SEMIACTIVE CONTROL OF BASE ISOLATED STRUCTURES WITH ACTUATOR DYNAMICS

Rodolfo Villamizar ◇, Ningsu Luo ◇, Josep Vehí ◇, José Rodellar †

◇ Departament d'Electrònica, Informàtica i Automàtica, Universitat de Girona, Girona, Spain
† Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya, Barcelona, Spain

Keywords: Semiactive control, base isolated structures, vibration reduction, parametric uncertainty, actuator dynamics

Abstract

In this paper, a new semiactive control approach is presented to stabilize a base isolated structure subjected to parametric uncertainties and unknown disturbances. In the controller design, the actuator dynamics (time delay and frictional effects) are taken into account. The ultimate boundedness is achieved in the closed-loop system. Numerical simulation is done for a 10 story base isolated building, with two semiactive controllers being put on the base and the first floor, to illustrate the effectiveness of the proposed semiactive control scheme.

1 Introduction

In recent years, different (passive, active and semiactive) control approaches have been proposed in order to attenuate the structural vibration in high rise buildings and long span bridges caused by the strong earthquake and wind [1]. In general, the application of active control force to the structure can achieve an important improvement of the structural behavior compared with the traditional passive controlled structures [2]−[5]. Some successful application of active control of structures can be found in Japan, China, etc. However, one of the main problems associated with the active structural control is the need of high electric energy for its correct operation, which could be failed during the strong seismic excitation. Semiactive control strategies become very promising for vibration suppression in flexible structures due to the requirement of low electric supply and the facility of maintenance [6]−[7]. In a semiactive control system, on-line adjustment of the damping and/or stiffness of adaptable devices are done according to feedback signals and control commands. In general, a semiactive controller can act in a desirable fashion in both a passive and a feedback control mode, with its performance generally enhanced in this mode. The use of semiactive devices in combination with base isolation systems has been also considered within this context. In the design of semiactive controller up to now, the actuator dynamics have not been considered but just being included in the validation of the controller implementation. In this paper, a new semiactive controller is presented for achieving the ultimate boundedness of structural performance in the presence of seismic excitation. The controller design is made based on the Lyapunov theory and the actuator dynamics is taken into account in order that the obtained results give a better approximation to the real conditions. Numerical simulation is done with a 10 story base isolated building to show the effectiveness of the proposed control strategy.

2 Problem Formulation

Consider a nonlinear base isolated building structure as shown in Figure 1, whose dynamic behavior can be described by means of a model composed of two coupled subsystems, namely, the main structure (S_r) and the base isolation (S_c):

$$\begin{aligned} & S_r : \boldsymbol{M}\ddot{\boldsymbol{q}}_r + \boldsymbol{C}\dot{\boldsymbol{q}}_r + \boldsymbol{K}\boldsymbol{q}_r = [c_1, 0, ..., 0]^T \dot{q}_c + [k_1, 0, ..., 0]^T q_c \,. \\ & S_c : m_0\ddot{q}_c + (c_0 + c_1)\dot{q}_c + (k_0 + k_1)q_c - c_1\dot{q}_{r1} - k_1 q_{r1} \\ & = -c_0\dot{d} - k_0 d + f_N \,. \\ & f_N = -sgn(\dot{q}_c - \dot{d})[\mu_{max} - \Delta\mu e^{-\nu|\dot{q}_c - \dot{d}|}]G \,. \end{aligned} \tag{1}$$

This model assumes that the structure has a linear behavior due to the effect of the base isolation. This behavior is represented by the positive definite mass, damping and stiffness matrices $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K} \in \boldsymbol{R}^{n\times n}$ respectively.

$$\boldsymbol{M} = diag(m_i) \quad ; \quad (i = 1, 2, ..., n) \tag{2}$$

$$\boldsymbol{C} = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 & \cdots & 0 & 0 \\ -c_2 & c_2 + c_3 & -c_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -c_n & c_n \end{bmatrix} \tag{3}$$

$$\boldsymbol{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & \cdots & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -k_n & k_n \end{bmatrix} \tag{4}$$

$\boldsymbol{q}_r = [\, q_{r1}\, , \, q_{r2}\, , \, \cdots \, , \, q_{rn}\,]^T \in \boldsymbol{R}^n$ represents the horizontal displacements of each floor with respect to an inertial frame. The base isolation is described as a single degree of freedom with horizontal displacement $q_c \in \boldsymbol{R}$. It is assumed to exhibit a linear behavior characterized by mass, damping and stiffness m_0, c_0 and k_0, respectively, plus a nonlinear behavior represented by a force f_N supplied by a frictional isolator with G being the force normal to the friction surface, μ the friction coefficient, ν a constant, μ_{max} the coefficient for high sliding velocity and $\Delta\mu$ the difference between μ_{max} and the friction coefficient for low sliding velocity. The term $-c_0\dot{d} - k_0 d$ is a dynamic excitation force acting on the base due to the horizontal seismic ground motion represented by inertial displacement $d(t)$ and velocity $\dot{d}(t)$ at each time instant t.

In general, the base isolator (passive control device) can achieve satisfactory performance if its resonance frequency is well tuned. It is very difficult to make such tuning in practice due to the lack of information on the forthcoming earthquake [8]. Another serious problem is that sometimes the peak response of absolute base displacement is so large as to exceed the elastic limit of the base isolator. The main purpose for the use of active and semiactive controllers in combination with the passive controller (base isolator) is to reduce the peak response of the absolute base displacement so that the base isolator works always in the elastic region and also to attenuate the dependence of structural performance on the resonance frequency of the base isolator.

3 Controller Design

Usually, the semiactive control devices have to be installed in all stories of the building to guarantee the global stability of the whole base/structure system. In this paper, we only use semiactive controllers at the base and the first floor to adjust the stiffness $k_i(t)$ and the damping $c_i(t)$ $(i = 0, 1)$, as illustrated in Figure 1. In this way, the number of semiactive control devices is significantly reduced. The following equations of motion of the base and the first floor will be used in the controller design:

$$\begin{aligned} S_{r1} &: \quad m_1\ddot{q}_{r1} + c_1\dot{q}_{r1} + k_1 q_{r1} = \alpha + \beta\,. && (5)\\ S_c &: \quad m_0\ddot{q}_c + [c_0 + c_1]\dot{q}_c + [k_0 + k_1]q_c = c_1\dot{q}_{r1} + k_1 q_{r1} \\ &\quad\ -c_0\dot{d} - k_0 d + f_N\,. && (6)\end{aligned}$$

where

$$\begin{cases} \alpha =: c_1\dot{q}_c + k_1 q_c\,. \\ \beta =: c_2[\dot{q}_{r2} - \dot{q}_{r1}] + k_2[q_{r2} - q_{r1}]\,. \end{cases} \qquad (7)$$

It is well accepted that the movement of the building S_r is very close to the one of a rigid body due to the base isolation [8]. Then it is reasonably to assume that the inter-story motion of the building will be much smaller than the absolute motion of the base. Hence, the right-hand terms of the eqn.(5) can be simplified as

$$\alpha + \beta \approx \alpha = c_1\dot{q}_c + k_1(t) q_c\,. \qquad (8)$$

A numerical verification of the above assumption can be found in Figure 2. Consequently, the following simplified equation of motion of the first floor can be used in the subsequent controller design:

$$S_{r1}: \qquad m_1\ddot{q}_{r1} + c_1\dot{q}_{r1} + k_1 q_{r1} = c_1(t)\dot{q}_c + k_1 q_c\,. \quad (9)$$

The semiactive controller is designed to provide adaptive damping and stiffness as being functions of the absolute motion. Concretely, the operation of control system is based on the on-line modification of the stiffness and the damping parameters of both the base $(k_0(t); c_0(t))$ and the first floor $(k_1(t); c_1(t))$. It is assumed that these parameters can take any value within prescribed bounds. That is,

$$k_i(t) \in \left[k_i^-, k_i^+\right] \quad ; \quad c_i(t) \in \left[c_i^-, c_i^+\right] \quad ; \quad i = 0, 1 \qquad (10)$$

where $k_i^{\pm}$ and $c_i^{\pm}$ are known constants (prescribed bounds). Suppose that $k_i(t)$ and $c_i(t)$ can be adjusted by control signals $u_i^k(t)$ and $u_i^c(t)$ $(i = 0, 1)$. For instance, without loss of generality, let

$$\begin{aligned} k_i(t) = k_i^* + \delta_i^k u_i^k(t) \ ; \ c_i(t) = c_i^* + \delta_i^c u_i^c(t) \ ; && (11)\\ u_i^k(t) \in [-1, 1] \ ; \ u_i^c(t) \in [-1, 1] && (12)\end{aligned}$$

where

$$\begin{aligned} k_i^* &= \tfrac{1}{2}(k_i^+ + k_i^-) \quad c_i^* = \tfrac{1}{2}(c_i^+ + c_i^-) && (13)\\ \delta_i^k &= \tfrac{1}{2}(k_i^+ - k_i^-) \quad \delta_i^c = \tfrac{1}{2}(c_i^+ - c_i^-) && (14)\end{aligned}$$

with k_i^* and c_i^* being considered generally as the nominal values of $k_i(t)$ and $c_i(t)$.

By taking into account the actuator dynamics, such as time delay and frictional force, the real control forces $v_i^k(t)$ and $v_i^c(t)$ generated by the semiactive controllers to the structure are given as follows

$$\begin{aligned} v_0^k &= \delta_0^k u_0^k q_c - \tau_0^k \dot{k}_0 q_c + P_{a_0}^k \dot{q}_c + k_0^* q_c && (15)\\ v_0^c &= \delta_0^c u_0^c \dot{q}_c - \tau_0^c \dot{c}_0 \dot{q}_c + P_{a_0}^c \dot{q}_c + c_0^* \dot{q}_c && (16)\\ v_1^k &= \delta_1^k u_1^k (q_c - q_{r1}) - \tau_1^k \dot{k}_1 (q_c - q_{r1}) \\ &\quad + P_{a_1}^k (\dot{q}_c - \dot{q}_{r1}) + k_1^* (q_c - q_{r1}) && (17)\\ v_1^c &= \delta_1^c u_1^c (\dot{q}_c - \dot{q}_{r1}) - \tau_1^c \dot{c}_1 (\dot{q}_c - \dot{q}_{r1}) \\ &\quad + P_{a_1}^c (\dot{q}_c - \dot{q}_{r1}) + c_1^* (\dot{q}_c - \dot{q}_{r1}) && (18)\end{aligned}$$

with

$$k_i = \delta_i^k u_i^k - \tau_i^k \dot{k}_i \quad ; \quad c_i = \delta_i^c u_i^c - \tau_i^c \dot{c}_i \qquad (19)$$

i.e.,

$$u_i^k = \frac{1}{\delta_i^k}\left[k_i + \tau_i^k \dot{k}_i\right] \quad ; \quad u_i^c = \frac{1}{\delta_i^c}\left[c_i + \tau_i^c \dot{c}_i\right] \qquad (20)$$

where τ_i^k and τ_i^c are time constants of the actuator dynamics for the stiffness and damping changing, $P_{a_i}^k$ and $P_{a_i}^c$ are the parameters related to the frictional forces existed in the actuator.

By substituting the above control laws into the the dynamic equations of the base(eqn.(6)) and the first floor (eqn.(9)), we obtain

$$\begin{aligned} & m_0\ddot{q}_c + (c_0^* + c_1^* + P_{a_0}^k + P_{a_0}^c + P_{a_1}^k + P_{a_1}^c)\dot{q}_c + (k_0^* + \\ & k_1^*)q_c - (c_1^* + P_{a_1}^k + P_{a_1}^c)\dot{q}_{r_1} - k_1^* q_{r_1} = f(q_c, \dot{q}_c, d, \dot{d}) \\ & -u_0^k \delta_0^k q_c + \tau_0^k \dot{k}_0 q_c - u_0^c \delta_0^c \dot{q}_c + \tau_0^c \dot{c}_0 \dot{q}_c - \delta_1^k u_1^k (q_c - q_{r1}) \\ & + \tau_1^k \dot{k}_1 (q_c - q_{r1}) - \delta_1^c u_1^c (\dot{q}_c - \dot{q}_{r1}) + \tau_1^c \dot{c}_1 (\dot{q}_c - \dot{q}_{r1}) \quad (21)\end{aligned}$$

$$\begin{aligned} & m_1\ddot{q}_{r_1} + (c_1^* + P_{a_1}^k + P_{a_1}^c)\dot{q}_{r_1} + k_1^* q_{r_1} - (c_1^* + P_{a_1}^k + \\ & P_{a_1}^c)\dot{q}_c - k_1^* q_c = \delta_1^k u_1^k (q_c - q_{r1}) - \tau_1^k \dot{k}_1 (q_c - q_{r1}) \\ & + \delta_1^c u_1^c (\dot{q}_c - \dot{q}_{r1}) - \tau_1^c \dot{c}_1 (\dot{q}_c - \dot{q}_{r1}) \quad (22)\end{aligned}$$

Now, define $\boldsymbol{x}(t) = [q_{r_1}(t), \dot{q}_{r_1}(t), q_c(t), \dot{q}_c(t)]^T$,$\boldsymbol{u}(t) = [u_1^k(t), u_1^c(t), u_0^k(t), u_0^c(t)]^T$ and $z(t) = [\dot{k}_1, \dot{c}_1, \dot{k}_0, \dot{c}_0]^T$. Then, the following state equation is obtained

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{u}(t) + C(\boldsymbol{x},t)\boldsymbol{z}(t) + \boldsymbol{F}(\boldsymbol{x},t) \quad (23)$$

where

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1^*}{m_1} & -\frac{c_1^* + P_{a1}^k + P_{a1}^c}{m_1} & \frac{k_1^*}{m_1} & \frac{c_1^* + P_{a1}^k + P_{a1}^c}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1^*}{m_0} & \frac{c_1^* + P_{a1}^k + P_{a1}^c}{m_0} & -\frac{k_0^* + k_1^*}{m_0} & -\frac{c_0^* + c_1^* + P_{a0}^k + P_{a0}^c + P_{a1}^k + P_{a1}^c}{m_0} \end{pmatrix} \quad (24)$$

$$\boldsymbol{B}(\boldsymbol{x},t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{\delta_1^k(q_c - q_{r_1})}{m_1} & \frac{\delta_1^c(\dot{q}_c - \dot{q}_{r_1})}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\delta_1^k(q_c - q_{r1})}{m_0} & -\frac{\delta_1^c(\dot{q}_c - \dot{q}_{r1})}{m_0} & -\frac{\delta_0^k q_c}{m_0} & -\frac{\delta_0^c \dot{q}_c}{m_0} \end{pmatrix} \quad (25)$$

$$\boldsymbol{C}(\boldsymbol{x},t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\frac{\tau_1^k(q_c - q_{r1})}{m_1} & -\frac{\tau_1^c(\dot{q}_c - \dot{q}_{r1})}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\tau_1^k(q_c - q_{r1})}{m_0} & \frac{\tau_1^k(\dot{q}_c - \dot{q}_{r1})}{m_0} & \frac{\tau_0^k q_c}{m_0} & \frac{\tau_0^c \dot{q}_c}{m_0} \end{pmatrix} \quad (26)$$

$$\boldsymbol{F}(\boldsymbol{x},t) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_0} \end{bmatrix} f[q_c(t), \dot{q}_c(t), d(t), \dot{d}(t)] \quad (27)$$

Suppose that the seismic excitation $(d, \dot{d})$ is unknown but bounded,

$$\|f[q_c(t), \dot{q}_c(t), d(t), \dot{d}(t)]\| \leq \phi_0\,, \quad (28)$$

where ϕ_0 is a known constant. Then

$$\|\boldsymbol{F}(\boldsymbol{x},t)\| \leq \frac{1}{m_0}\|f[q_c(t), \dot{q}_c(t), d(t), \dot{d}(t)]\| \leq F_0 \quad (29)$$

consequently $F_0 = \phi_0/m_0$ is a known constant.
Define the Lyapunov function candidate as

$$V(\boldsymbol{x},t) = \frac{1}{2}\boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{x}(t) \quad (30)$$

where $\boldsymbol{P} = \in R^{4\times4}$ is the positive definite solution of the Lyapunov equation

$$\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{Q} = 0 \quad (31)$$

for a given symmetric positive definite matrix $\boldsymbol{Q}$. By using eqns. (27)–(29), the derivative of $V(\boldsymbol{x},t)$ is obtained

$$\begin{aligned} \dot{V}(\boldsymbol{x},t) = & -\frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{b}_0^k u_0^k + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{b}_0^c u_0^c + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{b}_1^k u_1^k \\ & + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{b}_1^c u_1^c + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{c}_0^k \dot{k}_0 + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{c}_0^c \dot{c}_0 + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{c}_1^k \dot{k}_1 \\ & + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{c}_1^c \dot{c}_1 + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{F} \end{aligned} \quad (32)$$

where

$$\boldsymbol{b}_0^k = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\delta_0^k q_c/m_0 \end{pmatrix} \quad \boldsymbol{b}_1^k = \begin{pmatrix} 0 \\ \delta_1^k(q_c - q_{r_1})/m_1 \\ 0 \\ -\delta_1^k(q_c - q_{r1})/m_0 \end{pmatrix}$$

$$\boldsymbol{b}_0^c = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\delta_0^c \dot{q}_c/m_0 \end{pmatrix} \quad \boldsymbol{b}_1^c = \begin{pmatrix} 0 \\ \delta_1^c(\dot{q}_c - \dot{q}_{r_1})/m_1 \\ 0 \\ -\delta_1^c(\dot{q}_c - \dot{q}_{r1})/m_0 \end{pmatrix}$$

$$\boldsymbol{c}_0^k = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tau_0^k q_c/m_0 \end{pmatrix} \quad \boldsymbol{c}_1^k = \begin{pmatrix} 0 \\ -\tau_1^k(q_c - q_{r1})/m_1 \\ 0 \\ -\tau_1^k(q_{r1} - q_c)/m_0 \end{pmatrix}$$

$$\boldsymbol{c}_0^c = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \tau_0^c \dot{q}_c/m_0 \end{pmatrix} \quad \boldsymbol{c}_1^c = \begin{pmatrix} 0 \\ -\tau_1^c(\dot{q}_c - \dot{q}_{r_1})/m_1 \\ 0 \\ -\tau_1^c(\dot{q}_{r1} - \dot{q}_c)/m_0 \end{pmatrix}$$

It can be verified from the above relations that

$$\boldsymbol{c}_i^k = -\frac{\tau_i^k}{\delta_i^k}\boldsymbol{b}_i^k\;; \quad \boldsymbol{c}_i^c = -\frac{\tau_i^c}{\delta_i^c}\boldsymbol{b}_i^c\;; \quad (i = 0, 1) \quad (33)$$

The control objective is to minimize $\dot{V}(\boldsymbol{x},t)$ for every $(\boldsymbol{x},t)$. The semiactive control signals that result in the minimum of $\dot{V}(\boldsymbol{x},t)$ for $u_i^k(t) \in [-1,1]$ and $u_i^c(t) \in [-1,1]$ are

$$u_i^k = -sgn(\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{b}_i^k) \quad ; \quad u_i^c = -sgn(\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{b}_i^c) \quad (34)$$

Now, rewrite the expression of $\dot{V}(\boldsymbol{x},t)$ into the following form

$$\begin{aligned} \dot{V} = & -\frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{b}_0^k(\delta_0^k u_0^k - \tau_0^k \dot{k}_0) + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{b}_0^c(\delta_0^c u_0^c \\ & -\tau_0^c \dot{c}_0) + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{b}_1^k(\delta_1^k u_1^k - \tau_1^k \dot{k}_1) + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{b}_1^c(\delta_1^c u_1^c \\ & -\tau_1^c \dot{c}_1) + \boldsymbol{x}^T\boldsymbol{P}\boldsymbol{F} \end{aligned} \quad (35)$$

By applying the semiactive control laws in eqn.(34), we can show that

$$\boldsymbol{x}^T \boldsymbol{P}\boldsymbol{b}_0^k(\delta_0^k u_0^k - \tau_0^k \dot{k}_0) < 0 \quad (36)$$
$$\boldsymbol{x}^T \boldsymbol{P}\boldsymbol{b}_1^k(\delta_1^k u_1^k - \tau_1^k \dot{k}_1) < 0 \quad (37)$$
$$\boldsymbol{x}^T \boldsymbol{P}\boldsymbol{b}_0^c(\delta_0^c u_0^c - \tau_0^c \dot{c}_0) < 0 \quad (38)$$
$$\boldsymbol{x}^T \boldsymbol{P}\boldsymbol{b}_1^c(\delta_1^c u_1^c - \tau_1^c \dot{c}_1) < 0 \quad (39)$$

In fact, if $\boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{b}_0^k(\boldsymbol{x}, t) > 0$ for $t \geq t_s$ then $u_0^k(t) = -1$. In this case, we get from eqn.(19) that

$$k_0(t) = -\delta_0^k \left(1 - e^{-(t-t_s)/\tau_0^k}\right) \quad (40)$$

$$\dot{k}_0(t) = -\frac{\delta_0^k}{\tau_0^k} e^{-(t-t_s)/\tau_0^k} \geq -\frac{\delta_0^k}{\tau_0^k} \quad (41)$$

thus the relation eqn.(36) is accomplished. If $\boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{b}_0^k(t) < 0$ that implies $u_0^k(t) = -1$, then we obtain that $k_0(t) \leq \delta_0^k/\tau_0^k$. Therefore, the relation eqn.(36) is also accomplished. The relations eqns.(37)-(39) can be proved in a similar way.
Denote that

$$\begin{aligned}\theta(\boldsymbol{x}) \;=\;& \boldsymbol{x}^T \boldsymbol{P}\boldsymbol{b}_0^k(\delta_0^k u_0^k - \tau_0^k \dot{k}_0) + \boldsymbol{x}^T \boldsymbol{P}\boldsymbol{b}_0^c(\delta_0^c u_0^c - \tau_0^c \dot{c}_0) \\ &+ \boldsymbol{x}^T \boldsymbol{P}\boldsymbol{b}_1^k(\delta_1^k u_1^k - \tau_1^k \dot{k}_1) + \boldsymbol{x}^T \boldsymbol{P}\boldsymbol{b}_1^c(\delta_1^c u_1^c - \tau_1^c \dot{c}_1)\end{aligned}$$

then, $\theta(\boldsymbol{x}) < 0$ and the equation (35) can be rewritten as

$$\dot{V} = -\frac{1}{2}\boldsymbol{x}^T \boldsymbol{Q}\boldsymbol{x} + \theta(\boldsymbol{x}) + \boldsymbol{x}^T \boldsymbol{P}\boldsymbol{F} \quad (42)$$

Since $\boldsymbol{Q}$ and $\boldsymbol{P}$ are positive definite matrices, using (29) we may write

$$\dot{V} \leq -\frac{1}{2}\lambda_{min}(\boldsymbol{Q})||\boldsymbol{x}(t)||^2 + \theta[\boldsymbol{x}(t)] + \lambda_{max}(\boldsymbol{P})F_0||\boldsymbol{x}(t)|| \quad (43)$$

where λ_{min} and λ_{max} represent the minimum and maximum eigenvalue, respectively.

The compact set $\mathcal{K} = \{\boldsymbol{x} \in \boldsymbol{R}^4 \,|\, V(\boldsymbol{x}) \leq \gamma\}$ is a global uniform attractor for the semiactively controlled system (43), where

$$\gamma = \max\{V(\boldsymbol{x}) | \boldsymbol{x} \in \boldsymbol{R}^4, \psi(\boldsymbol{x}) \leq 0\},$$

with

$$\psi(\boldsymbol{x}) = \frac{1}{2}\lambda_{min}(\boldsymbol{Q})\|\boldsymbol{x}\|^2 - \theta(\boldsymbol{x}) - \lambda_{max}(\boldsymbol{P})F_0\|\boldsymbol{x}\| \quad (44)$$

By using the property that

$$\frac{\lambda_{min}(\boldsymbol{P})}{2}\|\boldsymbol{x}\|^2 \leq V(\boldsymbol{x}, t) \leq \frac{\lambda_{max}(\boldsymbol{P})}{2}\|\boldsymbol{x}\|^2 \quad (45)$$

it is easy to find that the set $\mathcal{B} = \{\boldsymbol{x} \in \boldsymbol{R}^4 \,|\, \|\boldsymbol{x}\| \leq \rho\}$, with

$$\rho = \sqrt{\frac{2\gamma}{\lambda_{min}(\boldsymbol{P})}}, \quad (46)$$

is the smallest ball that contains the attractor $\mathcal{K}$. This is called the *ball of ultimate boundedness* in the literature[9]. In control practical terms, this is a ball such that any trajectory entering at certain time T remains there for all $t > T$.

4 Numerical Example

As an application example, a 10-story base isolated building structure is considered in the numerical simulation. The mass of each floor, including that of the base, is 6×10^5 kg. The stiffness of the base is $1.184 \times 10^7 N/m$ and its damping ratio is 0.1. The stiffness of the structure varies in $5 \times 10^7 N/m$ between floors, from $9 \times 10^8 N/m$ the first one to $4.5 \times 10^8 N/m$ the top one with damping ratio 0.05. A frictional device is used for the base isolation, where the nonlinear force f_N is described by the next equation

$$f_N(q_c, \dot{q}_c, d, \dot{d}) = -sgn(\dot{q}_c - \dot{d})[\mu_{max} - \Delta\mu e^{-\nu|\dot{q}_c - \dot{d}|}]G \quad (47)$$

with $G = \sum_{i=1}^{10} m_i$, $\mu = 0.1$, $\nu = 2.0$, $\mu_{max} = 0.185$ and $\Delta\mu = 0.09$. In the simulation, the seismic excitation has been that of the El Centro (1940) earthquake as shown in Figure 3.

The semiactive controller laws in equations (40), (17)-(18) and (21)-(22) have been used with

$$\boldsymbol{P} = \begin{bmatrix} 116120000 & -194.07 & -103520000 & -194.07 \\ -194.07 & 0.69176 & 196.67 & 0.56275 \\ -103520000 & 196.67 & 104270000 & 194.12 \\ -194.07 & 562.75 & 194.12 & 0.56282 \end{bmatrix}$$

The semiactive magnetorheological device is used with $\tau_i^k = \tau_i^c = 1\,ms$, $\delta_0^k = 1.184 \times 10^7\,N/mV$, $\delta_0^c = 2.176 \times 10^5\,Ns/mV$, $\delta_1^c = 9.487 \times 10^5\,Ns/mV$, $\delta_1^k = 9.0 \times 10^8\,N/mV$, $P_{a0}^c = P_{a0}^k = 2.176 \times 10^4\,m^2$, $P_{a1}^c = P_{a1}^k = 9.487 \times 10^4\,m^2$. Both passive case (pure base isolation) and hybrid case (base isolation plus semi-active control) are studied. The time history of the absolute displacement of the base is shown in Figure 4. The interstory response between the 9th and 10th floors and the absolute acceleration of the top floor are shown in the figures 5 and 6, respectively.

It is observed that the semiactive controller that takes into account the actuator dynamic is effective and improves the structural performance as compared with the purely passive controlled case. It is seen from Figure 4 that the semiactive controllers reduce the peak response of absolute displacements of the base from a margin of $\pm 5.5cm$ (a reduction about 42.7%) so as to maintain the base isolator working in its elastic region. It is also observed from Figures 5 and 6 that the good dynamic performances achieved by the base isolator have been kept in the semiactive controlled structure.

5 Conclusions

As a novelty in the design of semiactive controllers, the control scheme proposed in this paper has taken into account the actuator dynamics so that controlled structural performance has better approximation to the real operation conditions such as the effects of time delay and frictional forces. It has been shown that the ultimate boundedness is achieved in the semiactive controlled structures subject to unknown seismic excitation. The numerical simulation has illustrated the effectiveness of the semiactive controller for a 10-story frictional base isolated structure. The peak response of the absolute movements of the base and main structure has been significantly reduced as compared with the purely passive controlled case.

6 Acknowledgements

This work has been funded by the Commission of Science and Technology of Spain (CICYT) under the Project DPI2002-04018.

References

[1] G.W. Housner, L.A. Bergman,T.K. Caughey, et al., Structural control: past, present and future, Journal of Engineering Mechanics, ASCE, 123, 897-971, 1997.

[2] J. Rodellar, G. Leitmann and E.P. Ryan, Output feedback control of uncertain coupled systems, International Journal of Control, 58, 2, 445-457, 1993.

[3] A.H. Barbat, J. Rodellar, E.P. Ryan and N. Molinares, Active control of nonlinear base-isolated buildings, Journal of Engineering Mechanics, ASCE, 121, 676-684, 1995.

[4] N. Luo, J. Rodellar and M. De la Sen, Composite robust active control of seismically excited structures with actuator dynamics, Earthquake Engineering and Structural Dynamics, 27, 301-311, 1998.

[5] N. Luo, J. Rodellar, M. De la Sen and J. Vehí, Output feedback sliding mode control of base isolated structures, Journal of The Franklin Institute, 337, 5, 555-577, 2000.

[6] G. Leitmann and E. Reithmeier, Semiactive control of a vibrating system by means of electrorheological fluids, Dynamics and Control, 3, 7-33, 1993.

[7] N. Luo, J. Rodellar, J. Vehí, M. De la Sen, Composite Semiactive control of a class of seismically excited structures, Special Issue on Control of Structural and Mechanical System, Journal of The Franklin Institute, 338, 3, 225-240, 2001.

[8] A.H. Barbat and L.M. Bozzo, Seismic analysis of base isolated buildings, Arch. Computational Methods in Engineering, 4, 153-192, 1997.

[9] M. Corless and G. Leitmann, Continuous state feedback guaranteeing uniform ultimate boundedness for uncertain dynamic systems, IEEE Transactions on Automatic Control, AC-26, 1139-1144, 1981 .

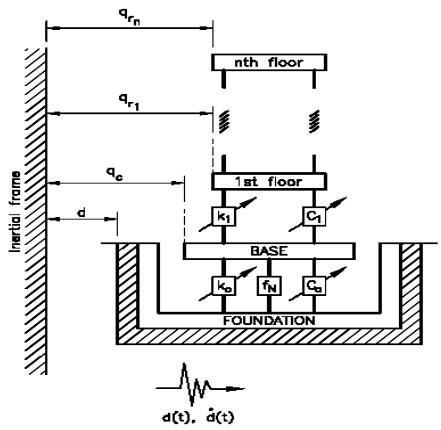

Figure 1: Base isolated structure with semiactive components

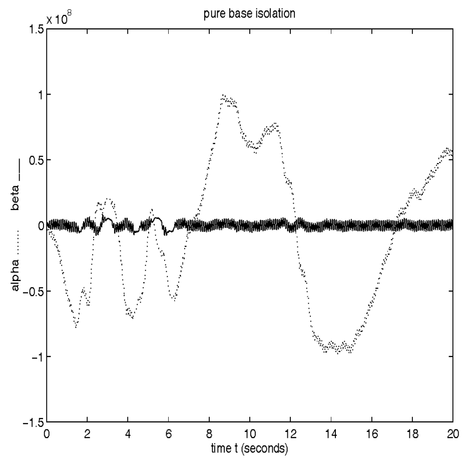

Figure 2: Dynamics of α and β

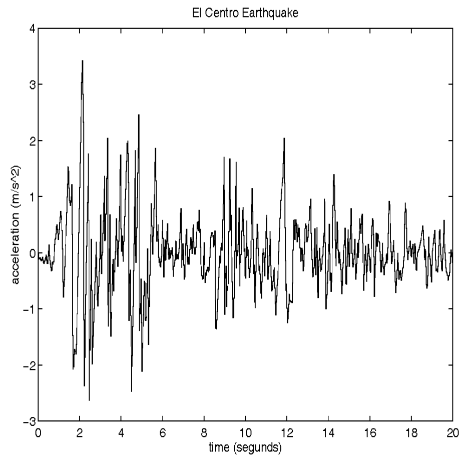

Figure 3: Accelerogram of *El Centro* Earthquake

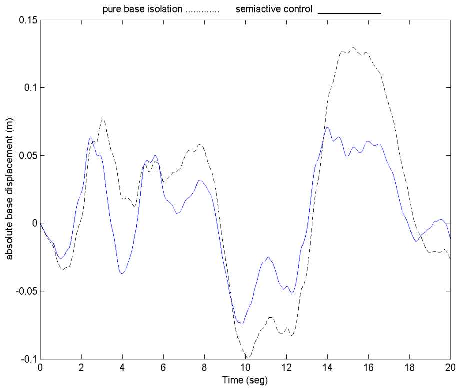

Figure 4: Absolute base displacement

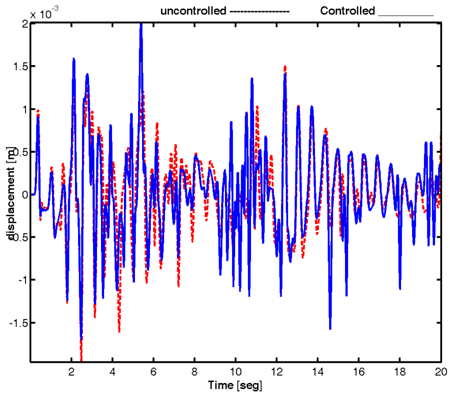

Figure 5: Relative displacement between 9th and 10th floors

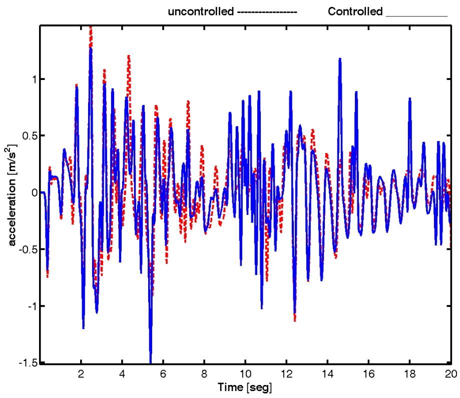

Figure 6: Absolute acceleration of the 10th floor