COLLABORATION BETWEEN BRAKING TORQUES AND ACTIVE SUSPENSION FORCES TO CONTROL A VEHICLE

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Abstract

In previous papers [2, 3], we studied the problem of making a vehicle brake in a corner with stability, by elaborating suitable independent braking torques for the four wheels.

In addition to the previous problem, we are now also interested in using active suspension forces to regulate the vertical dynamics of the vehicle. Then we show how the braking torques and the suspension forces can collaborate to control the global car's dynamics.

1 Introduction

We consider the problem for a car to track given yaw rate and longitudinal acceleration reference trajectories, while regulating to zero the pitch, roll and vertical velocities by using both braking torques and active suspension forces. The tyres, and therefore the car, are subject to interaction forces with the ground, modeled by Pacejka formulas (see [8]).

We suppose that the car is equipped with an electronic system producing independent braking torques on the four wheels and with active suspensions.

In **section 2** we recall how we can control the horizontal dynamics (namely the yaw rate and the longitudinal acceleration), using the braking torques which are elaborated from a nonlinear constrained optimization problem, combined with singular perturbation theory (see e.g. [2, 3]). We use the fact that the wheels' dynamics is much faster than the cars' one and therefore, that the longitudinal slip ratios at each wheel can be considered as intermediate control variables. The desired longitudinal slip ratios are then "realized" by the braking torques applied to the wheels which are the physical control variables. This leads to a two levels hierarchical structure for the braking control.

In **section 3** we are concerned with the vertical dynamics and more precisely with the problem of stabilizing to the origin the pitch, roll and vertical velocities, while minimizing the vertical acceleration, by means of the suspension forces.

For that purpose, we use again a time-scale decomposition property, noticing that the non suspended masses dynamics is faster than the cars' one. The vertical forces applied to the wheels can then be seen as intermediate control variables, linked to the physical active suspension forces through an algebraic relation. This leads once more to a two levels structure for the suspension control.

Then, in **section 4**, we combine the braking torques and the active suspension forces. Using results from singular perturbation theory, we study the stability of the global closed-loop dynamics of the vehicle.

Finally, the validity of our approach is illustrated by some simulation results in **section 5** and we conclude.

2 Control of the horizontal dynamics using the braking torques

2.1 The horizontal control model

To develop our control strategy for the elaboration of the braking torques, we have begun to consider a simple plane horizontal model describing the car (see Figure 1), given by the following mechanical equations:

$$
\begin{array}{l}
M(\dot{V}_x - \dot{\psi}\, V_y) = M\gamma_x = \\
\quad \cos\alpha_v\,(F_{x1}+F_{x2}) + \cos\alpha_r\,(F_{x3}+F_{x4}) \\
\quad -\sin\alpha_v\,(F_{y_1}+F_{y_2}) - \sin\alpha_r\,(F_{y_3}+F_{y_4}) \\
M(\dot{V}_y + \dot{\psi}\, V_x) = M\gamma_y = \\
\quad \sin\alpha_v\,(F_{x1}+F_{x2}) + \sin\alpha_r\,(F_{x3}+F_{x4}) \\
\quad +\cos\alpha_v\,(F_{y_1}+F_{y_2}) + \cos\alpha_r\,(F_{y_3}+F_{y_4}) \\
I_z\ddot{\psi} = L_1\,(\sin\alpha_v\,(F_{x1}+F_{x2}) + \cos\alpha_v\,(F_{y_1}+F_{y_2})) \\
\quad +l\,(\cos\alpha_v\,(F_{x2}-F_{x1}) + \sin\alpha_v\,(F_{y_1}-F_{y_2})) \\
\quad -L_2\,(\sin\alpha_r\,(F_{x3}+F_{x4}) + \cos\alpha_r\,(F_{y_3}+F_{y_4})) \\
\quad +l\,(\cos\alpha_r\,(F_{x4}-F_{x3}) + \sin\alpha_r\,(F_{y_3}-F_{y_4})) \\
I_r\dot{\omega}_i = -RF_{xi} + C_{mi} - C_{f_i}\,,\ i = 1, 4.
\end{array}
\tag{1}
$$

where $\dot{\psi}$ denotes the yaw rate, V_x (respectively V_y) the longitudinal (resp. lateral) velocity of the center of gravity of the car, γ_x its longitudinal acceleration, γ_y its lateral acceleration,

M the total mass of the vehicle, I_z its vertical inertia, L_1 the part of the wheelbase in front of the center of gravity, L_2 the other part of the wheelbase, l the half gauge, α_v (resp. α_r) the front (resp. rear) steering angle, F_{xi} (resp. F_{y_i}) the longitudinal (resp. lateral) component of the interaction force between wheel i and the ground, ω_i the rotation velocity of wheel i, R the wheel radius, I_r the wheel inertia, C_{mi} (resp. C_{f_i}) the motor torque (resp. braking torque) applied to wheel i.

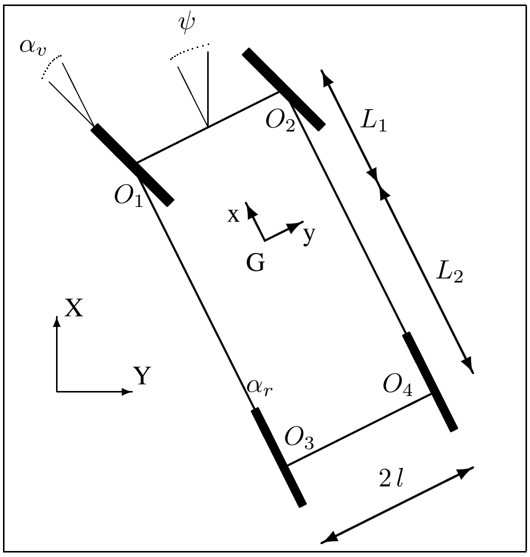

Figure 1: The horizontal plane model of the car

Remark 1 *We can consider that the constant rear steering angle α_r is equal to zero.*

Remark 2 *F_{xi} and F_{y_i} are described by the Pacejka formulas, depending nonlinearly and in a coupled way on the longitudinal slip ratio τ_i, on the slip angle δ_i (angle between the wheel plane and its velocity) and on the vertical load applied to wheel i (see e.g. [8]).*

The longitudinal slip ratio τ_i is given as follows:

$$\tau_i = \frac{R\omega_i - V_{xi}}{V_{xi}}, \tag{2}$$

V_{xi} being the longitudinal component of the velocity of the center O_i of wheel i (resp. V_{y_i} its lateral component). For example, we have for wheel 1:

$$\left\{ \begin{array}{l} V_{x1} = (V_x - l\dot{\psi}) \cos\alpha_v + (V_y + L_1\dot{\psi}) \sin\alpha_v \\ V_{y_1} = -(V_x - l\dot{\psi}) \sin\alpha_v + (V_y + L_1\dot{\psi}) \cos\alpha_v. \end{array} \right. \tag{3}$$

For wheel 2 we replace l by $-l$, for wheel 3 we replace L_1 by $-L_2$ and α_v by α_r and to pass from wheel 2 to wheel 4 we replace L_1 by $-L_2$ and α_v by α_r.

In equations (1), the car dynamics can be viewed as a **slow** subsystem where the control variables are F_{xi} and F_{y_i}, themselves depending on τ_i and consequently on ω_i.

Then, system (1) has a "cascaded" structure since, if we consider the τ_i or equivalently the ω_i as control variables for the car, they are related to the true physical control variables $(C_{mi} - C_{f_i})$ through a linear 1st order differential equation, i.e. the **fast** subsystem given by the wheels dynamics (see e.g. [4, 7, 10, 11] for this notion of cascaded system). This two-levels control structure or in "singular perturbation form" (see e.g. [6]) is reasonable since the car mass $M = 1800\,kg$ and the car inertia $I_z = 3000\,kg.m^2$ are quite greater than the wheels inertia $I_r = 1.2\,kg.m^2$. This structure has been successfully used to elaborate our control strategy as brie¤y recalled in the next subsection.

But before, let us express our **control problem**: general yaw rate and longitudinal acceleration reference trajectories being given, can we determine braking torques to make the yaw rate and the longitudinal acceleration of the vehicle follow these prescribed reference trajectories $\dot{\psi}_{ref}(t)$ and $\gamma_{xref}(t)$?

2.2 Elaboration of the "two-levels" control strategy

High level controls First we de£ne high level controls u_ψ and u_x ensuring a tracking of yaw rate and longitudinal acceleration references. Considering the longitudinal dynamics in (1), we simply take, in a 1st step:

$$u_x = M\gamma_x = M\gamma_{xref}, \tag{4}$$

$\gamma_{xref}(t)$ being the longitudinal reference acceleration.

To track the yaw rate reference, we consider the yaw rate dynamics in (1) and we take:

$$u_\psi = I_z\ddot{\psi} = I_z\ddot{\psi}_{ref} - \alpha_\psi I_z(\dot{\psi} - \dot{\psi}_{ref})\ ,\ \alpha_\psi > 0. \tag{5}$$

We will elaborate suitable interaction forces F_{xi} and F_{y_i} by means of suitable longitudinal slip ratios τ_i, to satisfy (4) and (5).

Then, using the fast wheels dynamics, the expected τ_i will be realized by means of suitable braking torques as explained in the next subsection.

For the moment, let us explain the method we have developed to compute the longitudinal slip ratios τ_i satisfying (4) and (5).

In the sequel, we will denote f_i (resp. g_i) the contribution of F_{xi} and F_{y_i} applied to wheel i in the expression (4) of u_x (resp. (5) of u_ψ). For example, using (1) we easily obtain:

$$f_1 = F_{x1}\cos\alpha_v - F_{y_1}\sin\alpha_v$$

and

$$g_1 = F_{x1}(L_1\sin\alpha_v - l\cos\alpha_v) + F_{y_1}(L_1\cos\alpha_v + l\sin\alpha_v).$$

The reader can obviously compute the other expressions of f_i and g_i.

In fact, four intermediate control variables τ_i, $i = 1, \cdots, 4$ are available to realize the two constraints (4) and (5). Therefore, this is an under-determined system which has been solved by considering the following nonlinear constrained optimization problem:

$$\min_{\tau_i} \mathcal{F} \text{ with } \mathcal{F} = \frac{1}{2}\Sigma_{i=1}^{4}\tau_i^2, \tag{6}$$

under the constraints (4) and (5), which can respectively be written:

$$f_1 + f_2 + f_3 + f_4 = u_x \text{ and} \tag{7}$$

$$g_1 + g_2 + g_3 + g_4 = u_\psi. \tag{8}$$

In fact, equation (7) can be splitted in the following two constraints:

$$f_1 + f_2 = pu_x \text{ and} \tag{9}$$

$$f_3 + f_4 = (1-p)u_x \text{ with } 0 \le p \le 1, \tag{10}$$

p being a parameter characterizing the brake power distribution front/rear. In the simulations presented at section 5, p is equal to 0.5.

The τ_i, $i = 1, \cdots, 4$, solutions of the minimization problem (6) under the constraints (9), (10) and (8) will be denoted τ_{di} in the sequel.

Remark 3 *The interest of minimizing function $\mathcal{F}$, is that the resulting longitudinal slip ratios τ_{di} are as small as possible and this property increases safety, the car being in a situation where sliding effects are small when braking, and can be considered as "pseudo-sliding" effects (see [1]).*

Computation of the motor and braking torques Computing the time derivative of the τ_i given by (2) and using the fast wheels dynamics gives:

$$\dot{\tau}_i = \frac{1}{V_{xi}}\left[\frac{R}{I_r}(-RF_{xi} + C_{mi} - C_{f\,i}) - \dot{V}_{xi}(\tau_i + 1)\right].$$

We cannot ensure $\tau_i = \tau_{di}$, $i = 1, \cdots, 4$ at each time instant, but we can produce the errors $(\tau_i - \tau_{di})$ exponentially tend to zero, as follows:

$$\dot{\tau}_i = \dot{\tau}_{di} - \alpha_\tau(\tau_i - \tau_{di}),\ \alpha_\tau > 0, \tag{11}$$

by choosing the following torques:

$$C_{mi} - C_{f\,i} = RF_{xi} + \frac{I_r}{R}(\dot{\tau}_{di} - \alpha_\tau V_{xi}(\tau_i - \tau_{di}) + V_{xi}\dot{\tau}_{di} + (\tau_i + 1)\dot{V}_{xi}). \tag{12}$$

Dynamics (11) can be chosen more faster than the car's one. Therefore, using arguments from singular perturbation theory (see e.g. [6]), we will prove in the next proposition, that our control strategy makes the yaw rate follow the reference trajectory and the longitudinal acceleration bounded.

Proposition 1 *If the initial conditions are such that $\mid \dot{\psi}(0) - \dot{\psi}_{ref}(0) \mid < \epsilon_{\dot{\psi}}$, $\mid \gamma_x(0) - \gamma_{xref}(0) \mid < \epsilon_{\gamma_x}$, $\mid \tau_i(0) - \tau_{di}(0) \mid < \nu$, where the positive constants ϵ_i and ν are sufficiently small, then the torques given by (12) with high level controls u_x and u_ψ given by (4) and (5) lead to:*

there exists $\epsilon^\star$ such that if $\epsilon_\tau = 1/\alpha_\tau < \epsilon^\star$:

- *i) $(\dot{\psi}(t) - \dot{\psi}_{ref}(t))$ and $(\tau_i(t) - \tau_{di}(t))$, $i = 1 \sim 4$, exponentially tend to 0;*
- *ii) the approximation $\gamma_x(t) = \gamma_{xref}(t) + O(\epsilon_\tau)$ is valid for all $t \in [0, T]$ and therefore $\gamma_x(t)$ is bounded on $[0, T]$.*

Proof: We use results from singular perturbation theory (see e.g. [6, 7]). The boundary layer system (or fast subsystem) is given by eq. (11) and the error $(\tau_i - \tau_{di})$ exponentially converges to 0 with a rate of convergence given by α_τ, so the origin of the boundary layer system is exponentially stable. Denoting $\epsilon_\tau = 1/\alpha_\tau$, the gain α_τ being arbitrarily chosen in (12), ϵ_τ can be taken sufficiently small to be considered as the "small parameter" in the singular perturbation approach.

Moreover, when $\epsilon_\tau = 0$, the unique solution of the fast subsystem is given by:

$$\tau_i = \tau_{di},\ i = 1 \sim 4.$$

So the complete system is in standard form (see [6]). Concerning the reduced subsystem, made of V_x, V_y and $\dot{\psi}$ in (1) when $\epsilon_\tau = 0$, we can see, using the control u_ψ given by (5), that the yaw rate error $(\dot{\psi} - \dot{\psi}_{ref})$ exponentially converges to 0. Therefore point i) results e.g. from [6, Thm. 9.3].

On the other hand, when $\epsilon_\tau = 0$, the constraint (4) leads to $\gamma_x(t) = \gamma_{xref}(t)$, $\forall\, t \ge 0$, but we cannot say that $\gamma_x(t)$ asymptotically converges to $\gamma_{xref}(t)$. Then, using Tikhonov's theorem (see e.g. [6, Thm. 9.1]), we can only conclude to the approximation given by ii), approximation in $O(\epsilon_\tau)$ valid on finite time intervals $[0, T]$ and not uniformly with respect to T. $\diamondsuit$

Remark 4 *In proposition 1, nothing can be said asymptotically about V_x and V_y. Nevertheless, we have shown in [3], that if reference trajectories $V_{xref}(t)$ and $V_{yref}(t)$ for the longitudinal and lateral velocity were also available, we could obtain, by slightly modifying the high level control u_x as follows:*

$$u_x = M\gamma_{xref} - \beta M\tilde{V}_x - M(\dot{\psi}V_y - \dot{\psi}_{ref}V_{yref}),\ \beta > 0, \tag{13}$$

that $(V_x - V_{xref})(t)$ exponentially converges to 0 and $V_y(t)$ remains bounded $\forall\, t \ge 0$.

We also have proposed in [3], a method to elaborate these trajectories from a simpler model. It is important to point out that if $\dot{\psi}_{ref}(t)$ and $\gamma_{xref}(t)$ are given, this simpler reference model can be integrated off-line, independently of the wheels' dynamics.

3 Control of the vertical dynamics through the active suspension forces

Before presenting our method to compute suspension forces, we describe the simple model that we have used to model the vertical dynamics.

3.1 The vertical control model

Applying the fundamental principle of mechanics to the vector: $(\ \theta\ \ \phi\ \ V_z\ )^T$, where θ denotes the roll angle, ϕ the pitch angle and V_z the vertical velocity of the center of mass expressed in the referential linked to the car, we obtain after some simplications (see e.g. [5]):

$$\left\{\begin{array}{lcl} I_x\ddot{\theta} & = & l_1F_{z1} - l_2F_{z2} + l_3F_{z3} - l_4F_{z4} + z_G M\gamma_y \\ & & -((l_1 - l_2)m_{av} + (l_3 - l_4)m_{ar})\,g \\ & & +c_{yz}\dot{\phi}\dot{\psi} + \hat{a}(V_x\dot{\psi} - V_z\dot{\theta}) \\ I_y\ddot{\phi} & = & -L_1F_{z1} - L_2F_{z2} + L_3F_{z3} + L_4F_{z4} \\ & & -z_G M\gamma_x + ((L_1 + L_2)m_{av} - (L_3 + L_4)m_{ar})\,g \\ & & +c_{zx}\dot{\theta}\dot{\psi} + \hat{a}(V_y\dot{\psi} - V_z\dot{\phi}) \\ M_S\gamma_z & = & \sum_{i=1}^4 F_{zi} - Mg, \end{array}\right. \tag{14}$$

where M_S denotes the suspended mass of the car, z_G the vertical coordinate of the center of mass G, γ_z the vertical accelereration of G, L_i (respectively l_i) are the wheelbase (resp. gauge) related to wheel i w.r.t. G, I_x (resp. I_y) is the inertia of the car relatively to axis x (resp. y), m_{ri} is the mass of wheel i, with: $m_{av} = m_{r1} = m_{r2}$ and $m_{ar} = m_{r3} = m_{r4}$, z_{ri} is the radius of wheel i and:

$$\begin{array}{lcl} \hat{a} & = & \sum_{i=1}^4 (z_G - z_{ri})m_{ri} \\ \hat{b} & = & \sum_{i=3}^4 L_i m_{ri} - \sum_{i=1}^2 L_i m_{ri} \\ c_{yz} & = & I_y - I_z \\ c_{zx} & = & I_z - I_x \\ c_{xy} & = & I_x - I_y. \end{array} \tag{15}$$

Finally, it remains to introduce the F_{zi}, $i = 1 \sim 4$, which are the vertical forces applied to the wheels. They linearly appear in equation (14), and can be considered as control variables for the vertical dynamics. In fact, this vertical dynamics should be controlled via **the active suspension forces** F_i, $i = 1 \sim 4$. **But**, we have considered that the dynamics of the non suspended masses (i.e. of the wheels) is faster than the car's one. Let us recall this dynamics of the non suspended masses. It can be written as follows, assuming that pitch and roll angles are sufficiently small:

$$m_{ri}\ddot{z}_{ri} = -m_{ri}g - F_i - A_i - F_{badi} + F_{zi}\,,\ i = 1 \sim 4, \tag{16}$$

where the A_i denote the connection forces applied to the wheels by the front and rear axle units and F_{badi} are the forces exerted by mechanical "anti-rolling" bars.

Therefore, neglecting the dynamics of the non suspended masses means that we can take $m_{ri}\ddot{z}_{ri} = 0$ in (16). This is reasonable since the mass of the wheels is quite smaller than the car's one. Then, F_i and F_{zi} are linked through the following algebraic equation:

$$F_i = -m_{ri}g - A_i - F_{badi} + F_{zi}\,,\ i = 1 \sim 4. \tag{17}$$

Then, as for longitudinal slip ratios when controlling the horizontal dynamics, these vertical forces F_{zi} can be considered here as **intermediate** control variables for the vertical dynamics, control variables which can be related to the physical **active** suspension forces via the algebraic relation (17).

This time-scale decomposition argument, together with a singular perturbation approach, will be used in the next section to analyse the behavior of the complete closed-loop system.

Remark 5 *Let us point out that, in the case of* **passive** *suspensions, the vertical forces F_{zi} applied to the wheels can be modelled as follows:*

$$F_{zi} = k_p(r_l - z_{ri})\,,\ i = 1 \sim 4,$$

where r_l is the free radius of the wheels, i.e. the radius of the wheel when it is not charged and k_p denotes the radial tyre stiffness coefficient.

3.2 Computation of the active suspension forces

As previously explained, we want the pitch, roll and vertical velocities asymptotically tend to zero. Then, we can choose the following high-level controls:

$$\begin{array}{ll} u_\theta & = I_x\ddot{\theta} = -\alpha_\theta I_x\dot{\theta}\,,\ \alpha_\theta > 0 \\ u_\phi & = I_y\ddot{\phi} = -\alpha_\phi I_y\dot{\phi}\,,\ \alpha_\phi > 0 \\ u_z & = M_S\gamma_z = -\alpha_z M_S(dz_G/dt)\,,\ \alpha_z > 0. \end{array} \tag{18}$$

But these 3 equations are not sufficient to determine the four intermediate control variables F_{zi}. We can solve this problem by mimimizing in addition the following function:

$$\int_0^t \gamma_z(\tau)\,d\tau\,, \tag{19}$$

this will lead to improve comfort, since we minimize the pumping acceleration.

4 Control of the complete model via both the braking torques and the suspension forces

4.1 Collaboration of braking torques and active suspension forces

Of course, the choice of the F_{zi} changes the interaction forces F_{xi} and F_{yi} which acts on the yaw dynamics. The computation of the high level controls u_x and u_ψ (and therefore of the $(C_{mi} - C_{fi})$) are changed through the corresponding evolution of the F_{xi} and F_{yi}.

Conversely, F_{xi} and F_{yi} act on the vertical dynamics of $\ddot{\theta}$ and $\ddot{\phi}$ through the accelerations γ_x and γ_y. The global algorithm is illustrated in the Figure below.

In fact, we have the following proposition, concerning the closed-loop behavior of system (Σ_{10}) described by the ten state variables V_x, V_y, V_z, ω_i, $i = 1 \sim 4$, $\dot{\psi}$, $\dot{\theta}$, $\dot{\phi}$ satisfying dynamical equations (1) and (14) :

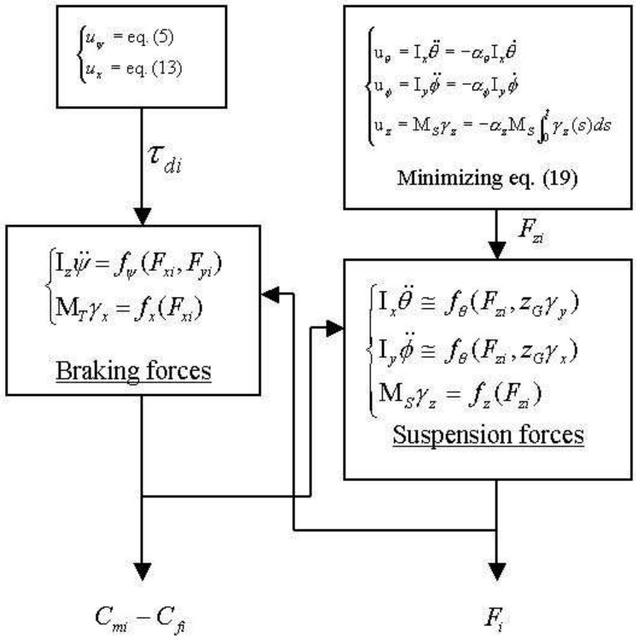

Figure 2: Control of the global dynamics

Proposition 2 *If the initial conditions are such that* $|\ \dot{\psi}(0) - \dot{\psi}_{ref}(0)\ | < \epsilon_{\dot{\psi}}$, $|\ V_x(0) - V_{xref}(0)\ | < \epsilon_{v_x}$, $|\ V_y(0) - V_{yref}(0)\ | < \epsilon_{v_y}$, $|\ \gamma_x(0) - \gamma_{xref}(0)\ | < \epsilon_{\gamma_x}$, $|\ \tau_i(0) - \tau_{di}(0)\ | < \nu$, $|\ \dot{\phi}(0)\ | < \epsilon_{\dot{\phi}}$, $|\ \dot{\theta}(0)\ | < \epsilon_{\dot{\theta}}$, $|\ \dot{z}_G(0)\ | < \epsilon_{\dot{z}_G}$, *where the positive constants* ϵ_i *and* ν *are sufficiently small, then* **the application of** *the braking torques given by (12) with high level controls* u_x *and* u_ψ *given by (13) and (5) and the active suspension forces solution of (17) where the* F_{zi} *satisfy equations (18) and minimize (19)* **leads to***:*

there exists $\epsilon^\star$ *such that if* $\epsilon_\tau = 1/\alpha_\tau < \epsilon^\star$*:*

- *i)* $(\dot{\psi}(t) - \dot{\psi}_{ref}(t))$, $(\tau_i(t) - \tau_{di}(t))$, $i = 1 \sim 4$ *and* $(V_x(t) - V_{xref}(t))$ *exponentially tend to 0;*
- *ii) the approximation* $\gamma_x(t) = \gamma_{xref}(t) + O(\epsilon_\tau)$ *is valid for all* $t \in [0, T]$ *and therefore* $\gamma_x(t)$ *is bounded on* $[0, T]$ *and the same holds for* $V_y(t)$*;*
- *iii)* $\dot{\phi}$, $\dot{\theta}$ *and* $\dot{z}_G$ *exponentially tend to 0.*

Proof: The proof of proposition 2 is totally similar to the one of proposition 1. $\diamondsuit$

4.2 Stability analysis of the complete model

But, the global model of the vehicle, developed by PSA-Peugeot-Citroën and denoted (Σ_{14}) is described by the state variables of (Σ_{10}) completed with the dynamical equations of the non suspended masses (16) which can be seen as fast dynamics w.r.t. the car's one.

To put the closed-loop system in the framework of singular perturbation theory, we have to slightly modify from (17) the computation of the F_i, $i = 1 \sim 4$, as follows:

$$F_i = F_{zdi} - A_i - F_{badi} - m_{ri}g - \sqrt{m_{ri}}\alpha_i\dot{z}_{ri} - m_{ri}\ddot{z}_{rdi},\ \alpha_i > 0, \qquad (20)$$

the desired suspension forces F_{zdi} being solution of the problem described in section 3.2, and z_{rdi} being the corresponding radius of wheel i vertically charged by F_{zdi} (i.e. $F_{zdi} = k_p(r_l - z_{rdi})$). Therefore, the closed-loop equation of the non suspended masses takes the following form:

$$m_{ri}(\ddot{z}_{ri} - \ddot{z}_{rdi}) + \sqrt{m_{ri}}\alpha_i(\dot{z}_{ri} - \dot{z}_{rdi}) + k_p(z_{ri} - z_{rdi}) = 0, \qquad (21)$$

which means that the errors $(z_{ri} - z_{rdi})$, $i = 1 \sim 4$ exponentially tend to zero.

Moreover, let us now prove that the complete closed-loop system is in standard form. If we denote:

$$y_i = \begin{pmatrix} z_{ri} - z_{rdi} \\ \epsilon(\dot{z}_{ri} - \dot{z}_{rdi}) = \epsilon\dot{z}_{ri} \end{pmatrix},\ \text{with } \epsilon = \sqrt{m_{ri}}, \qquad (22)$$

ϵ can be reasonably interpreted as a small parameter, compared to the mass and inertias of the car. Then using (21) and (22) leads to the following fast subsystem (assuming that the four wheels have the same mass $m_{ri} = \epsilon^2$):

$$\epsilon\dot{y}_i = \begin{pmatrix} 0 & 1 \\ -k_p & -\alpha_i \end{pmatrix} y_i,\ i = 1 \sim 4. \qquad (23)$$

Then the complete system is in standard form since if we make $\epsilon = 0$, the dynamics of the non suspended mass has the unique solution:

$$z_{ri} = z_{rdi},\ i = 1 \sim 4.$$

Moreover, the origin of the fast subsystem (23) is clearly exponentially stable.

Let us point out that we have added a **damping term** in (20) to make the origin of the fast subsystem exponentially stable so that we can apply Tichonov's theorem.

On the other hand, for $\epsilon = 0$, if we consider as reduced (or slow) subsystem, the following closed-loop equations for which the active suspension forces were given by (17):

$$\begin{array}{l} \ddot{\theta} + \alpha_\theta\dot{\theta} = 0\ ,\ \ddot{\phi} + \alpha_\phi\dot{\phi} = 0\ ,\ \gamma_x - \gamma_{xref} = 0 \\ (\ddot{\psi} - \ddot{\psi}_{ref}) + \alpha_\psi(\dot{\psi} - \dot{\psi}_{ref}) = 0 \\ (\dot{V}_x - \dot{V}_{xref}) + \beta(V_x - V_{xref}) = 0 \\ \gamma_z + \alpha_z\int_0^t \gamma_z(s)ds = 0 \\ (\dot{\tau}_i - \dot{\tau}_{di}) + \alpha_\tau(\tau_i - \tau_{di}) = 0,\ i = 1 \sim 4. \end{array} \qquad (24)$$

Then, we can apply Tichonov's theorem e.g. [6, Thm. 9.3] and the following proposition holds for the complete closed-loop system (Σ_{14}).

Proposition 3 *Under the assumptions of proposition 2, the application of the braking torques (12) with high level controls* u_x *and* u_ψ *given by (13) and (5) and of the active suspension forces given by (20) leads for* (Σ_{14}) *to:*

- *i)* $|\ \dot{\theta}(t)\ |$, $|\ \dot{\phi}(t)\ |$, $|\ \dot{\psi}(t) - \dot{\psi}_{ref}(t)\ |$, $|\ V_x(t) - V_{xref}(t)\ |$, $|\ dz_G/dt(t)\ |$ *and* $|\ \tau_i - \tau_{di}\ |$, $i = 1 \sim 4$, *exponentially tend to 0;*

- *ii)* $\mid z_{ri} - z_{rdi} \mid$, $i = 1 \sim 4$, *exponentially tend to 0;*

- *iii) there exists* $\epsilon^\star$ *such that if* $\epsilon_\tau = 1/\alpha_\tau < \epsilon^\star$, *the approximation* $\gamma_x(t) = \gamma_{xref}(t) + O(\epsilon_\tau)$ *is valid for all* $t \in [0, T]$. *The same holds for* $(V_y(t) - V_{yref}(t))$.

5 Simulation results and conclusion

5.1 Simulation results

We have applied our global control strategy to system (Σ_{14}) from $t = 1\,s$. The yaw rate reference is the one of an "ideal" vehicle, i.e. without overshoot. The longitudinal reference acceleration is shown in Figure 4.

As displayed in Figures 3 and 4, we can observe as expected from proposition 3, that $(\dot\psi - \dot\psi_{ref})$ exponentially converges to 0 and that $(\gamma_x - \gamma_{xref})$ remains bounded. In fact, γ_x and γ_{xref} are quasi superposed and the same holds for $\dot\psi$ and $\dot\psi_{ref}$.

Moreover, we see that the vertical, pitch and roll velocities exponentially converge to zero. There are small oscillations when the control is applied and these oscillations can be reduced by increasing the damping gains α_i in (20). The closed-loop system transient behavior with the proposed suspension forces is clearly better than in the passive case (dotted line curve). Finally, the behavior of the braking torques and the suspension forces is quite satisfying as displayed in Figure 4.

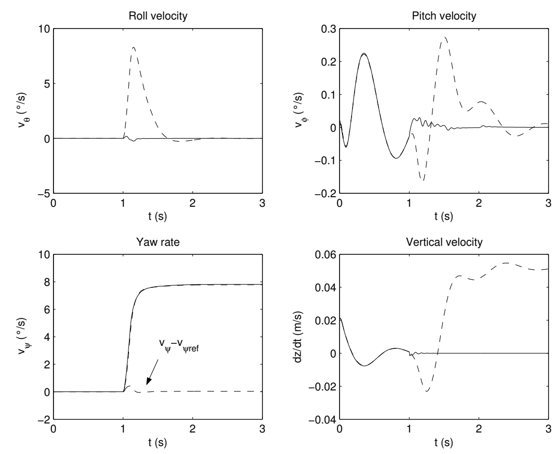

Figure 3: Yaw rate and vertical dynamics

5.2 Conclusion

To **conclude**, we can say that from quite simple control models, we have proposed a global control strategy using the natural time-scale decompositions of the system and making active suspension forces and braking torques collaborate. The first results we have obtained are quite satisfying. Moreover, preliminary studies show that our control is quite robust w.r.t. the suspended mass M_S which can be under-estimated up to 50%. But, the control laws need some state variables which are not

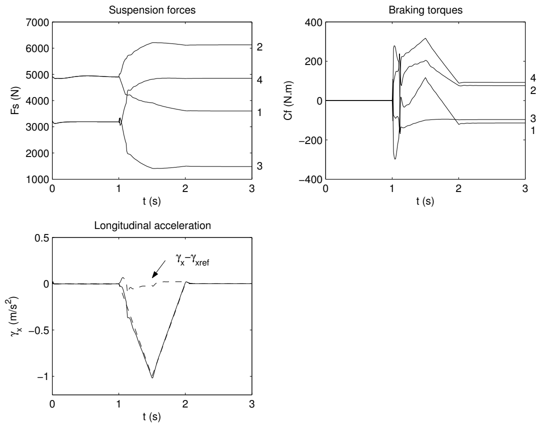

Figure 4: Longitudinal acceleration, braking torques and suspension forces

directly measured, such as velocities V_x and V_y as well as the longitudinal slip ratios τ_i. Therefore, from a real-time implementation point of view, some nonlinear observers have been designed to realize observer-based controllers (see e.g. [9]).

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