NEUAL STATE SPACE MODEL BASED APPROXIMATION POLE ASSIGNMENT CONTROL FOR A CLASS OF UNKNOWN NONLINEAR SYSTEMS

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Abstract

In this paper, an extended linearized neural state space (ELNSS) model is proposed and used to design an approximate pole assignment control strategy for a class of nonlinear systems. At first, the applicability of the ELNSS model to approximate affine nonlinear systems is studied, where the extended Kalman filter (EKF) algorithm is employed to train the weights of the ELNSS model. It has been shown that such a training algorithm can guarantee the convergence of the network weights. Using the trained weights in the ELNSS model, the design of an approximate pole assignment controller is performed using a state feedback framework. The convergence of the approximate pole assignment adaptive control algorithm is also analyzed.

1. Introduction

It is well known that in the neural network based controller design ([1]-[3]), the primary task is to construct a neural network of a suitable type and architecture. Once this is completed, the controller design and system analysis, such as the convergence and stability of the closed loop system, can be readily carried out. In this context, a number of neural network based control structures have been developed. Examples are feedforward neural network control schemes ([1]-[3], [16]) and radial basis function based modelling and control methods ([14]-[16]). In 1995, a neural state space model for the identification and robust control design of nonlinear systems was proposed [1]. From then on, many researchers have adopted this neural state space structure to model and analyze the system, and to design various controllers for nonlinear systems [2~4]. Despite many reported successful applications of the neural state space model to nonlinear system identification and control, the actual procedure of designing a controller using the neural state space models is similar to the formulations of other neural networks based controllers and in general complicated procedures are involved.

In this paper, an extended linearized neural state space model is proposed. The ELNSS model is of a controllable canonical form which simplifies the required controller design. Moreover, the weights in the ELNSS model are composed of multi-layers neurons which enables the ELNSS model to effectively approximate general unknown nonlinear functions. The questions to be asked here is whether this new type of neural state space model ([1]) can be used for the identification of nonlinear systems in the same way as that of a multiplayer feedforward neural network (MFNN). In this paper, the feasibility of using ELNSS to model affine nonlinear systems is investigated. Indeed, the weights training for the ELNSS model employs the well-known extended Kalman filter (EKF) algorithm. This procedure is the same as the situation when the EKF method is applied to the training of MFNNs. It has been shown that the convergence of the training algorithm can be guaranteed if the associated learning rate is correctly chosen.

It is well known that the pole assignment method is an effective and simple scheme in linear control system design, where the controller can be obtained so as to guarantee the stability of the closed loop system. For nonlinear systems, several pole assignment control methods have been developed through the concept of closed loop linearizations [5~9]. Since the ELNSS model exhibits a quasi-linear character ([1]), the pole assignment controller design can be also applied to the ELNSS models. In this paper, a nonlinear approximate pole assignment control algorithm is proposed using an ELNSS model and the convergence of the closed loop system is analyzed.

2. Modelling using an ELNSS model

Consider the following discrete nonlinear dynamic system

$$\begin{aligned} x(k+1) &= g(x(k),u(k)), \qquad x(0)=x_0 \\ y(k) &= h(x(k),u(k)) \end{aligned} \tag{1}$$

where $x(k)\in R^n$ is the state vector, $u(k)\in R^m$ is the input vector and $y(k)\in R^r$ is the output vector.

This is a general nonlinear expression for nonlinear systems whose direct control formulation is difficult to perform. To overcome this difficulty, the following extended linearization model of nonlinear system was proposed in [12].

$$\begin{aligned} x(k+1) &= A(x(k)) + B(x(k))u(k) \\ y(k) &= C(k)x(k) \end{aligned} \tag{2}$$

where $A(x(k))$ and $B(x(k))$ are general nonlinear functions of the state $x(k)$ at discrete sample time k. For system (2), an extended linearized neural state space model is established, which is in fact a recurrent network model of the following form

$$\begin{aligned} X(k+1) &= A(X(k))X(k) + B(X(k))U(k) \\ Y(k) &= CX(k) \\ A(X(k)) &= \sigma(X(k),W_a) \\ B(X(k)) &= \sigma(X(k),W_b) \\ \sigma(x) &= \frac{1-exp(x)}{1+exp(x)} \end{aligned} \tag{3}$$

where $X(k)$, $Y(k)$ and $U(k)$ are state, output and input, respectively, and $\sigma(\bullet)$ is a sigmoid function. Consider an SISO nonlinear system and suppose that matrix A is of a controllable companion canonical form

$$A(\bullet)=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 1 \\ a_1(k) & \cdots & \cdots & \cdots & a_n(k) \end{bmatrix} \quad B(\bullet)=\begin{bmatrix} b_1(k) \\ \vdots \\ \vdots \\ b_n(k) \end{bmatrix} \tag{4}$$

$$C=[c_1,\cdots,\cdots,c_n]$$

where $\{ a_i(k), b_i(k) \}$ are nonlinear functions of the state vector X with the following neural networks form

$$a_i(k), b_i(k) = \sigma(\sum_{j=1}^{n} W_{ij}X + \xi_i) \tag{5}$$

and W_{ij} are the weights in the ELNSS model, and ξ_i is the threshold. The advantage of the ELNSS model is that each element of A and B is the nonlinear combination of the state vector with the form that is similar to a neural network. As a result, the well-established linear system approaches can be readily extended to the controller design for the nonlinear systems represented by ELNSS models.

It is well known that the nonlinear discrete systems described by (1) can be approximated by the dynamic systems constituted by neural networks with arbitrary accuracy. Since the ELNSS model is a special recurrent neural network, the feasibility of using ELNSS model for the identification of nonlinear discrete systems can be proved similarly. In the rest of this paper, we consider the following affine unknown nonlinear system

$$\begin{aligned} y_{k+1} = {} & f(y_k,\cdots y_{k-n+1}, u_{k-1},\cdots u_{k-m}) \\ & + g(y_k,\cdots y_{k-n+1}, u_{k-1},\cdots u_{k-m})u_k \end{aligned} \tag{6}$$

where the purpose of system identification is to use the measured inputs and the outputs to estimate (i.e., to model) the unknown nonlinear functions $f(\bullet)$ and $g(\bullet)$. By choosing the state as follows

$$x_i(k) = y_{k-n+i}, \quad x_{n+j}(k) = u_{k-m+j}, \quad (i = 1,\cdots n, \quad j = 1,\cdots m)$$

then equation (6) can be described in the state space form to give

$$\begin{cases} x_1(k+1) = x_2(k) \\ \quad \vdots \\ x_n(k+1) = f(x(k)) + g(x(k))u_k \\ x_{n+1}(k+1) = x_{n+2}(k+1) \\ \quad \vdots \\ x_{n+m}(k+1) = u_k \\ y(k) = x_n(k) \end{cases} \tag{7}$$

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When BP feedforward neural networks with more than one hidden layers are adpoted to model unknown nonlinear functions $f(\bullet)$ and $g(\bullet)$, respectively, there exist weight matrices W^* and V^* such that

$$\begin{aligned} f(x(k)) &\doteq \hat{f}(x(k),W^*) = \\ &= \sum_{i=1}^{p} w_i^* H(\sum_{j=1}^{n+m} w_{ij}^* x_j + w_{i0}^*) \\ g(x(k)) &\doteq \hat{g}(x(k), V^*) = \\ &= \sum_{i=1}^{q} v_i^* H(\sum_{j=1}^{n+m} v_{ij}^* x_j + v_{i0}^*) \end{aligned} \tag{8}$$

where W^* and V^* are constant vectors constitued by w_i^*, w_{ij}^*, v_i^* and v_{ij}^*, respectively, and $H(\bullet)$ is a sigmoid function. Thus, the system described by (6) can be represented as

$$y_{k+1} = \hat{f}(x(k),W^*) + \hat{g}(x(k),V^*)u_k \tag{9}$$

When ELNSS model is adopted to estimate the unknown $f(\bullet)$ and $g(\bullet)$, it can be rewritten as

$$\begin{cases} x_1(k+1) = x_2(k) \\ \vdots \\ x_n(k+1) = \tilde{f}(x(k)) + \tilde{g}(x(k))u_k \\ = f_1(x(k))x_1(k) + \cdots + f_n(x(k))x_n(k) + \\ \quad + \tilde{g}(x(k))u_k \\ x_{n+1}(k+1) = x_{n+2}(k+1) \\ \vdots \\ x_{n+m}(k+1) = u_k \\ \hat{y}(k) = x_n(k) \end{cases} \qquad (10)$$

where $f_i(x(k))$ and $\tilde{g}(x(k))$ can be represented as $\hat{f}(x(k), W^*)$ and $\hat{g}(x(k), V^*)$ in (9), respectively. $\tilde{f}(x(k))$ can be considered as the output of a new neural network whose output of hidden layer is $f_i(x(k))$, and output of the output layer is $x_i(k)$. As such, there exists a weight W^{**} vector that makes $\tilde{f}(x(k), W^{**})$ be equivalent to $\hat{f}(x(k), W^*)$. Since the neural network described by (9) can identify the nonlinear systems represented by (6), the ElNSS model can be also used for the identification of the nonlinear systems described by equation (6). Similar to other neural networks, the ELNSS model can be trained by using several training methods, such as the well known back propagation algorithms, the conjugate gradient method [10], Levenberg-Marquardt optimisation algorithm [11] and methods based on genetic algorithms [12]. In this study, the extended Kalman filter (EKF) algorithm [16] is adopted to train the ELNSS model so as to ensure fast convergence. It should be noticed that the learning rate in EKF could be made adaptive. Therefore the problem of a properly tuning of the learning rate can be avoided. The principle of EKF is as follows:

Considering the nonlinear system described by (1) that is modeled by an ELNNS model. Denote the parameters of the ELNNS model as θ, and then the nonlinear state space equation of the ELNNS model can be rewritten as

$$\theta(k+1)=\theta(k) \qquad (11)$$

$$y_m(k)=f(\theta(k))+v(k) \qquad (12)$$

where $v(k)$ is a noise. With the EKF training algorithm, the ELNSS model parameters are updated as follows:

$$\theta(k) = \theta(k-1) + \eta K(k)[y(k) - y_m(k)] \qquad (13)$$

$$K(k) = P(k)H(k)[H^T(k)P(k)H(k) + R(k)]^{-1} \qquad (14)$$

$$P(k+1) = P(k) - K(k)H^T(k)P(k) \qquad (15)$$

where $K(k)$ is the Kalman gain matrix, $P(k)$ is the estimated error covariance matrix, and $R(k)$ is the estimated covariance matrix of noise $v(k)$. For SISO systems $R(k) = r(k)$ is an estimate of covariance of noise $v(t)$. In this context, it can be further obtained by the following recursive calculation

$$r(k) = r(k-1) + [(y(k) - y_m(k)) - r(k-1)]/k \qquad (16)$$

$$\theta = [c_i(k) \quad W_{aij}(k) \quad W_{bij}(k)]^T \qquad (17)$$

$$P_{ij}(k) = \frac{\partial y(k)}{\partial W_{aij}} \qquad Q_{ij}(k) = \frac{\partial y(k)}{\partial W_{bij}} \qquad (18)$$

$$\begin{aligned} P_{ij}(k) &= \sigma'(a_i(k-1))\hat{x}_i(k-1)\hat{x}_j(k-1) \\ &+ a_n(k+1)P_{ij}(k-1) \end{aligned} \qquad (19)$$

$$\begin{aligned} Q_{ij}(k) &= c_i\sigma'(b_i(k-1))\hat{x}_j(k-1)u(k-1) \\ &+ c_n a_n(k-1)\sigma'(b_i(k-2))\hat{x}_j(k-2)u(k-2) \end{aligned} \qquad (20)$$

where $H(k)$ is a Jacoobian matrix of output $y(k)$ with respect to parameter vector $\theta(k)$ at time k. This means that $H(k)$ can be calculated from

$$H(k) = \left[\frac{\partial y(k)}{\partial \theta}\right]_{\theta = \theta(k-1)} = \begin{cases} \hat{x}_j(k-1) & if\ \theta_j = c_j \\ c_n P_{ij}(k) & if\ \theta_j = W_{aij} \\ Q_{ij}(k) & if\ \theta_j = W_{bij} \end{cases} \qquad (21)$$

With respect to the convergence of learning process, the following theorem can be established.

***Theorem 1*:** The convergence of the ELNSS model (11)-(12) can be guaranteed if η in (13) is chosen as

$$0 < \eta < \frac{2}{g_{\max}} \qquad (22)$$

$$\begin{aligned} g_{\max} &= \max|g_m(k)| = \\ &= \max_t \left|H^T(k)P(k)H(k)A(t)\right| \end{aligned} \qquad (23)$$

$$A(k) = [R(k) + H^T(k)P(k)H(k)]^{-1} \qquad (24)$$

and the optimal η is given by

$$\eta^* = 1/g_{\max} \qquad (25)$$

Proof: Denote $e(k) = y(k) - y_m(k)$, then one can choose the following Lyapunov function

$$V(k) = \frac{1}{2}e^T(k)e(k) \qquad (26)$$

Assuming the modelling error is approximated as

$$\Delta e(k) = [\frac{\partial e(k)}{\partial \theta}]^T \Delta \theta$$
$$= -H^T(k)P(k)H(k)A(k)e(k) \quad (27)$$
$$= -\eta(k)g_m(k)e(k)$$

it can be shown that

$$\Delta V(k) = \Delta e(k)[e(k) + \tfrac{1}{2}\Delta e(k)]$$
$$= -\eta(k)g_m(k)[1 - 1/2\eta(k)g_m(k)]e^2(k) \quad (28)$$

If condition (22) is satisfied, then from equation (28) it can be seen that $\Delta V(k) < 0$.

3. Nonlinear system approximate

Because the ELNSS model can approximate a class of nonlinear systems and represent pseudo-linearity, control design approaches, such as model reference adaptive control and pole assignment method, can be applied to the nonlinear system modelled by the ELNSS scheme. In this section, a new nonlinear pole assignment control method is introduced. Different from other nonlinear pole assignment method through output feedback linearizations, the purpose of the new nonlinear pole assignment method is to ensure that the closed loop state space equation of the ELNSS model has the same set of poles as the desired linear system controlled by a state feedback scheme. The procedures of controller design are as follows:

Step 1: An ELNSS model with following form is used for the modelling of a nonlinear system.

$$A(\bullet) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 1 \\ a_1(k) & \cdots & \cdots & \cdots & a_n(k) \end{bmatrix}$$

$$B(\bullet) = [0, \cdots 0, b(k)]^T \qquad C = [0 \ \cdots \ 0 \ \ 1] \qquad (29)$$

If $b(k) \neq 0$ and not all $a_i(k)$ are zero, system $\{A, B\}$ is completely controllable at each sample time. From the feasibility proof of using the ELNSS model for the identification of affine nonlinear systems in section 2, it can be seen that there exist model parameters $\{W_a^*, W_b^*\}$, which make the ELNSS model be equivalent to the identified nonlinear system. In this case, A^* is constituted by $a_i^*(k)$, and B^* is constituted by $b^*(k)$ and $a_i^*(k)$ and $b^*(k)$ are nonlinear functions constituted by $\{W_a^*(k), \mathbf{x}^*(k)\}$ and $\{W_b^*(k), \mathbf{x}^*(k)\}$, respectively.

Step 2: Define a state feedback gain matrix as K, which is a vector in SISO system.

$$K = [k_1(X(k)), k_2(X(k)), \cdots\cdots k_n(X(k))]$$

then it can be seen that each element in K vector is a function of state X and weight W.

Step 3: The output of the controller is as follows

$$u(k) = -KX(k) + r(k) \qquad (30)$$

which leads to the following ELNSS model based closed loop system

$$X(k+1) = \tilde{A}X(k) + \tilde{B}r(k); \quad \tilde{B} = B^* \qquad (31)$$

$$\tilde{A} = A^* - B^*K$$
$$= \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots\cdots & 1 \\ a_1^* - b^*k_1 & a_2^* - b^*k_2 & \cdots\cdots & a_n^* - b^*k_n \end{bmatrix} \qquad (32)$$

Step 4: Set the desired pole vector of the closed loop system as $\hat{P} = [\lambda_1, \lambda_2, \cdots\cdots \lambda_n]$ then the expected linear system state space model is given by

$$X(k+1) = \Phi X(k) + \Gamma r(k) \qquad (33)$$

$$\Phi = \begin{bmatrix} 0 & 1 & 0 \cdots & 0 \\ 0 & 0 & 1 \cdots & 0 \\ \vdots & \vdots & \vdots & 1 \\ \lambda_1 & \lambda_2 \cdots & & \lambda_n \end{bmatrix} \qquad \Gamma = B^* \qquad (34)$$

To design the controller, it requires that $\tilde{A} = \Phi$. This means that

$$k_i(k) = b^*(k)^{-1}(a_i^*(k) - \lambda_i) \qquad (i = 1 \sim n) \qquad (35)$$

$$a_i^*(k) = \sum_{j=1}^{n}(w_{aj}^* x_j^* + w_{a0}^*); \quad b_i^*(k) = \sum_{j=1}^{n}(w_{bj}^* x_j^* + w_{b0}^*)$$

In this context, the ELNSS model based closed loop system is made completely equal to the linear system with expected eigenvalues. Therefore, the dynamic performance of nonlinear system can be determined by altering the poles of expected linear system (33). In terms of stability, since the ELNSS model is an extended linearization of the identified nonlinear system, the ELNSS model based closed loop system is stable. This indicates that the nonlinear system represented by ELNSS model is stable if all the poles are within the unit circle. As for a time-variant nonlinear system, a nonlinear pole assignment adaptive algorithm can be similarly developed. In the adaptive framework, the parameters $\{W_a(k), W_b(k)\}$ in the ELNSS model are learnt online. When the output of ELNSS model tracks the output of real system with satisfactory accuracy, the ELNSS model is the instantaneous linearization model of the real system ([19]). The state feedback gain vector at each sample time k can be calculated by using $W_a(k)$, $W_b(k)$ and equations (31)~(35) as follows.

$$k_i(k) = b(k)^{-1}(a_i(k) - \lambda_i) \qquad (i = 1 \sim n) \qquad (36)$$

$$a_i(k) = \sum_{j=1}^{n}(w_{aj}x_j + w_{a0})$$

$$b_i(k) = \sum_{j=1}^{n}(w_{bj}x_j + w_{b0})$$

The resulting controller can therefore be obtained by substituting (36) into (30). In terms of the convergence for the adaptive controller given by equation (36), the following theorem can be established:

Theorem 2**:** If an ELNSS model is trained by EKF method and condition (22) is satisfied, then the adaptive ELNSS model based approximate pole assignment control algorithm (36) is convergent.

Proof**:** Let an ELNSS model to be the identification model of the nonlinear system, then the parameters $\{W_a(k), W_b(k)\}$ in the ELNSS model at each sample time can be obtained by the EKF training method, where the state feedback vector at each sample time is calculated using equation (36).

Following the proof of theorem 1, it can be concluded that there exist parameters $\{W_a^*, W_b^*\}$ and state vector x^*, such that the ELNSS model $\{A^*, B^*\}$ can be made equivalent to the identified nonlinear system. By choosing the same Lyapunov function as (26) and following the convergence proof of the training process in theorem 1, it can be seen that condition (22) also guarantees.

$$W_a(k) \to W_a^*, \quad W_b(k) \to W_b^*, \quad x \to x^* \qquad (37)$$

thus

$$a_i(k) \to a_i^*, b(k) \to b^* \qquad (38)$$

Substituting equations (37) and (38) into equation (36), it can be further obtained that

$$\begin{aligned}&\lim_{k\to+\infty}[k_i(k) - k_i^*(k)] = \\ &= \lim_{k\to+\infty}[b(k)^{-1}(a_i(k) - \lambda_i^*(k)) - b^*(k)^{-1}(a_i^*(k) - \lambda_i^*(k))] \\ &= 0\end{aligned} \qquad (39)$$

and in terms of the pole assignment, we have

$$\begin{aligned}&\lim_{k\to+\infty}[\lambda_i(k) - \lambda_i^*(k)] \\ &= \lim_{k\to+\infty}[(a_i(k) - b(k)k_i) - (a_i^*(k) - b^*(k)k_i^*(k))] \\ &= 0\end{aligned} \qquad (40)$$

Equation (40) indicates that the poles of the closed loop system is convergent to those of the desired linear system. The stability of desired closed loop system is therefore guaranteed when its poles are placed inside the unit circle. As such, the approximate pole assignment adaptive control algorithm (36) is convergent.

4. An example

Consider an affine nonlinear system of the following form:

$$y(k) = \frac{y(k-1)y(k-2)}{1 + y(k-1)^2 + y(k-2)^2} + u(k-1) \qquad (41)$$

The architecture of the ELNSS model is selected whose input is given by $I(t)=[x_1(t), x_2(t), x_3(t), u(t-1)]^T$. Suppose that the initial EKF identification parameters are $P(0)=10^5*I$, $R(0)=2*10^{-5}$, then after 18 training epochs, the mean squared error between the output of the ELNSS model and the system (41) is $2.69*10^{-6}$. The weights of the ELNSS model are therefore obtained. The training data is input again to test the performance of identification, where the mean squared error between the output of ELNSS model and the output of the system (41) is $3.72*10^{-5}$.

Using the established ELNSS model, an approximate pole assignment controller is designed according to (30) and (35) with the poles of the desired closed loop linear system being as follows:

$$p = [0.8\times10^{-3}, 0.8\times10^{-3}, 0.8\times10^{-5}]$$

The reference trajectory is described by the following equation

$$\begin{aligned}r_2(k) = 0.1\sin(2\pi k/100) + \\ 0.1\sin(2\pi k/150) + 0.2\end{aligned} \qquad (42)$$

The simulated closed loop response is shown in Figure 1, where the mean squared of tracking error is $6.72*10^{-5}$.

5. Conclusions

In this paper, an extended linearized neural state space model is developed and used to model a class of nonlinear dynamic systems. Since the ELNSS model exhibits pseudo-linearity nature, most linear system controller design approaches can be applied to the ELNSS model represented systems. In particular, the ELNSS model is an approximation of Volterra basis neural networks in some cases. Therefore, the ELNSS model is suitable to model the system with polynomial characteristics. In the paper the extended Kalman filtering based learning algorithm ([12]) is adopted to estimate the parameters of the ELNSS model. The applicability of using ELNSS to model a class of nonlinear systems has been proved. The nonlinear system approximate pole assignment control approach was proposed using the ELNSS model. The convergence of corresponding adaptive algorithm was investigated. In fact, the closed loop system stability is guaranteed by choosing a suitable set of poles of an expected linear system.

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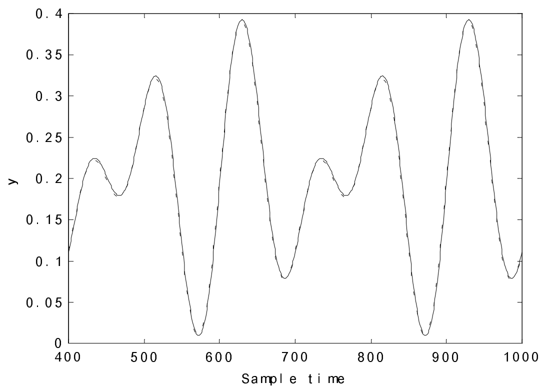

Fig 1. Closed loop system response of example 1.