NEW NUMERICAL METHOD FOR THE POLYNOMIAL POSITIVITY INVARIANCE UNDER COEFFICIENT PERTURBATION

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Abstract

In this paper the robust positivity of polynomials under coefficient perturbation is investigated. This robust positivity of polynomials can be used for polynomial systems in order to determine the robust asymptotic stability of the system. We assume that the polynomials under investigation depend linearly on some parameters. Our aim is to determine the parameter perturbation region as a hypercube, for which the polynomial is globally positive. We use the theorem of Ehlich and Zeller to achieve this aim. This theorem enables us to give conditions in the parameter space for global positivity. These conditions are linear inequalities. By means of these inequalities we calculate inner and outer approximations to the relevant perturbation region which is a hypercube. One nontrivial example concludes the paper and shows the effectiveness of the presented method.

1 Introduction

In this paper the problem of global positivity of polynomials depending linearly on uncertain parameters [1], [2], [6], [7] will be dealt with. Generally a polynomial can be written as

$$p(x) \;=\; \sum_{i=1}^{s} p_{\alpha_i}\, x^{\alpha_i} \;,\;\; x \in \mathbb{R}^m \tag{1}$$

where $x^{\alpha_i} = \prod_{j=1}^{m} x_j^{\alpha_{ij}}$ is the i-th monomial of the polynomial $p(x)$, p_{α_i} is the coefficient of the i-th monomial and s is the number of the monomials in the polynomial. We define the degree of the i-th monomial and the degree of the polynomial $p(x)$ as

$$\mid \alpha_i \mid \;=\; \sum_{j=1}^{m} \alpha_{ij} \;, \tag{2}$$

$$deg\, p(x) \;=\; max \mid \alpha_i \mid \,, \quad i = 1,\, \ldots\,, s \tag{3}$$

respectively, where α_{ij} is either a positive entire number or zero. Such a polynomial that depends linearly on some parameters can be written with respect to the uncertainties at the parameters as

$$p(x) \;=\; p_0(x) \;+\; \sum_{i=1}^{r} (k_{in} + \delta k_i)\, p_i(x) \;,\;\; x \in \mathbb{R}^m \tag{4}$$

where k_{in} denotes the known nominal value of the i-th parameter and δk_i represents the uncertainty at the i-th parameter. The investigations in this paper are necessary, for example, when analyzing global asymptotical stability of polynomial dynamical systems [4], [10]. In this paper we will use the theorem of Ehlich and Zeller [3] and develop an algorithm in order to compute the maximum domain as a hypercube in the parameter space for which a given polynomial is globally positive. We define this region of the parameter space as follows.

$$\Omega := \{\delta k \;\mid\; -\varepsilon \;\leq\; \delta k_i \;\leq\; \varepsilon \;,\;\; i = 1, \ldots, r\} \;,\;\; \delta k \in \mathbb{R}^r \tag{5}$$

where ε is a positive number and Ω is a hypercube in the parameter space for which the polynomial $p(x)$ (4) is globally positive.

$$p(x) > 0 \;\;\forall\, x \in \mathbb{R}^m \;, \;\;\forall\, \delta k \in \Omega \tag{6}$$

We introduce first the theorem of Ehlich and Zeller and then discuss polynomial homogenization which is necessary for the application of this theorem. By means of homogenization we reduce the whole $\mathbb{R}^m$-space (m is the number of the variables in the polynomial) into a hyperrectangle in the $\mathbb{R}^{m+1}$ space. If the homogenized polynomial is positive on a subset of the boundary of the hyperrectangle, the original polynomial is globally positive. We will prove this property and use it in order to determine the parameter region Ω for which the polynomial is globally positive. We present our algorithm and show an example. Conclusions and an outlook will finish the paper.

2 Theorem of Ehlich and Zeller

In this section we will closely follow the corresponding section in [3], [8], [9]. In the following, $J = [a, b]$ denotes a nonempty compact real interval with $J \subset \mathbb{R}$. We define the set of Chebychev points in J for a given natural number $N > 0$ by

$$x(N, J) \;:=\; \{\, x_1 \,,\, x_2 \,,\, \ldots \,,\, x_i \,,\, \ldots \,,\, x_N \,\} \tag{7}$$

where

$$x_i \ := \ \frac{a+b}{2} \ + \ \frac{b-a}{2}\cos\left(\frac{(2i-1)\pi}{2N}\right) \ . \qquad (8)$$

For a continuous function h defined on a set I we define the norm

$$||h||^I \ := \ \max_{x\in I} |h(x)| \qquad (9)$$

which is the usual maximum norm. Let p_n be the set of polynomials p in one variable with $deg\ p = n$. Then the following inequality

$$\|p\|^J \ \le \ C\left(\frac{n}{N}\right) \|p\|^{x(N,J)} \qquad (10)$$

with $N > n$ and

$$C(q) \ := \ \left[\cos(q\frac{\pi}{2})\right]^{-1} \ , \ 0 < q < 1 \ , \qquad (11)$$

is valid for every $p \in p_n$ and every nonempty compact interval J. Inequality (10) is remarkable because the norm $\|p\|^{x(N,J)}$ on the right hand side of (10) depends on the values of p at the Chebychev points only. This result was given by Ehlich and Zeller in [3]. Using (10) the following inequalities

$$p^J_{min} \ \ge \ \frac{1}{2}\left\{\left(C\left(\frac{n}{N}\right)+1\right)p^{x(N,J)}_{min} \ - \right.$$
$$\left.\left(C\left(\frac{n}{N}\right)-1\right)p^{x(N,J)}_{max}\right\}, \qquad (12)$$

$$p^J_{max} \ \le \ \frac{1}{2}\left\{\left(C\left(\frac{n}{N}\right)+1\right)p^{x(N,J)}_{max} \ - \right.$$
$$\left.\left(C\left(\frac{n}{N}\right)-1\right)p^{x(N,J)}_{min}\right\} \qquad (13)$$

which are valid for every $p \in p_n$ and $N > n$ are given by Gärtel in [5]. In the inequalities $p^J_{min} := \min_{x\in J} p(x)$ and $p^J_{max} := \max_{x\in J} p(x)$ are the minimum and maximum of p in the set J respectively. Similarly $p^{x(N,J)}_{min} := \min_{x\in x(N,J)} p(x)$ and $p^{x(N,J)}_{max} := \max_{x\in x(N,J)} p(x)$ are the minimum and maximum of p in the set of Chebychev points respectively. For trigonometric polynomials and for rational functions similar inequalities are given by Gärtel [5].

The inequalities (10),(12),(13) are valid for polynomials in one variable. They are extended to polynomials of several variables using the following replacements. The interval J is replaced by

$$\hat{J} \ = \ [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_m, b_m] \qquad (14)$$

which represents a hyperrectangle. For the degree of p with respect to the i-th variable x_i we introduce the abbreviation n_i and the set of Chebychev points in $\hat{J}$ is given by

$$x(\hat{N}, \hat{J}) \ := \ x(N_1, [a_1, b_1]) \times \cdots \times x(N_m, [a_m, b_m]) \qquad (15)$$

where N_i is the number of Chebychev points for the i-th variable x_i in the interval $[a_i, b_i]$. Then the inequalites

$$p^{\hat{J}}_{min} \ \ge \ \frac{1}{2}\left\{(K+1)\,p^{x(\hat{N},\hat{J})}_{min} \ - \ (K-1)\,p^{x(\hat{N},\hat{J})}_{max}\right\} \qquad (16)$$

$$p^{\hat{J}}_{max} \ \le \ \frac{1}{2}\left\{(K+1)\,p^{x(\hat{N},\hat{J})}_{max} \ - \ (K-1)\,p^{x(\hat{N},\hat{J})}_{min}\right\} \qquad (17)$$

with

$$K \ = \ \prod_{i=1}^{m} C\left(\frac{n_i}{N_i}\right) \qquad (18)$$

under the conditions $N_i > n_i, \ i = 1, ..., m$, are valid. We use and apply the theorem of Ehlich and Zeller in the next section. We will show that if a homogenized polynomial is positive on a subset of the boundary of the hyperrectangle, the original polynomial is globally positive. By means of this property the uncertain parameter region Ω for which the polynomial $p(x)$ (4) is globally positive can be determined.

3 Approximation Method

The theorem of Ehlich and Zeller helps us analyzing the positivity of polynomials on finite intervals. Investigating the positivity of a polynomial $p(x)$ on $\mathbb{R}^m$ is our goal and we have to do some calculations in order to apply the theorem of Ehlich and Zeller in this case. The main tool is homogenization, i.e. for every polynomial $p(x)$ we introduce the polynomial $\tilde{p}(x)$ which is defined according to the following expression.

$$\tilde{p}(x_0, \ \ldots, \ x_m) \ := \ {x_0}^{deg\ p(x)}\, p(\frac{x_1}{x_0}, \ \ldots, \ \frac{x_m}{x_0}) \ , \qquad (19)$$

$$\tilde{p}(x) \ = \ \sum_{i=1}^{s} \tilde{p}_{\alpha_i}\, x^{\tilde{\alpha}_i} \ , \ \ x \in \mathbb{R}^{m+1} \qquad (20)$$

Each monomial of the polynomial $\tilde{p}(x)$ has the same degree

$$|\ \tilde{\alpha}_i\ | = \ deg\, p(x) \ , \quad i = 1, \ \ldots \ , s \ . \qquad (21)$$

Here $|\ \tilde{\alpha}_i\ |$ represents the degree of the i-th monomial in the polynomial $\tilde{p}(x)$. The positivity of $\tilde{p}(x)$ and $p(x)$ is related by the following implication.

$$\tilde{p}(x) \ > \ 0 \ \forall\, x \in \mathbb{R}^{m+1} \ \text{with} \ x_0 \ > \ 0 \quad \Longleftrightarrow$$
$$p(x) \ > \ 0 \ \forall\, x \in \mathbb{R}^{m} \qquad (22)$$

Thus, in order to test $p(x)$ for positivity in $\mathbb{R}^m$ we can alternatively test $\tilde{p}(x)$ for positivity in $\mathbb{R}^{m+1}$ under the further condition $x_0 > 0$. The following equation

$$\tilde{p}(\lambda\, x) \ = \ \lambda^{deg\ p(x)}\, \tilde{p}(x) \ , \quad \forall\, \lambda \in \mathbb{R} \qquad (23)$$

is valid due to homogeneity of $\tilde{p}(x)$. It follows that if $\lambda > 0$, then $\tilde{p}(\lambda\, x)$ and $\tilde{p}(x)$ have the same sign. If we choose

$$\lambda \ := \ \left(\max_{i=0,\ \ldots,\ m} \ |\ x_i\ |\right)^{-1} \ , \qquad (24)$$

the vector $(\lambda\, x)$ is on the boundary of the hypercube H defined by

$$H \ := \ H^-_0 \cup H^+_0 \cup \cdots \cup H^-_j \cup H^+_j \cup \cdots \cup H^-_m \cup H^+_m \qquad (25)$$

with

$$H_j^- \;:=\; \{\, x \in \mathbb{R}^{m+1} \mid x_j = -1 \,,\;\; -1 \le x_i \le 1 \,, \\ i \neq j \,,\;\; i = 0, \ldots, m \,\} \tag{26}$$

$$H_j^+ \;:=\; \{\, x \in \mathbb{R}^{m+1} \mid x_j = +1 \,,\;\; -1 \le x_i \le 1 \,, \\ i \neq j \,,\;\; i = 0, \ldots, m \,\} \,. \tag{27}$$

Thus the test of $\tilde{p}(x)$ for positivity in $\mathbb{R}^{m+1}$ under the condition $x_0 > 0$ is reduced to test positivity of $\tilde{p}(x)$ of that part of the boundary of H for which $x_0 > 0$ is fulfilled. This boundary consists of a finite number of hyperrectangles defined by

$$H_0 \;:=\; \{\, x \in \mathbb{R}^{m+1} \mid x_0 = 1 \,,\; -1 \le x_i \le 1 \,, \\ i = 1, \ldots, m \,\} \tag{28}$$

$$H_{j0}^{\mp} \;:=\; \{\, x \in \mathbb{R}^{m+1} \mid x_j = \mp 1 \,,\;\; 0 < x_0 \le 1 \,, \\ -1 \le x_i \le 1 \,,\;\; i \neq j \,,\;\; i = 1, \ldots, m \,\} \,, \\ j = 1, \ldots, m \tag{29}$$

and thus the theorem of Ehlich and Zeller can be applied to every part of this boundary. Inequality (16) is used to ensure the positivity of $\tilde{p}(x)$ on H_0,H_{10}^+,H_{10}^-, ... ,H_{m0}^+,H_{m0}^-. If for every hyperrectangle $\hat{J} \in \{H_0, H_{10}^+, H_{10}^-, \ldots, H_{m0}^+, H_{m0}^-\}$ in this boundary the inequality

$$(K+1)\, \tilde{p}_{min}^{x(\hat{N},\hat{J})} \;-\; (K-1)\, \tilde{p}_{max}^{x(\hat{N},\hat{J})} \;>\; 0 \tag{30}$$

is fulfilled, the polynomial $\tilde{p}(x)$ is positive definite on the boundary of the hyperrectangle in $\mathbb{R}^{m+1}$ for which x_0 is greater than zero. Due to (22) and (23) the polynomial $p(x)$ is global positive in $\mathbb{R}^m$. In case that

$$(K+1)\, \tilde{p}_{min}^{x(\hat{N},\hat{J})} \;-\; (K-1)\, \tilde{p}_{max}^{x(\hat{N},\hat{J})} \;>\; 0 \tag{31}$$

on the set $\hat{J}$, the inequalities

$$(K{+}1)\, \tilde{p}(x_i) - (K{-}1)\, \tilde{p}(x_j) \;>\; 0 \;,\;\; i, j = 1, \ldots, \hat{N} \tag{32}$$

are valid for all i, j due to fact that

$$\tilde{p}_{min}^{x(\hat{N},\hat{J})} \;\le\; \tilde{p}(x_i) \;\le\; \tilde{p}_{max}^{x(\hat{N},\hat{J})} \,, \qquad i = 1, \ldots, j, \ldots, \hat{N} \tag{33}$$

where x_i, $x_j \in X(\hat{N}, \hat{J})$ are two Chebychev points in the same hyperrectangle. For $\hat{N}$ Chebychev points in one hyperrectangle we have $\hat{N}^2$ inequalities of type (32) which are equivalent to (30). Since there are $(2m+1)$ hyperrectangles to be checked, the total number of the inequalities is $(2m+1)\hat{N}^2$. If the polynomial $p(x)$ depends linearly on some uncertain parameters k_1, ..., k_r as in (4), then the polynomial $\tilde{p}(x)$ can be written as

$$\tilde{p}(x) \;=\; \tilde{p}_0(x) \;+\; \sum_{i=1}^{r} \delta k_i\, \tilde{p}_i(x) \tag{34}$$

and the inequalities (32) can be represented as

$$a_r^{i,j}\, \delta k_r \;+\; a_{r-1}^{i,j}\, \delta k_{r-1} \;+\; \ldots \;+\; a_1^{i,j}\, \delta k_1 \;+\; a_0^{i,j} \;>\; 0 \\ ,\;\; i, j = 1, \ldots, \hat{N} \tag{35}$$

with

$$a_t^{i,j} \;=\; (K+1)\, \tilde{p}_t(x_i) \;-\; (K-1)\, \tilde{p}_t(x_j) \;, \\ t \;=\; 0 \,, \ldots \,, r \;,\;\; i,j \;=\; 1 \,, \ldots \,, \hat{N} \tag{36}$$

where δk_t, $t = 1, \ldots, r$, denotes the uncertainty at the parameter k_t in the polynomial $p(x)$ and $a_t^{i,j}$, $t = 0, \ldots, r$, $i, j = 1, \ldots, \hat{N}$, is constant. For the values of the δk_t's that satisfy the $\hat{N}^2$ inequalities in (35) for each hyperrectangle H_0,H_{10}^+,H_{10}^-, ... ,H_{m0}^+,H_{m0}^-, the polynomial $p(x)$ is globally positive definite. From the inequalities (35) we get an inner approximation to the convex set

$$\Omega \;=\; \{\, \delta k \mid -\varepsilon \;\le\; \delta k_i \;\le\; \varepsilon \,,\;\; i = 1, \ldots, r \,\} \,. \tag{37}$$

Because the inequalities (35) are the sufficient conditions for the strict positivity of the polynomial $p(x)$. An outer approximation to Ω is achieved if only the $\hat{N}$ inequalities

$$\begin{aligned} \tilde{p}(x_i) \;&=\; \tilde{p}_r(x_i)\delta k_r \;+\; \tilde{p}_{r-1}(x_i)\delta k_{r-1} \;+\; \ldots \\ &\quad + \tilde{p}_1(x_i)\delta k_1 \;+\; \tilde{p}_0(x_i) \\ &=\; a_r^i\, \delta k_r \;+\; a_{r-1}^i\, \delta k_{r-1} \;+\; \ldots \\ &\quad + a_1^i\, \delta k_1 \;+\; a_0^i \;>\; 0 \,,\;\; i = 1, \ldots, \hat{N} \end{aligned} \tag{38}$$

for each hyperrectangle H_0,H_{10}^+,H_{10}^-, ... ,H_{m0}^+,H_{m0}^- are taken into account at the Chebychev points. The total number of the inequalities for the outer approximation is $(2m+1)\hat{N}$. Since the inequalities (38) are the necessary conditions for the strict positivity, by means of the solutions of the inequalities in (38) we get an outer approximation to the set Ω. Thus, using the theorem of Ehlich and Zeller we are able to compute inner and outer approximations to Ω.

The inequalities in (35) and (38) are in the form

$$a_r\, \delta k_r \;+\; a_{r-1}\, \delta k_{r-1} \;+\; \ldots \;+\; a_1\, \delta k_1 \;+\; a_0 \;>\; 0 \tag{39}$$

where the coefficient a_j, $j = 0, \ldots, r$, is known and constant. Because it is a function of the Chebychev points on the hyperrectangles H_0,H_{10}^+,H_{10}^-, ... ,H_{m0}^+,H_{m0}^-.

Our aim in this paper is to determine the set Ω which is the largest hypercube in the parameter space so that for the values of the parameter vector $\delta k \in \Omega$ the polynomial $p(x)$ is globally positive. The hypercube has the origin of the parameter space $\mathbb{R}^r$ as its center. To do this it is necessary to parametrise the parameter uncertainties $\delta k_1, \ldots, \delta k_r$ along the 2^r rays starting at the origin and passing through the hypercube's 2^r vertices.

Before we do this we want to define the function

$$b_2 \text{ div } b_1 \tag{40}$$

where b_1 is a natural number and b_2 is a natural number or zero. The number b_2 can be described according to the number b_1 as

$$b_2 \;=\; c_1\, b_1 \;+\; c_0 \tag{41}$$

where c_1 and c_0 are two positive entire numbers or zero. Then the arithmetic operation $(b_2 \text{ div } b_1)$ gives us the number c_1.

$$b_2 \text{ div } b_1 \ = \ c_1 \tag{42}$$

Now we can parametrise the parameter vector δk along the 2^r rays as follows.

$$\delta k^i \ = \ \Delta k^i \begin{pmatrix} (-1)^{\ (i-1)} \\ \\ (-1)^{\ [\ (i-1) \text{ div } 2\ ]} \\ \\ \vdots \\ \\ (-1)^{\ [\ (i-1) \text{ div } 2^{r-2}\ ]} \\ \\ (-1)^{\ [\ (i-1) \text{ div } 2^{r-1}\ ]} \end{pmatrix} \tag{43}$$

where $\delta k^i \in \mathbb{R}^r$, $i = 1, \ldots, 2^r$, denotes the parameter vector along the i-th ray passing through the i-th vertex and Δk^i is scalar and positive. Let us consider the inequality in (39). We can rewrite this inequality according to vectors in (43) with the following expression.

$$\begin{gathered} \left(\sum_{j=2}^{r} a_j \ (-1)^{[(i-1) \text{ div } 2^{j-1}]} \right) \ \Delta k^i \ + \\ a_1 (-1)^{i-1} \, \Delta k^i \ + \ a_0 \ > \ 0 \, , \qquad i = 1, \, \ldots \, , 2^r \end{gathered} \tag{44}$$

Let be our aim to determine the largest hypercube in the parameter space for which the inequality (39) is valid. It means the inequality must be solved according to vectors in (43). For the minimum positive value of the Δk^i that satisfies the inequalities (44) we get the largest hypercube for which the inequality (39) is valid. If the inequality (39) is the $j - th$ inequality under the conditions in (35) or in (38), we define the minimum positive value of the Δk^i for this inequality as

$$\varepsilon_j \ := \ min(\Delta k^i) \, , \qquad i = 1, \, \ldots \, , 2^r \, . \tag{45}$$

The largest hypercube

$$\Omega_{in} := \{ \delta k \ | \ -\varepsilon_{min} \ \leq \ \delta k_i \ \leq \ \varepsilon_{min} \, , \ \ i = 1, \ldots, r \} \tag{46}$$

in the parameter space that satisfies the sufficient conditions in (35) can be found by

$$\varepsilon_{min} \ = \ min(\varepsilon_j) \, , \qquad j = 1, \, \ldots \, , (2m+1)\hat{N}^2 \, . \tag{47}$$

The largest hypercube

$$\Omega_{out} := \{ \delta k \ | \ -\varepsilon_{max} \ \leq \ \delta k_i \ \leq \ \varepsilon_{max} \, , \ \ i = 1, \ldots, r \} \tag{48}$$

in the parameter space that satisfies the necessary conditions in (38) can be found by

$$\varepsilon_{max} \ = \ min(\varepsilon_j) \, , \qquad j = 1, \, \ldots \, , (2m+1)\hat{N} \, . \tag{49}$$

The value of ε lies in the interval

$$\varepsilon_{min} \ \leq \ \varepsilon \ \leq \ \varepsilon_{max} \tag{50}$$

and the set Ω lies between the sets Ω_{in} and Ω_{out}.

$$\Omega_{in} \ \subseteq \ \Omega \ \subseteq \ \Omega_{out} \tag{51}$$

We want to give an example now in order to illustrate our algorithm.

4 Example

In the following example the number of variables of the main polynomial is one and the relevant hyperrectangles are a subset of the edges of a square. They are given by

$$\begin{aligned} &(1) \quad H_0 &&= \{ x \ | \quad x_0 = 1, \ -1 \leq x_1 \leq 1 \} \, , && (52) \\ &(2) \quad H_{10}^{+} &&= \{ x \ | \quad x_1 = 1, \ 0 < x_0 \leq 1 \} \, , && (53) \\ &(3) \quad H_{10}^{-} &&= \{ x \ | \quad x_1 = -1, \ 0 < x_0 \leq 1 \} \, . && (54) \end{aligned}$$

We will test our numerical method with the polynomial

$$\begin{aligned} p(x) \ &= \ (k_{1n} + \delta k_1) x_1^4 \ - \ (k_{2n} + \delta k_2) x_1^3 \ + \\ &\quad\ (k_{3n} + \delta k_3) x_1^2 \ + \ (k_{4n} + \delta k_4) \\ \\ &= \ (1 + \delta k_1) x_1^4 \ - \ (2 + \delta k_2) x_1^3 \ + \\ &\quad\ (1 + \delta k_3) x_1^2 \ + \ (5 + \delta k_4) \end{aligned} \tag{55}$$

that is taken from [1].

After the transformation we get the homogenized polynomial

$$\begin{aligned} \tilde{p}(x) \ &= \ (1 + \delta k_1) x_1^4 \ - \ (2 + \delta k_2) x_0 x_1^3 \ + \\ &\quad\ (1 + \delta k_3) x_0^2 x_1^2 \ + \ (5 + \delta k_4) x_0^4 \\ \\ &= \ (x_1^4 \ - \ 2 x_0 x_1^3 \ + \ x_0^2 x_1^2 \ + \ 5 x_0^4) \ + \ \delta k_1 x_1^4 \ - \\ &\quad\ \delta k_2 x_0 x_1^3 \ + \ \delta k_3 x_0^2 x_1^2 \ + \ \delta k_4 x_0^4 \, . \end{aligned} \tag{56}$$

Fig.1 shows the inner and outer approximations to the value of ε depending on the number of Chebychev points on the sets H_0, H_{10}^{+} and H_{10}^{-}. For 100 Chebychev points per variable the following interval

$$\varepsilon_{min} = 0.297520663 \ \leq \ \varepsilon \ \leq \ \varepsilon_{max} = 0.299647902 \tag{57}$$

was found for the value of ε.

5 Conclusions and Outlook

In this paper the positivity of polynomials depending on uncertain parameters have been investigated. By means of the theorem of Ehlich and Zeller we have developed a new algorithm that defines the parameter region as a hypercube where a polynomial $p(x)$ is positive definite. The example presented

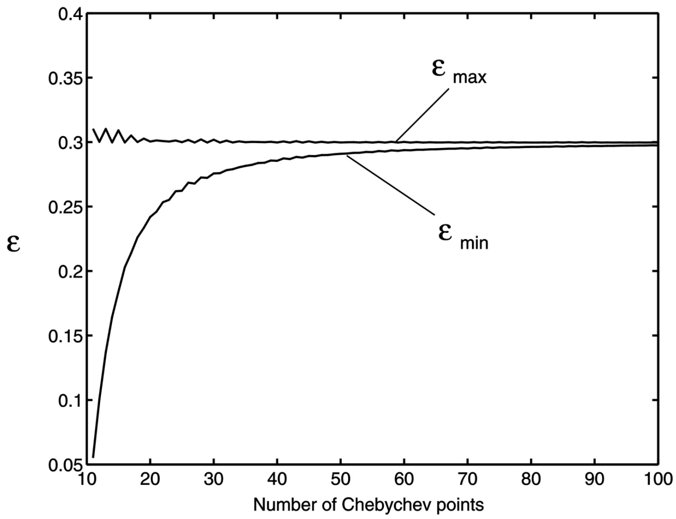

Figure 1: Inner and outer approximation to ε.

in this paper illustrates the result which can be achieved with this new algorithm. In contrast to other methods our method is able to produce inner and outer approximations to the set Ω and relies entirely on linear inequalities. This offers the posibility of using methods from linear programming in the case of higher parameter dimension. This will be the focus of future research.

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