Problem Formulation

Consider a SISO Wiener system (Figure 1) composed of a linear dynamic system followed by a static nonlinear element.

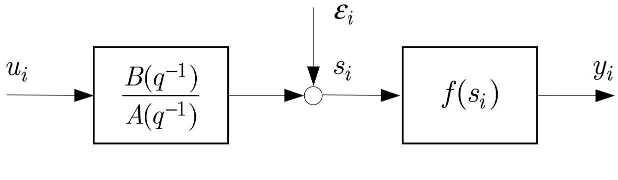

Figure 1: The Wiener system.

The output y_i to the input u_i at time i is

$$y_i = f\left[\frac{B(q^{-1})}{A(q^{-1})}u_i + \varepsilon_i\right], \tag{1}$$

where

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_{na} q^{-na}, \tag{2}$$

$$B(q^{-1}) = b_1 q^{-1} + \cdots + b_{nb} q^{-nb}, \tag{3}$$

and $f(\cdot)$ is the nonlinear element characteristic, q^{-1} is the backward shift operator, a_1,. . ., a_{na}, b_1,. . ., b_{nb}, are the unknown parameters of the linear dynamic system, and ε_i is the disturbance. The following assumptions are made about the system:

1. The linear dynamic system is asymptotically stable.
2. The nonlinear function $f(\cdot)$ is invertible and its inverse nonlinear function $f^{-1}(\cdot)$ can be expressed by the polynomial of the order r

$$f^{-1}(y_i) = \gamma_0 + \gamma_1 y_i + \gamma_2 y_i^2 + \cdots + \gamma_r y_i^r. \tag{4}$$

3. The polynomial orders r, na and nb are known.

The identification problem can be formulated as follows. Given the sequence of the Wiener system input and output measurements $\{u_i, y_i\}$, $i = 1, \ldots, N$, estimate parameters of the linear dynamic system and the inverse nonlinear element minimizing the following criterion

$$J_N = \frac{1}{2}\sum_{j=1}^{N} e_i^2, \tag{5}$$

where e_i is the one step prediction error of the system ouptut.

3 Least squares approach to identification of Wiener systems

For Wiener systems, both the parallel and series-parallel polynomial models are nonlinear functions of model parameters. Moreover, the series-parallel model contains not only a model of the nonlinear element but also its inverse [7].
Consider the parallel model of the Wiener system given by

$$\hat{y}_i = \hat{f}\left[\frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})}u_i\right], \tag{6}$$

with

$$\hat{A}(q^{-1}) = 1 + \hat{a}_1 q^{-1} + \cdots + \hat{a}_{na} q^{-na}, \tag{7}$$

$$\hat{B}(q^{-1}) = \hat{b}_1 q^{-1} + \cdots + \hat{b}_{nb} q^{-nb}, \tag{8}$$

where the estimated nonlinear function $f(\cdot)$, the estimated polynomials $A(q^{-1})$, $B(q^{-1})$ and the estimated parameters of the linear dynamic system are denoted with the hat symbol. If $\hat{f}(\cdot)$ is invertible, (6) can be written as

$$\hat{f}^{-1}(\hat{y}_i) = \frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})}u_i. \tag{9}$$

Assume that the function $\hat{f}^{-1}(\cdot)$ has the form of a polynomial of the order r

$$\hat{f}^{-1}(\hat{y}_i) = \hat{\gamma}_0 + \hat{\gamma}_1 \hat{y}_i + \hat{\gamma}_2 \hat{y}_i^2 + \cdots + \hat{\gamma}_r \hat{y}_i^r. \tag{10}$$

Assume also that $\hat{\gamma}_1 \neq 0$. Then combining (9) and (10), the output of the model can be expressed as

$$\hat{y}_i = \frac{1}{\hat{\gamma}_1}\left[\frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})}u_i - \Delta\hat{f}^{-1}(\hat{y}_i)\right], \tag{11}$$

where

$$\Delta\hat{f}^{-1}(\hat{y}_i) = \hat{\gamma}_0 + \hat{\gamma}_2 \hat{y}_i^2 + \hat{\gamma}_3 \hat{y}_i^3 + \cdots + \hat{\gamma}_r \hat{y}_i^r. \tag{12}$$

The model (11) can be written as

$$\begin{aligned}\hat{y}_i =& \big[1 - \hat{A}(q^{-1})\big]\hat{y}_i + \frac{1}{\hat{\gamma}_1}\big[\hat{B}(q^{-1})u_i \\ &- \hat{A}(q^{-1})\Delta\hat{f}^{-1}(\hat{y}_i)\big].\end{aligned} \tag{13}$$

Replacing $\hat{y}_i$ by y_i on the r.h.s. of (13), the following modified series-parallel model can be obtained [9]

$$\begin{aligned}\hat{y}_i =& \big[1 - \hat{A}(q^{-1})\big]y_i + \frac{1}{\hat{\gamma}_1}\big[\hat{B}(q^{-1})u_i \\ &- \hat{A}(q^{-1})\Delta\hat{f}^{-1}(y_i)\big].\end{aligned} \tag{14}$$

The modified series-parallel model (Figure 2) differs from both the series-parallel model, which contains the model of the nonlinear element and its inverse, and the inverse series parallel model [10]. Applying (14), the following definition of the prediction error can be introduced

$$\begin{aligned}e_i &= y_i - \hat{y}_i \\ &= \hat{A}(q^{-1})y_i - \frac{1}{\hat{\gamma}_1}\big[\hat{B}(q^{-1})u_i - \hat{A}(q^{-1})\Delta\hat{f}^{-1}(y_i)\big].\end{aligned} \tag{15}$$

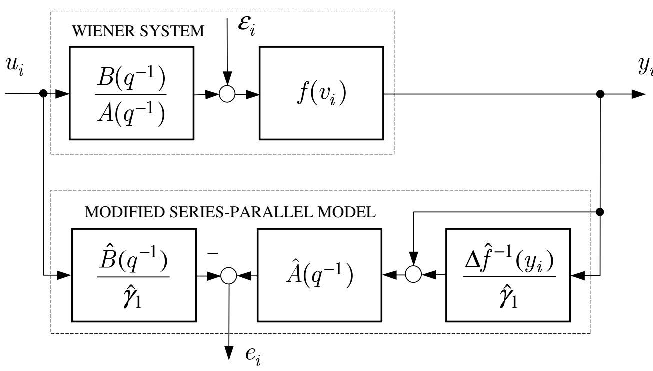

Figure 2: Identification error definition for Wiener systems with the first order term.

3.1 Wiener system with the first order term

Assuming that the identified Wiener system has an invertible inverse nonlinear characteristic with $\gamma_1 \neq 0$, we will formulate the identification problem as a linear in-parameters one. The model (14) can be written in the following linear in-parameters form

$$\hat{y}_i = \mathbf{x}_i^T \boldsymbol{\theta}, \tag{16}$$

with the parameter vector $\boldsymbol{\theta}$ and the regression vector $\mathbf{x}_i$

$$\boldsymbol{\theta} = \left[\hat{a}_1 \ldots \hat{a}_{na}\ \hat{\beta}_1 \ldots \hat{\beta}_{nb}\ \hat{\alpha}_{0,0}\ \hat{\alpha}_{2,0} \ldots \hat{\alpha}_{r,na}\right]^T, \tag{17}$$

$$\begin{aligned} \mathbf{x}_i = \big[& -y_{i-1} \ldots -y_{i-na}\ u_{i-1} \ldots u_{i-nb}\ \ 1 \\ & -y_i^2 \ldots -y_{i-na}^r\big]^T, \end{aligned} \tag{18}$$

where

$$\hat{\beta}_k = \frac{\hat{b}_k}{\hat{\gamma}_1},\ k = 1, \ldots, nb, \tag{19}$$

$$\hat{\alpha}_{j,k} = \begin{cases} \dfrac{\hat{\gamma}_j}{\hat{\gamma}_1}, & k = 0,\ j = 0,\ 2, 3, \ldots, r, \\ \hat{a}_k \dfrac{\hat{\gamma}_j}{\hat{\gamma}_1}, & k = 1, \ldots, na,\ j = 0,\ 2,\ 3, \ldots, r. \end{cases} \tag{20}$$

Minimizing (5), the parameter vector $\boldsymbol{\theta}$ can be obtained with the least squares (LS) method. Note that the number of parameters in (14) is $na + nb + r(na + 1)$ while the number of parameters of $\hat{A}(q^{-1})$, $\hat{B}(q^{-1})$, and $\hat{f}(\cdot)$ is $na + nb + r + 1$. Therefore, to obtain a unique solution, methods similar to these proposed for identification of Hammerstein systems by Eskinat *et al.* [4] can be employed.

3.2 Wiener system without the first order term

Consider a Wiener system that fulfills the following conditions:

1. The function $f(\cdot)$ is invertible on the interval $[a, b]$.
2. $\gamma_1 = 0$.
3. $\gamma_2 \neq 0$.

In this case, the following modified series-parallel model can be defined

$$\begin{aligned} \hat{y}_i^2 = {} & \big[1 - \hat{A}(q^{-1})\big] y_i^2 + \frac{1}{\hat{\gamma}_2}\big[\hat{B}(q^{-1}) u_i \\ & - \hat{A}(q^{-1}) \Delta \hat{f}^{-1}(y_i)\big], \end{aligned} \tag{21}$$

where

$$\Delta \hat{f}^{-1}(\hat{y}_i) = \hat{\gamma}_0 + \hat{\gamma}_3 \hat{y}_i^3 + \hat{\gamma}_4 \hat{y}_i^4 + \cdots + \hat{\gamma}_r \hat{y}_i^r, \tag{22}$$

and the definition of the prediction error (Figure 3) has the form

$$\begin{aligned} e_i &= y_i^2 - \hat{y}_i^2 \\ &= \hat{A}(q^{-1}) y_i^2 - \frac{1}{\hat{\gamma}_2}\big[\hat{B}(q^{-1}) u_i - \hat{A}(q^{-1}) \Delta \hat{f}^{-1}(y_i)\big]. \end{aligned} \tag{23}$$

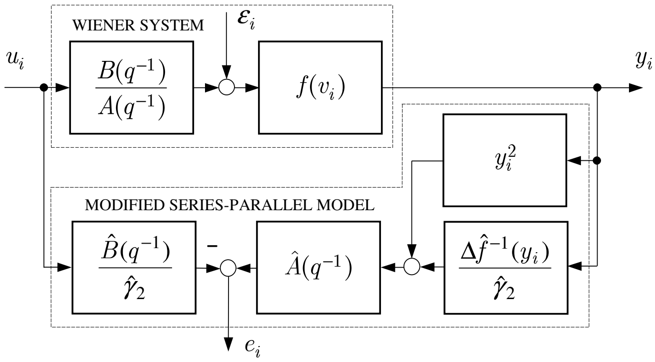

Figure 3: Identification error definition for Wiener systems without the first order term.

Then (21) can be written in the linear in-parameters form (16) with the parameter vector $\boldsymbol{\theta}$ and the regression vector $\mathbf{x}_i$ defined as

$$\boldsymbol{\theta} = \left[\hat{a}_1 \ldots \hat{a}_{na}\ \hat{\beta}_1 \ldots \hat{\beta}_{nb}\ \hat{\alpha}_{0,0}\ \hat{\alpha}_{3,0} \ldots \hat{\alpha}_{r,na}\right]^T, \tag{24}$$

$$\begin{aligned} \mathbf{x}_i = \big[& -y_{i-1}^2 \ldots -y_{i-na}^2\ u_{i-1} \ldots u_{i-nb}\ \ 1 \\ & -y_i^3 \ldots -y_{i-na}^r\big]^T, \end{aligned} \tag{25}$$

where

$$\hat{\beta}_k = \frac{\hat{b}_k}{\hat{\gamma}_2},\ k = 1, \ldots, nb, \tag{26}$$

$$\hat{\alpha}_{j,k} = \begin{cases} \dfrac{\hat{\gamma}_j}{\hat{\gamma}_2}, & k = 0,\ j = 0,\ 3,\ 4, \ldots, r, \\ \hat{a}_k \dfrac{\hat{\gamma}_j}{\hat{\gamma}_2}, & k = 1, \ldots, na, j = 0,\ 3,\ 4, \ldots, r. \end{cases} \tag{27}$$

As in the previous case, the parameter vector $\boldsymbol{\theta}$ can be obtained minimizing (5) with the LS method.

3.3 Asymptotic bias error of parameter estimates

Consider the polynomial Wiener system (1) containing the linear term, i.e., $\gamma_1 \neq 0$, and its modified series-parallel Wiener model (16). We will show now that the parameter estimates of the modified series-parallel Wiener model obtained with the LS method are non-consistent, i.e., asymptotically biased, even if the additive disturbance ε_i is

$$\varepsilon_i = \frac{\epsilon_i}{A(q^{-1})}, \tag{28}$$

where ϵ_i is the discrete white noise.

Theorem 1. Let $\boldsymbol{\theta}$ denote the vector of parameter estimates, defined by (17) and $\boldsymbol{\theta}_0$ is the corresponding true parameter vector of the Wiener system. Then the the LS estimate of $\boldsymbol{\theta}_0$ is asymptotically biased, i.e., $\boldsymbol{\theta}$ does not converge (with probability 1) to true parameter vector $\boldsymbol{\theta}_0$.

Proof: The output y_i of the Wiener system, defined by (1), (4) and (28) is

$$y_i = \big[1 - A(q^{-1})\big]y_i + \frac{1}{\gamma_1}\big[B(q^{-1})u_i - A(q^{-1})\Delta f^{-1}(y_i) + \epsilon_i\big], \tag{29}$$

where $\Delta f^{-1}(y_i) = f^{-1}(y_i) - \gamma_1 y_i$. Introducing the true parameter vector $\boldsymbol{\theta}_0$,

$$\boldsymbol{\theta}_0 = \big[a_1 \dots a_{na}\ \beta_1 \dots \beta_{nb}\ \alpha_{0,0}\ \alpha_{2,0} \dots \alpha_{r,na}\big]^T, \tag{30}$$

where

$$\beta_k = \frac{b_k}{\gamma_1},\ k = 1, \dots, nb, \tag{31}$$

$$\alpha_{j,k} = \begin{cases} \dfrac{\gamma_j}{\gamma_1}, & k = 0,\ j = 0,\ 2, 3, \dots, r, \\ a_k \dfrac{\gamma_j}{\gamma_1}, & k = 1, \dots, na,\ j = 0,\ 2,\ 3, \dots, r, \end{cases} \tag{32}$$

the system output can be expressed as

$$y_i = \mathbf{x}_i^T \boldsymbol{\theta}_0 + \frac{1}{\gamma_1}\epsilon_i. \tag{33}$$

The solution to the LS estimation problem is given by

$$\boldsymbol{\theta} = \left[\frac{1}{N}\sum_{i=1}^{N} \mathbf{x}_i \mathbf{x}_i^T\right]^{-1}\left[\frac{1}{N}\sum_{i=1}^{N} \mathbf{x}_i y_i\right]. \tag{34}$$

From (33) and (34), it follows that the difference between the estimated and the true parameter vectors $\Delta\boldsymbol{\theta} = \boldsymbol{\theta} - \boldsymbol{\theta}_0$ is

$$\Delta\boldsymbol{\theta} = \left[\frac{1}{N}\sum_{i=1}^{N} \mathbf{x}_i \mathbf{x}_i^T\right]^{-1}\left[\frac{1}{N}\sum_{i=1}^{N} \mathbf{x}_i y_i - \left(\frac{1}{N}\sum_{i=1}^{N} \mathbf{x}_i \mathbf{x}_i^T\right)\boldsymbol{\theta}_0\right]$$
$$= \frac{1}{\gamma_1}\left[\frac{1}{N}\sum_{i=1}^{N} \mathbf{x}_i \mathbf{x}_i^T\right]^{-1}\left[\frac{1}{N}\sum_{i=1}^{N} \mathbf{x}_i \epsilon_i\right].$$

Therefore, if $N \to \infty$

$$\boldsymbol{\theta} - \boldsymbol{\theta}_0 \to \frac{1}{\gamma_1}\big[\mathrm{E}\big(\mathbf{x}_i \mathbf{x}_i^T\big)\big]^{-1}\big[\mathrm{E}\big(\mathbf{x}_i \epsilon_i\big)\big] \neq \mathbf{0},$$

as $\mathrm{E}\big[y_i^2 \epsilon_i\big] \neq 0, \dots, \mathrm{E}\big[y_i^r \epsilon_i\big] \neq 0$, and thus $\mathrm{E}\big[\mathbf{x}_i \epsilon_i\big] \neq \mathbf{0}$.

3.4 Instrumental variables method

To obtain asymptotically unbiased parameter estimates, the regression vector $\mathbf{x}_i$ should be be uncorrelated with the system disturbances ϵ_i. That is not the case if we use the modified series-parallel model. Instrumental variables methods are the well-known remedy for such a situation. Applying an instrumental variables method, the parameter estimation can be performed according to the following scheme:

1. Estimate parameters with the LS method.
2. Simulate the linear dynamic model.
3. Estimate parameters using the IV method with the instrumental variables $\mathbf{z}_i$.

The choice of instrumental variables is a vital design problem in any instrumental variables approach. Clearly, the best choice would be the undisturbed system outputs, but these are not available for measurement. Instead, we can employ the outputs $\hat{s}_i$ of the linear model obtained with the LS, and define the instrumental variables as

$$\mathbf{z}_i = \big[-\hat{s}_{i-1} \dots -\hat{s}_{i-na}\ u_{i-1} \dots u_{i-nb}\ 1\ -\hat{s}_i^2 \dots -\hat{s}_{i-na}^r\big]^T. \tag{35}$$

in the case of Wiener systems with the linear term or

$$\mathbf{z}_i = \big[-\hat{s}_{i-1}^2 \dots -\hat{s}_{i-na}^2\ u_{i-1} \dots u_{i-nb}\ 1\ -\hat{s}_i^3 \dots -\hat{s}_{i-na}^r\big]^T. \tag{36}$$

in the case of Wiener systems without the linear term. The instrumental variables $\mathbf{z}_i$ are uncorrelated with the system disturbances, i.e. $E[\mathbf{z}_i \epsilon_i] = \mathbf{0}$.

4 Simulation examples

Example 1. The Wiener system with the first order term, $\gamma_1 \neq 0$. The following Wiener system composed of the linear dynamic system

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{0.125q^{-1} - 0.025q^{-2}}{1 - 1.75q^{-1} + 0.85q^{-2}}, \tag{37}$$

and the nonlinear element given by $f(s_i) = \arcsin(s_i)$, $|s_i| \leqslant 1$, was used in the simulation study. The input sequence $\{u_i\}$ consisted of 50000 pseudo-random numbers uniformly distributed in $(-0.6125, 0.6125)$ and the additive system disturbances were given by $\varepsilon_i = [1/A(q^{-1})]\epsilon_i$ with $\{\epsilon_i\}$ – a pseudo-random sequence, uniformly distributed in $(-0.025, 0.025)$. Both the LS and IV parameter estimation was performed assuming $\hat{\gamma}_1 = 1$ and $r = 7$. The identification results, given in Tables 1 and 2 and illustrated in Figures 4 and 5, show a considerable improvement of the IV parameter estimates in comparison with the LS ones.

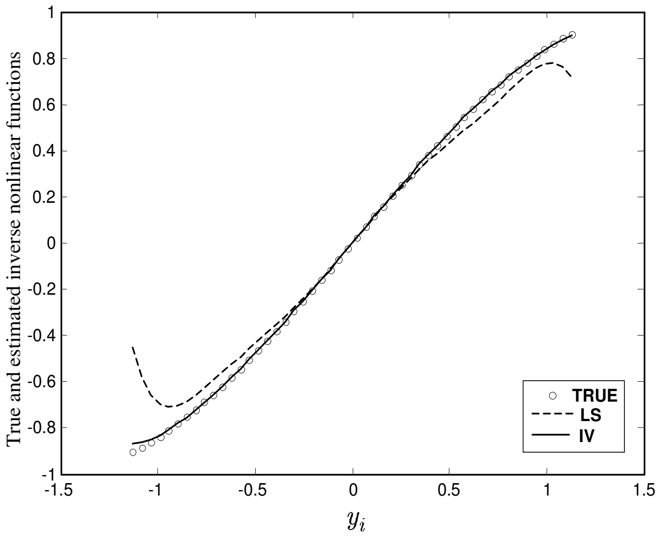

Figure 4: Wiener system with the first order term. The true $f^{-1}(y_i)$ and estimated $\hat{f}^{-1}(y_i)$ inverse nonlinear functions.

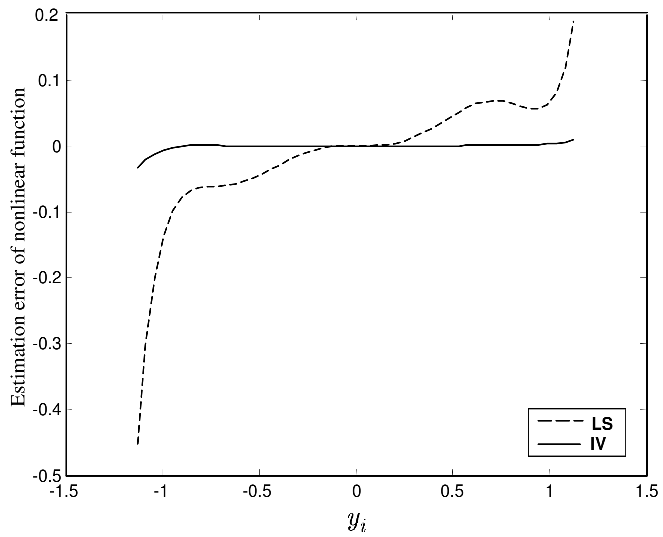

Figure 5: Wiener system with the first order term. The estimation error $f^{-1}(y_i) - \hat{f}^{-1}(y_i)$.

Parameter	True	LS	IV
$\hat{a}_1$	0.1250	0.1174	0.1249
$\hat{a}_2$	-0.0250	-0.0235	-0.0252
$\hat{b}_1$	-1.7500	-1.7143	-1.7526
$\hat{b}_2$	0.8500	0.8308	0.8517
$\hat{\gamma}_1$	1.0000	1.0000	1.0000
$\hat{\gamma}_2$	0.0000	0.0252	0.0069
$\hat{\gamma}_3$	-0.1667	-0.7992	-0.1792
$\hat{\gamma}_4$	0.0000	-0.1487	-0.0299
$\hat{\gamma}_5$	0.0083	1.2778	0.0495
$\hat{\gamma}_6$	0.0000	0.1649	0.0252
$\hat{\gamma}_7$	-0.0002	-0.7438	-0.0339

Table 1: Parameter estimates.

Performance index	LS	IV
$\frac{1}{4}\sum_{j=1}^{2}\left[(a_j-\hat{a}_j)^2+(b_j-\hat{b}_j)^2\right]$	4.27×10^{-4}	2.45×10^{-6}
$\frac{1}{6}\sum_{j=2}^{7}(\gamma_j - \hat{\gamma}_j)^2$	4.36×10^{-1}	7.60×10^{-4}
$\frac{1}{50}\sum_{i=1}^{50}\left[f^{-1}(y_i)-\hat{f}^{-1}(y_i)\right]^2$	1.01×10^{-2}	3.83×10^{-5}

Table 2: Comparison of estimation accuracy.

Example 2. The Wiener system without the first order term, $\gamma_1 = 0$ and $\gamma_2 \neq 0$. The linear dynamical system (37) along with the nonlinear function $f(s_i) = \sqrt{\sqrt{s_i} + 0.5}$, $s_i \geqslant 0$ were used in the example of a Wiener system with the second order term and without the first order term. The inverse nonlinear function $f^{-1}(y_i) = 0.25 - y_i^2 + y_i^4$ is invertible for $y_i \geqslant \sqrt{0.5}$. The input sequence $\{u_i\}$ contained 50000 pseudo-random numbers uniformly distributed in $(1.5, 5.25)$. The additive system disturbances were given by $\varepsilon_i = [1/A(q^{-1})]\epsilon_i$ with $\{\epsilon_i\}$ – a pseudo-random sequence, uniformly distributed in $(-0.025, 0.025)$. As in Example 1, the LS and IV parameter estimation was performed assuming $\hat{\gamma}_2 = 1$, $\hat{\gamma}_3 = 0$ and $r = 4$. The identification results, given in Tables 3 and 4 and illustrated in Figures 6 and 7, confirm the practical feasibility of the proposed approach.

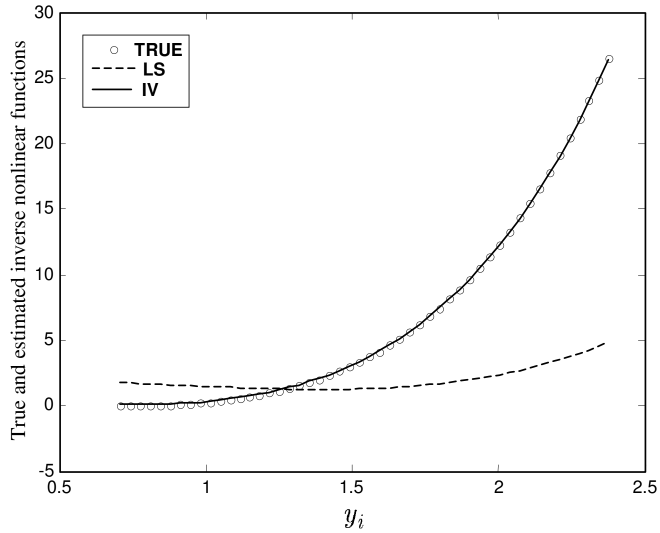

Figure 6: Wiener system without the first order term. The true $f^{-1}(y_i)$ and estimated $\hat{f}^{-1}(y_i)$ inverse nonlinear functions.

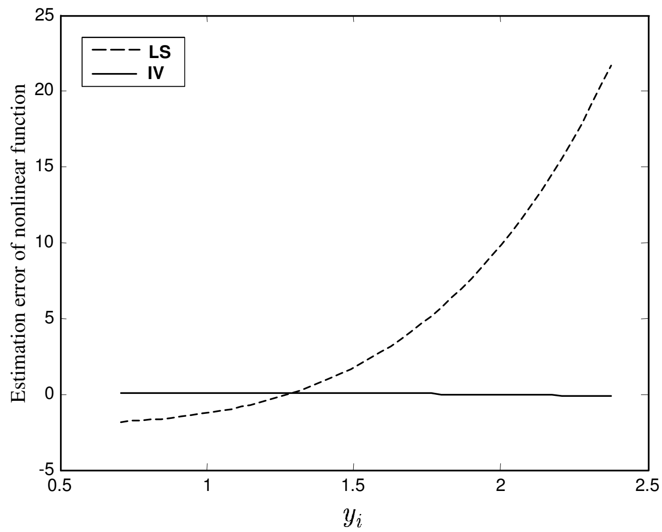

Figure 7: Wiener system without the first order term. The estimation error $f^{-1}(y_i) - \hat{f}^{-1}(y_i)$.

Parameter	True	LS	IV
$\hat{a}_1$	0.1250	-0.0068	0.1238
$\hat{a}_2$	-0.0250	0.0013	-0.0247
$\hat{b}_1$	-1.7500	-1.4988	-1.7655
$\hat{b}_2$	0.8500	0.6972	0.8725
$\hat{\gamma}_0$	0.2500	2.2297	0.3435
$\hat{\gamma}_\mathbf{2}$	1.0000	1.0000	1.0000
$\hat{\gamma}_4$	1.0000	0.2590	0.9925

Table 3: Parameter estimates.

Performance index	LS	IV
$\frac{1}{4}\sum_{j=1}^{2}\left[(a_j-\hat{a}_j)^2+(b_j-\hat{b}_j)^2\right]$	2.52×10^{-2}	1.87×10^{-4}
$\frac{1}{2}\left[(\gamma_0-\hat{\gamma}_0)^2+(\gamma_4-\hat{\gamma}_4)^2\right]$	2.23×10^{0}	4.40×10^{-3}
$\frac{1}{50}\sum_{i=1}^{50}\left[f^{-1}(y_i)-\hat{f}^{-1}(y_i)\right]^2$	6.94×10^{1}	5.28×10^{-3}

Table 4: Comparison of estimation accuracy.

5 Conclusions

This paper describes a combined least squares and instrumental variables approach to identification of polynomial Wiener systems. It is assumed that the inverse nonlinear element is described by a single-valued smooth function that can be approximated by a polynomial. Assuming that the linear dynamic system is modelled by the ARX model, a modified series parallel Wiener model is introduced. It is also shown that least squares parameter estimates of the polynomial Wiener model are non-consistent. To avoid the consistency problem, two identification procedures for systems with and without the first order term are considered. Both these procedures employ a model of the linear dynamic system obtained with the least squares method to generate instrumental variables. Although, only one technique of instrumental variables generation is discussed in the paper, the other known techniques can also be considered. Two simulation examples included in the paper illustrate practical effectiveness of the proposed identification procedures.

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