A NUMERICAL CONTROL DESIGN METHOD FOR PROTOTYPICAL AEROELASTIC WING SECTION WITH STRUCTURAL NON-LINEARITY

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Abstract

A comprehensive analysis of aeroelastic systems has shown that these systems exhibit a broad class of pathological response regimes when certain types of non-linearities are included. In this paper, we propose a design method of a state-dependent non-linear controller for aeroelastic systems that includes polynomial structural non-linearities. The method is based on recent numerical methods such as Tensor Product model transformation and Parallel Distributed Compensation. As an example, a controller is derived that ensures the quadratic stability of a prototypical aeroelastic wing section via one control surface. Numerical simulations are used to provide empirical validation of the control results. The effectiveness of the controller design is compared with former approaches.

1 Introduction

In the past few years various studies of aeroelastic systems have emerged. [1] presents a detailed background and refers to a number of papers dealing with the modelling and control of aeroelastic systems. The following provides a brief summary of this background.

Regarding the properties of aeroelastic systems one can find the study of free-play non-linearity by Tang and Dowell in[2, 3], by Price et al. in [4] and [5], by Lee et al. in [6], and a complete study of a class of non-linearities is in [7], [5]. O'Neil et al. [8] examined the continuous structural non-linearity of aeroelastic systems. These papers conclude that an aerolesatic system may exhibit a variety of control phenomena such as *limit cycle oscillation, flutter* and even *chaotic vibrations.*

Control strategies have also been derived for aeroelastic systems. [9] and these show that controllers, capable of stabilizing structural non-linearity over flow regimes, can be derived via classical multivariable control methods. However, while several authors have investigated the effectiveness of linear control strategies for aeroelastic systems, experimental evidence has shown that linear control methods may not be reliable when non-linear effects predominate. For example in the case of large amplitude limit cycle oscillation behaviour the linear control methodologies [9] do not stabilize aeroelastic systems consistently. [10] and [9] proposed non-linear feedback control methodologies for a class of non-linear structural effects of the wing section [8]. Papers [10, 11, 1] develop a controller, capable of ensuring local asymptotic stability, via partial feedback linearization. It has been shown that by applying two control surfaces global stabilization can be achieved. For instance, adaptive feedback linearization [12] and the global feedback linearization technique were introduced for two control actuators in the work of [1].

The primary goal of this paper is to develop non-linear state dependent control method capable of globally and quadratically stabilizing a given prototypical aerolelastic wing section via one control surface. The controller design is based on the Tensor Product (TP) transformation introduced in [13, 14] and Parallel Distributed Dompensation (PDC) [15]. Our model incorporates the essential and well-characterized structural non-linearities that yield limit cycle oscillation at low velocity. The control results are compared with the previously developed partial feedback linearization technique that also utilizes one control surface.

2 Nomenclature

This section is devoted to introduce the notations being used in this paper: $\{a, b, \ldots\}$: scalar values. $\{\mathbf{a}, \mathbf{b}, \ldots\}$: vectors. $\{\mathbf{A}, \mathbf{B}, \ldots\}$: matrices. $\{\mathcal{A}, \mathcal{B}, \ldots\}$: tensors. $\mathcal{R}^{I_1\times I_2\times\cdots\times I_N}$:vector space of real valued $(I_1\times I_2\times\cdots\times I_N)$-tensors. Subscript defines lower order: for example, an element of matrix $\mathbf{A}$ at row-column number i, j is symbolized as $(\mathbf{A})_{i,j} = a_{i,j}$. Systematically, the ith column vector of $\mathbf{A}$ is denoted as $\mathbf{a}_i$, i.e. $\mathbf{A} = \begin{bmatrix}\mathbf{a}_1 & \mathbf{a}_2 & \cdots\end{bmatrix}$. $\diamond_{i,j,n}, \ldots$: are indices. $\diamond_{I,J,N}, \ldots$: index upper bound: for example: $i = 1..I$, $j = 1..J$, $n = 1..N$ or $i_n = 1..I_n$. $\mathbf{A}_{(n)}$: n-mode matrix of tensor $\mathcal{A} \in \mathcal{R}^{I_1\times I_2\times\cdots\times I_N}$. $\mathcal{A}\times_n \mathbf{U}$: n-mode matrix-tensor product. $\mathcal{A}\otimes_n \mathbf{U}_n$: multiple product as $\mathcal{A}\times_1 \mathbf{U}_1 \times_2 \mathbf{U}_2 \times_3 .. \times_N \mathbf{U}_N$. Detailed discussion of tensor notations and operations is given in [16].

3 Equations of Motion

In this paper, we consider the problem of flutter suppression for the prototypical aeroelastic wing section as shown in Figure 1.

The aerofoil is constrained to have two degrees of freedom, the plunge h and pitch α. The equations of motion of the system have been derived in many references (for example, see [17], and [18]), and can be written as

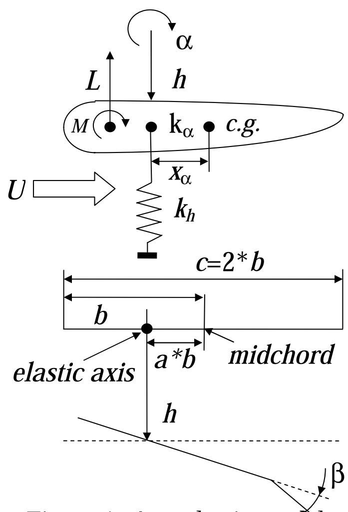

Figure 1: Aeroelastic model

$$\begin{pmatrix} m & mx_\alpha b \\ mx_\alpha b & I_alpha \end{pmatrix} \begin{pmatrix} \ddot{h} \\ \ddot{\alpha} \end{pmatrix} + \begin{pmatrix} c_h & 0 \\ 0 & c_\alpha \end{pmatrix} \begin{pmatrix} \dot{h} \\ \dot{\alpha} \end{pmatrix} + \qquad (1)$$

$$+ \begin{pmatrix} k_h & 0 \\ 0 & k_\alpha(\alpha) \end{pmatrix} \begin{pmatrix} h \\ \alpha \end{pmatrix} = \begin{pmatrix} -L \\ M \end{pmatrix},$$

where

$$L = \rho U^2 b c_{l_\alpha} \left(\alpha + \frac{\dot{h}}{U} + \left(\frac{1}{2} - a \right) b \frac{\dot{\alpha}}{U} \right) + \rho U^2 b c_{l_\beta} \beta \quad (2)$$

$$M = \rho U^2 b^2 c_{m_\alpha} \left(\alpha + \frac{\dot{h}}{U} + \left(\frac{1}{2} - a \right) b \frac{\dot{\alpha}}{U} \right) + \rho U^2 b c_{m_\beta} \beta,$$

and where x_α is the non-dimensional distance between elastic axis and the centre of mass; m is the mass of the wing; I_α is the mass moment of inertia; b is semi-chord of the wing, and c_α and c_h respectively are the pitch and plunge structural damping coefficients, and k_h is the plunge structural spring constant. Traditionally, there have been many ways to represent the aerodynamic force L and moment M, including steady, quasi-steady, unsteady and non-linear aerodynamic models. In this paper we assume the quasi-steady aerodynamic force and moment, see work [17]. It is assumed that L and M are accurate for the class of low velocities concerned. Wind tunnel experiments are carried out in [9]. In the above equation ρ is the air density, U is the free stream velocity, c_{l_α} and c_{m_α} respectively, are lift and moment coefficients per angle of attack, and c_{l_α} and c_{m_α}, respectively are lift and moment coefficients per control surface deflection, and a is non-dimensional distance from the mid-chord to the elastic axis. Several classes of non-linear stiffness contributions $k_\alpha(\alpha)$ have been studied in papers treating the open-loop dynamics of aeroelastic systems [2, 19, 20, 7]. For the purpose of illustration, we now introduce the use of polynomial non-linearities. The non-linear stiffness term $K_\alpha(\alpha)$ is obtained by curve-fitting the measured displacement-moment data for non-linear spring as [21]:

$$k_\alpha(\alpha) = 2.82(1 - 22.1\alpha + 1315.5\alpha^2 + 8580\alpha^3 + 17289.7\alpha^4).$$

The equations of motion derived above exhibit limit cycle oscillation, as well as other non-linear response regimes including chaotic response [21, 19, 7]. The system parameters to be used in this paper are given in [1] and are obtained from experimental models described in full detail in work by [21, 1].

With the flow velocity $u = 15(m/s)$ and the initial conditions of $\alpha = 0.1(rad)$ and $y = 0.01(m)$, the resulting time response of the non-linear system exhibits limit cycle oscillation, in good qualitative agreement with the behaviour expected in this class of systems. Papers[21, 8] have shown the relations between limit cycle oscillation, magnitudes and initial conditions or flow velocities.

Let the equations (1) and (2) be combined and reformulated into state-space model form:

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} h \\ \alpha \\ \dot{h} \\ \dot{\alpha} \end{pmatrix} \quad \text{and} \quad \mathbf{u} = \beta.$$

Then we have:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{p})\mathbf{x} + \mathbf{B}(\mathbf{p})\mathbf{u} = \mathbf{S}(\mathbf{p}) \begin{pmatrix} \mathbf{x} \\ \mathbf{u} \end{pmatrix}, \qquad (3)$$

where

$$\mathbf{A}(\mathbf{p}) = \begin{pmatrix} x_3 \\ x_4 \\ -k_1 x_1 - (k_2 U^2 + p(x_2))x_2 - c_1 x_3 - c_2 x_4 \\ -k_3 x_1 - (k_4 U^2 + q(x_2))x_2 - c_3 x_3 - c_4 x_4 \end{pmatrix}$$

$$\mathbf{B}(\mathbf{p}) = \begin{pmatrix} 0 \\ 0 \\ g_3 U^2 \\ g_4 U^2 \end{pmatrix},$$

where $\mathbf{p} \in \mathcal{R}^{N=2}$ contains values x_2 and U. The new variables are tabulated in Table 1. One should note that the equations of motion are also dependent upon the elastic axis location a.

4 Controller design method

The recently proposed very powerful numerical methods (and associated theory) for *convex optimization* involving Linear Matrix Inequalities (LMI) help us with the analysis and the design issues of dynamic systems models (3) in acceptable computational time [22, 23]. One direction of these analysis and design methods is based on LMI's and PDC techniques [15], and functions with the multiple-model form. In this paper we utilise the TP transformation and a PDC controller design technique to derive viable control methodologies for the non-linear aeroelastic system defined in the previous section. The key idea

Table 1: System variables

$d = m(I_\alpha - mx_\alpha^2 b^2)$
$k_1 = \frac{I_\alpha k_h}{d}$
$k_2 = \frac{I_\alpha \rho b c_{l_\alpha} + m x_\alpha b^3 \rho c_{m_\alpha}}{d}$
$k_3 = \frac{-m x_\alpha b k_h}{d}$
$k_4 = \frac{-m x_\alpha b^2 \rho c_{l_\alpha} - m \rho b^2 c_{m_\alpha}}{d}$
$p(\alpha) = \frac{-m x_\alpha b}{d} k_\alpha(\alpha)$
$q(\alpha) = \frac{m}{d} k_\alpha(\alpha)$
$c_1(U) = \frac{I_\alpha (c_h + \rho U b c_{l_\alpha}) + m x_\alpha \rho U^3 c_{m_\alpha}}{d}$
$c_2(U) = \frac{I_\alpha \rho U b^2 c_{l_\alpha}(\frac{1}{2} - a) - m x_\alpha b c_\alpha + m x_\alpha \rho U b^4 c_{m_\alpha}(\frac{1}{2} - a)}{d}$
$c_3(U) = \frac{-m x_\alpha b c_h - m x_\alpha \rho U b^2 c_{l_\alpha} - m \rho U b^2 c_{m_\alpha}}{d}$
$c_4(U) = \frac{m c_\alpha - m x_\alpha \rho U b^3 c_{l_\alpha}(\frac{1}{2} - a) - m \rho U b^3 c_{m_\alpha}(\frac{1}{2} - a)}{d}$
$g_3 = \frac{1}{d}(-I_\alpha \rho b c_{l_\beta} - m x_\alpha b^3 \rho c_{m_\beta})$
$g_4 = \frac{1}{d}(m x_\alpha b^2 \rho c_{l_\beta} + m \rho b^2 c_{m_\beta})$

of the proposed design method is that the TP transformation is utilized to represent the model (3) in multiple-model form with specific characteristics, whereupon PDC controller design techniques can immediately be executed. The detailed description of the TP transformation and PDC based designs is beyond the scope of this paper and can be found in [13, 14, 15]. First of all, let us define the multiple-model form.

4.1 Multiple-model

This subsection defines the multiple model form of (3) as:

$$\begin{pmatrix} \dot{\mathbf{x}} \\ \mathbf{y} \end{pmatrix} = \left(\sum_{r=1}^{R} w_r(\mathbf{p}) \mathbf{S}_r \right) \begin{pmatrix} \mathbf{x} \\ \mathbf{u} \end{pmatrix}. \qquad (4)$$

where basis functions fulfill:

$$\forall r, \mathbf{p} : w_r(\mathbf{p}) \in [0, 1]; \quad \text{and} \quad \forall \mathbf{p} : \sum_{r}^{R} w_r(\mathbf{p}) = 1. \qquad (5)$$

This defines a fixed polytope, where the system varies in: $\mathbf{S}(\mathbf{p}) \in \{\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_R\}$. Matrices $\mathbf{S}_r \in \mathcal{R}^{O \times I}$ are termed vertex systems. Further, (5) defines the convex hull of the vertex systems as:

$$\mathbf{S}(\mathbf{p}(t)) = co\{\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_R\}_{\mathbf{w}(\mathbf{p})},$$

where the row vector $\mathbf{w}(\mathbf{p}) \in \mathcal{R}^R$ contains the basis functions $w_r(\mathbf{p})$. In many cases the basis functions $w_r(\mathbf{p})$ are decomposed to dimensions, which leads to a higher structure of (4). Having the decomposed basis the multiple-model (4) can be written, in order to avoid complicated indexing, in terms of tensors as:

$$\begin{pmatrix} \dot{\mathbf{x}} \\ \mathbf{y} \end{pmatrix} = \mathcal{S} \overset{N}{\underset{n=1}{\otimes}} \mathbf{w}_n(p_n) \begin{pmatrix} \mathbf{x} \\ \mathbf{u} \end{pmatrix}. \qquad (6)$$

Here, the row vector $\mathbf{w}_n(p_n) \in \mathcal{R}^{I_n}$ contains the basis functions $w_{n,i_n}(p_n)$, the $N+2$ -dimensional coefficient tensor $S \in \mathcal{R}^{I_1 \times I_2 \times \cdots \times I_N \times O \times I}$ is constructed from the vertex system matrices $\mathbf{S}_{i_1, i_2, \ldots, i_N} \in \mathcal{R}^{O \times I}$. The first N dimensions of S are assigned to the dimensions of $\mathbf{p}$.

4.2 TP transformation to multiple model

The TP transformation has various options. Let us summarize here only those that have prominent roles in this work:

$$(\mathbf{w}_{n=1..N}(p_n), \mathcal{S}) = TP\_transf(\mathbf{S}(\mathbf{p}), \Omega, options), \qquad (7)$$

where $\mathbf{S}(\mathbf{p}) \in \mathcal{R}^{O \times I}$ is from the state-space model (3), and $\Omega \subset \mathcal{R}^N$ denotes the bounded domain which the transformation is performed over, and "*options*" is to define some characteristics of the basis. The transformation can generate "minimal", "convex" and "close-to-localised" basis. The "close-to-localised " basis means that the vertex models involved in the multiple-model form are the linear models of the given dynamic model $\mathbf{S}(\mathbf{p})$ over certain operation points, namely, the vertex systems are included in the dynamic model at certain points $\mathbf{p}$ or they are as close to the given model as possible. close-to-localised basis - option understood on a convex basis, required by the multiple-model form, see (5). In the control design of this paper (Section 5) we select the "close-to-localised"-basis option. Papers [13, 14] introduces the method of generating "minimal" and "convex" basis. The transformation to "close-to-localised " basis is introduced in [24] in a slightly different manner. Vectors $\mathbf{w}_n(p_n) \in \mathcal{R}^{I_n}$ and tensor $\mathcal{S}$ are defined at (6). At this point, we should describe briefly the existence of the exact TP transformation. In [25] it is shown that the multiple-model (6) is no-where dense in the modelling space if the number of basis functions is bounded, which is always the case in numerical implementations. The practical significance of this is that the transformed multiple-model is only an approximation in general cases:

$$\dot{\mathbf{x}} \underset{\varepsilon}{\approx} \mathcal{S} \overset{N}{\underset{n=1}{\otimes}} \mathbf{w}_n(p_n) \begin{pmatrix} \mathbf{x} \\ \mathbf{u} \end{pmatrix}. \qquad (8)$$

ε denotes the transformation error. It is zero if the given model can be transformed exactly to multiple-model form. If exact representation does not exist then we should employ as many basis functions as possible to ensure small ε. The TP transformation defines the relation between ε and the number of basis functions, which helps us with optimising the number of basis functions, subject to an acceptable error.

4.3 PDC controller design

The PDC design techniques determine one feedback to each vertex model:

$$\mathcal{K} = PDC(\mathcal{S}, stability\_theorem).$$

"stability_theorem" is a symbolic parameter. It specifies the stability criteria expressed in terms of matrix algebra or Linear Matrix Inequalities. The control performance depends on the selected criteria. For instance, the speed of response, constraints on the state vector or on the control value can also be set by properly selected LMI based stability theorems. A large collection of such theorems is presented in [15]. Under the framework of vertex feedback systems, one can define the control value as:

$$\mathbf{u} = -\mathcal{K} \bigotimes_{n=1}^{N} \mathbf{w}_n(p_n)\mathbf{x}.$$

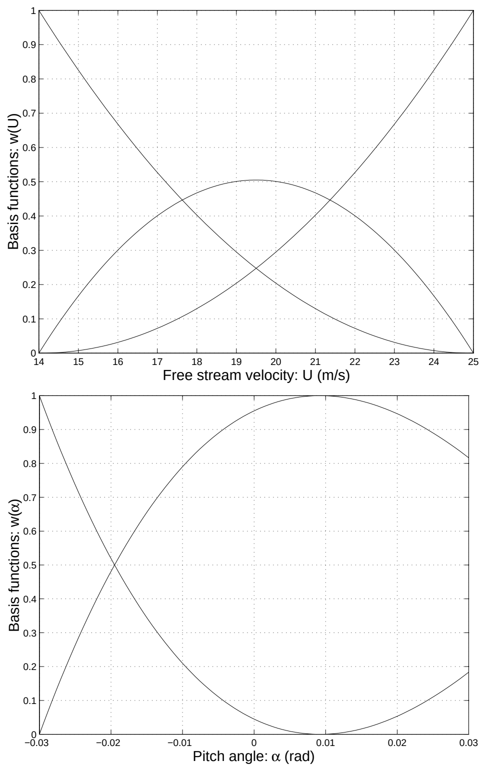

Figure 2: Basis functions on the dimensions α and U.

5 Quadratic stability of the aeroelastic wing section

This section is intended to perform the controller design method discussed in the previous section to the present aeroelastic system defined in (3). First of all let us define the transformation space Ω. We are interested in the interval $U \in [14, 25](m/s)$ and we presume that the interval $\alpha \in [-0.03, 0.03](rad)$ is sufficiently large enough (note that these intervals can arbitrarily be set). Therefore let:

$$\Omega : [14, 25] \times [-0.03, 0.03]$$

in the present example. Executing the TP transformation (7), yields that the dynamic model (3) can be represented exactly in TP model form (6) over a 3 times 2 "closeto localizes" basis. This means that the model in (3) can be described exactly ($\varepsilon = 0$ in (8)) by the convex combination of $3 \times 2 = 6$ linear vertex systems.

Note that, we may try to derive the basis functions analytically. The basis functions of α can easily be extracted from $k_\alpha(\alpha)$. However, finding the basis functions of U, seems to be rather complicated. TP transformation takes a few seconds to execute on a Pentium computer.

Having the multiple-model form we can execute the PDC design techniques. Let us select one of the simplest PDC techniques that does not consider any constraint on the speed of the controller, the state vector and the control values, and does not involve LMI's: First we execute pool-replacement technique to define the vertex feedback systems:

$$\mathcal{K} = Pool\_replacement(\mathcal{S}, pools), \tag{9}$$

then we utilise Theorem C14 of paper [26] to check the stability of the controlled multiple model:

Theorem 1 (Quadratic stability) *Dynamic system*

$$\dot{\mathbf{x}}(t) = \mathcal{S} \bigotimes_{n} \mathbf{w}_n(p_n) \begin{pmatrix} \mathbf{x} \\ \mathbf{u} \end{pmatrix}$$

where

$$\mathbf{u} = -\mathcal{K} \bigotimes_{n=1}^{N} w_n(p_n)\mathbf{x}.$$

is quadratically stable if and only if the following condition holds:

$$\mathrm{Re}\lambda_i(\mathbf{H}) \neq 0, \tag{10}$$

where $\mathbf{H}$ *is an indicator matrix. Its elements are detailed in [26].*

As a matter of fact the $pools$ in (9) cannot be arbitrarily set, but we can easily find densely located regions of those pools, which lead to (10). Furthermore, as one might expect, the effect of the selected pools on closed-loop controller performance (speed of response) and on the maximum value of $\mathbf{u}$. We may select the pools according to a desired controller speed.

6 Control results

To demonstrate the performance of the controlled system, numerical experiments are presented in this section. In order to be comparable to other published results, the numerical examples are performed with free stream velocity $U = 20m/s$, a velocity that exceeds the linear flutter velocity $U = 15.5m/s$.

6.1 Time response of controlled system

Figure 3 shows the control results for $U = 20m/s$ and for initials $h = 0.01$ and $\alpha = 0.1$.

6.2 Comparison to other solutions

For comparison, Figure 4 presents the time response of the controller developed, via exact feedback linearization, for $U = 20m/s$ and for initials $h = 0.01$ and $\alpha = 0.1$, see [1]. Comparing the results we can observe that the controller developed in this paper is faster and stabilises the system in about 3 sec whilst the maximum control values are significantly smaller than those shown in Figure 3.

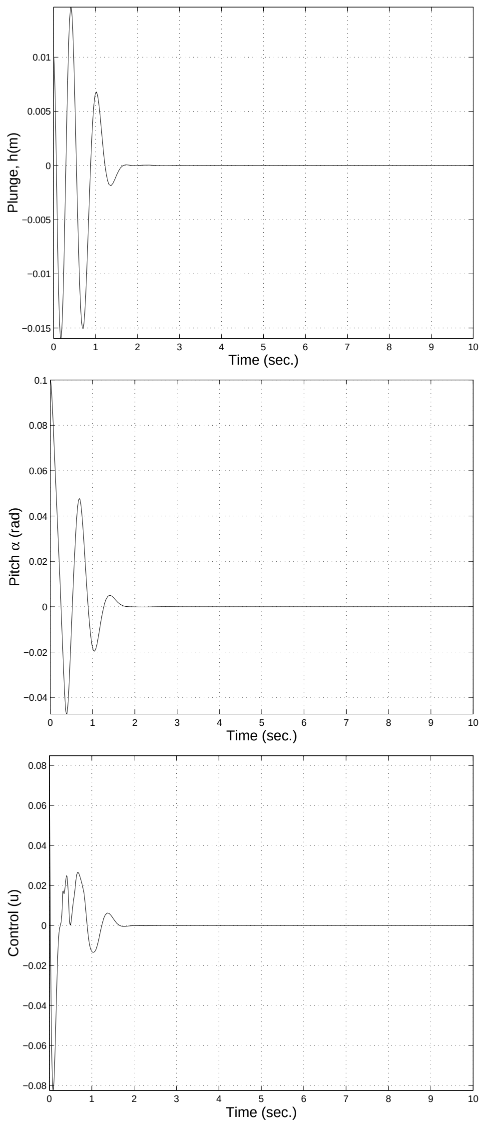

Figure 3: Time response of derived controller for $U = 20m/s$ and $a = -0.4$.

7 Conclusion

In this paper we have applied a numerical control design method which is based on the TP model transformation and PDC design methods, to design non-linear controllers for prototype aeroelastic wing sections that includes structural nonlinearity. The control design utilises one control surface. Without any control effort, or with linear controllers, the aeroelastic system reveals various kinds of non-linear phenomenon including limit cycle oscillation as noted in various text. The proposed controller design method quadratically stabilises the system and is based on numerical steps. The controller can thus be determined automatically and without analytic derivations. The effectiveness of the controller has been compared with an alternative another control solution. If the design requirements extend beyond stability, various performance specifications can be given by selecting proper PDC design theorems. As a further development of this work the authors plan to design controllers for advantageous control performance.

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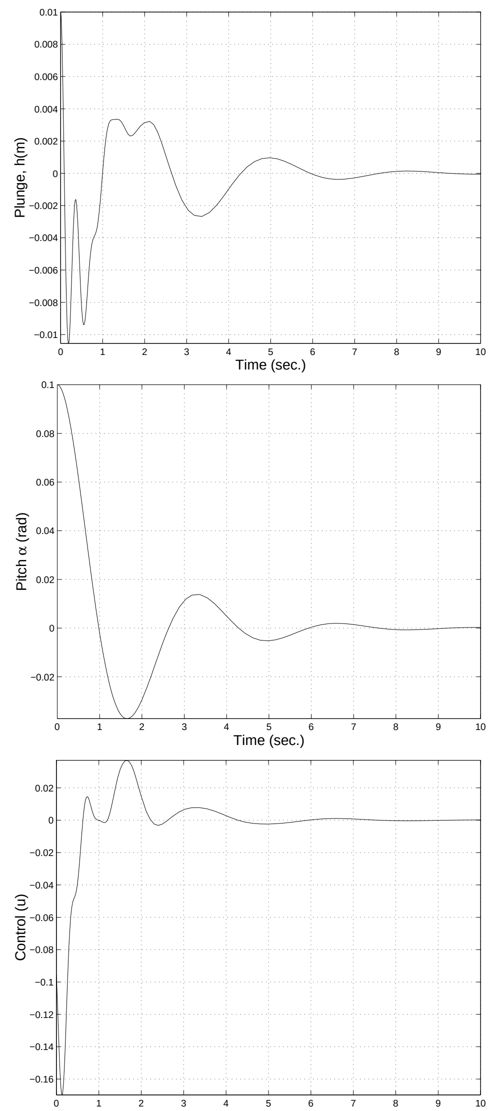

Figure 4: Time response of exact feedback linearization method for $U = 20m/s$ and $a = -0.4$.

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