MULTI-SCENARIO DATA DRIVEN FUZZY TSK NONHOLONOMIC MOBILE ROBOT MODELLING

J T Economou*, **A Tsourdos, P C K Luk, B A White**

Department of Aerospace, Power and Sensors,
CRANFIELD University-RMCS, Shrivenham, Wiltshire,
Swindon, SN6 8LA, UK
*email: J.T.Economou@rmcs.cranfield.ac.uk

Keywords: Fuzzy logic, Takagi-Sugeno-Kang, mobile-robot, subtractive clustering

Abstract

In this paper the problem of multi-scenario data driven fuzzy parameter estimation is considered. Experimental data are used from a small scale differentially steered four-wheel mobile robot "PROMETHEUS". In particular two key modes of operation were identified and the multi-model parameters were obtained using the subtractive clustering approach. The two modes of the mobile robot operation were blended using a suitable blending function. The robotic vehicle modes structure was of a 1-st order multivariate Takagi-Sugeno-Kang. The parameter estimation process also included a non-casual filtering approach which resulted in a reduced number of TSK rules.

1 Introduction

Differentially steered electrically-actuated wheeled robots which belong to the class of nonholonomic mobile robots, still remain a challenging modelling problem due to the complex interactions between the wheels and actuators, and the surface and the near-surface regions. The interactions are essential in obtaining a model which is valid for a wide range of input demands [10]. For the case of the differentially steered mobile robot, due to the tight turns that it can perform up to pivot turns, (depending on the actuator capabilities), it does share some of the steerability advantages of a holonomic mobile robot.

Fuzzy logic modelling belongs to the class of "intelligent" methods which are based on "knowledge based" methodologies. Conventional modelling methods, when applied to practical problems, have demonstrated the difficulty of representing accurately a complex process by a single mathematical model over a wide range of input demands. Over the course of time, there has been increasing interest in the application of intelligent methods in solving such practical problems. Several researchers have introduced the fuzzy modelling approaches from a control and engineering perspective [3], and in particular the use of fuzzy clustering techniques for fuzzy model identification [6]. However, the fuzzy modelling of real systems from experimental data often produces a large number of rules (models) which can be unrealistic for any control purposes. In [4] and [5] several approaches have been introduced which in effect reduce the number of rules within an error tolerance.

In this paper, a skid-steer electrically actuated mobile robot has been modelled using a number of Takagi-Sugeno-Kang fuzzy logic local models. The two inputs are the left and right armature voltage demands of the four d.c. motors which are initially crisp (i.e. non-fuzzy). All the TSK mobile robot rules are $1-st$ order multivariate models which are evaluated in parallel using fuzzy reasoning at every time instant. The rules results are combined and finally defuzzified to a give crisp (non-fuzzy) number which represents the mobile robot blended state vector ($\tilde{x}$) which represents the non-linear mobile robot model.

The wheeled differentially steered nonholonomic mobile robot employs wheel skidding for its turning function rather than the more conventional Ackerman steering geometry [4]. The experimental mobile robot can perform all turns from smooth high radius turns (HRTs) to low radius turns (LRTs) which include pivot turns.

The paper is organised as follows: Section 2 describes the dual-scenario mobile robot operation, the TSK problem formulation and the basic principle of the data driven subtractive clustering filtering algorithm. Section 3 presents the experimental results and the effectiveness of the fuzzy TSK mobile robot model. The concluding remarks are given in Section 4.

2 Mobile Robot Operational Scenario

The generic differentially-steered mobile robot state variables and dimensions are shown in figure 1. The mobile robot planar trajectory is divided to two main modes of operation:

- HRT: High Radius Turns
- LRT: Low Radius Turns

Assuming that the left and right mobile robot side actuators are operating at the same armature voltages, the desired family of linear model structures

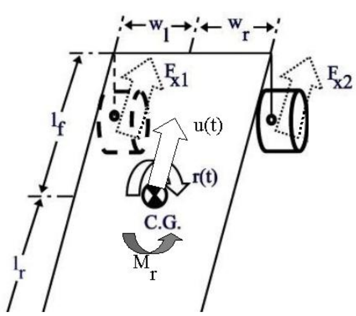

Figure 1: *Differentially Steered Mobile Robot.*

are shown in 1 and 2 for the two data experimental data sets $\xi = \{1, 2\}$.

i-th Rule: IF u is$(A_1^{j_1})_\xi$ AND r is $(A_2^{j_2})_\xi \dots$ AND V_{a1} is $(A_3^{j_3})_\xi$ AND V_{a2} is $(A_4^{j_4})_\xi$ THEN

$$\left\{\begin{array}{l} \dot{\tilde{u}}_\xi^i = f_\xi^i(u, r, V_{ai}) = v_{11}{}_\xi^i u + v_{12}{}_\xi^i r + v_{13}{}_\xi^i V_{a1} \quad + \\ v_{14}{}_\xi^i V_{a2} + v_{15\xi}^i \end{array}\right\} \tag{1}$$

i-th Rule: IF u is$(B_1^{j_1})_\xi$ AND r is $(B_2^{j_2})_\xi \dots$ AND V_{a1} is $(B_3^{j_3})_\xi$ AND V_{a2} is $(B_4^{j_4})_\xi$ THEN

$$\left\{\begin{array}{l} \dot{\tilde{r}}_\xi^i = g_\xi^i(u, r, V_{ai}) = v_{21}{}_\xi^i u + v_{22}{}_\xi^i r + v_{23}{}_\xi^i V_{a1} \quad + \\ v_{24}{}_\xi^i V_{a2} + v_{25\xi}^i \end{array}\right\} \tag{2}$$

2.1 Fuzzy TSK Model Structure

The two inputs are the left and right hand mobile robot d.c. motor armature voltages. The TSK_3 block requires as inputs the common-mode and differential voltage demands for the actuators which are defined in equation 3.

$$\left.\begin{array}{ll} \delta u_a^+ & = \frac{V_{a1}+V_{a2}}{2} \\ \delta u_a^- & = \frac{V_{a1}-V_{a2}}{2} \end{array}\right\} \tag{3}$$

The TSK mobile robot fuzzy rules consist of several local models, which are evaluated, in parallel using fuzzy reasoning for every time instant. The rules results are combined and finally defuzzified to give a crisp (non-fuzzy) number which represents the mobile robot blended state vector $\tilde{x} \triangleq [\tilde{u}, \quad \tilde{r}]^T$.

The defuzzified blended TSK model for $\xi = \{1, 2\}$ is given from equation 4 and 5.

$$\left.\begin{array}{l} \dot{\tilde{u}}_\xi = \dfrac{\Sigma_{i=1}^{i_{max}}\left(\prod_{k=1}^{k_{max}} \mu_{(A_k^{j_k})_\xi}(x_k)\right)^i \zeta_\xi^i(\underline{\lambda}, \underline{x}, \lambda_o)}{\Sigma_{i=1}^{i_{max}}\left(\prod_{k=1}^{k_{max}} \mu_{(A_k^{j_k})_\xi}(x_k)\right)^i} \\ \zeta_\xi^i(\underline{\lambda}, \underline{x}, \lambda_o) = \underline{\lambda}^i \underline{x}^T + \lambda_o^i = \\ (\lambda_4^i u + \lambda_3^i r + \lambda_2^i V_{a1} + \lambda_1^i V_{a2} + \lambda_0^i)_\xi \\ \text{Where } \underline{\lambda}_\xi \triangleq [\lambda_4^i, \lambda_3^i, \lambda_2^i, \lambda_1^i]_\xi = \\ [u_{11}{}_\xi^i, u_{12}{}_\xi^i, u_{13}{}_\xi^i, u_{14}{}_\xi^i] \\ \text{ and } \lambda_0{}_\xi^i = u_{15}{}_\xi^i \end{array}\right\} \tag{4}$$

$$\left.\begin{array}{l} \dot{\tilde{r}}_\xi = \dfrac{\Sigma_{i=1}^{i_{max}}\left(\prod_{k=1}^{k_{max}} \mu_{(B_k^{j_k})_\xi}(x_k)\right)^i \varphi_\xi^i(\underline{\tau}, \underline{x}, \tau_o)}{\Sigma_{i=1}^{i_{max}}\left(\prod_{k=1}^{k_{max}} \mu_{(B_k^{j_k})_\xi}(x_k)\right)^i} \\ \varphi_\xi^i(\underline{\tau}, \underline{x}, \tau_0) = \underline{\tau}^i \underline{x}^T + \tau_0^i = \\ (\tau_4^i u + \tau_3^i r + \tau_2^i V_{a1} + \tau_1^i V_{a2} + \tau_0^i)_\xi \\ \text{Where } \underline{\tau}_\xi \triangleq [\tau_4^i, \tau_3^i, \tau_2^i, \tau_1^i]_\xi = \\ [u_{21}{}_\xi^i, u_{22}{}_\xi^i, u_{23}{}_\xi^i, u_{24}{}_\xi^i] \\ \text{ and } \tau_0{}_\xi^i = u_{25}{}_\xi^i \end{array}\right\} \tag{5}$$

The model blending variable w_s depends on the differential and common-mode demands $(\delta u_a^-, \delta u_a^+)$ respectively.

$$\left.\begin{array}{l} \tilde{u} = w_s \tilde{u}_1 + (1 - w_s)\tilde{u}_2 \\ \tilde{r} = w_s \tilde{r}_1 + (1 - w_s)\tilde{r}_2 \end{array}\right\} \tag{6}$$

The associated fuzzy logic TSK membership functions for the inference engine are shown in figure 2. The inference rules are shown in figure 3 and result in the output surface shown in figure 4. The blending $0 - degree\ TSK_3$ output is given from equation 7.

$$\left.\begin{array}{l} w_s = \dfrac{\Sigma_{q=1}^{q_{max}}\left(\prod_{\rho=1}^{\rho_{max}} \mu_{D_\rho^{j_\rho}}(x_\rho)\right)^q h^q(\underline{C}, \underline{x}, C_o)}{\Sigma_{q=1}^{q_{max}}\left(\prod_{\rho=1}^{\rho_{max}} \mu_{D_\rho^{j_\rho}}(x_\rho)\right)^q} \\ h^q(\underline{C}, \underline{x}, C_o) = \underline{C}^q \underline{x}^T + C_o^q = C_0^q \\ \text{Where } \underline{C} \triangleq 0 \end{array}\right\} \tag{7}$$

The resulting multi-scenario mobile robot model is given from equations 8 and 4 and 5. The independent and blended HRT and LRT fuzzy logic regions are clearly shown in figure 4. Effectively, the zero order TSK output (w_s) in figure 4 is equivalent to a singleton Mamdani fuzzy logic inference engine.

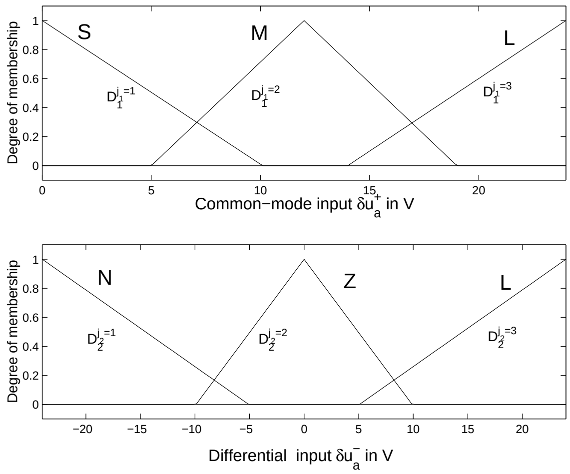

Figure 2: *TSK Membership Functions.*

$$
\left.
\begin{aligned}
\tilde{u} = & \frac{\Sigma_{q=1}^{q_{max}} \left(\prod_{\rho=1}^{\rho_{max}} \mu_{D_\rho^{j_\rho}}(x_\rho) \right)^q h^q(\underline{C}, \underline{x}, C_o)}{\Sigma_{q=1}^{q_{max}} \left(\prod_{\rho=1}^{\rho_{max}} \mu_{D_\rho^{j_\rho}}(x_\rho) \right)^q} \tilde{u}_1 \quad + \\
& (1 - \frac{\Sigma_{q=1}^{q_{max}} \left(\prod_{\rho=1}^{\rho_{max}} \mu_{D_\rho^{j_\rho}}(x_\rho) \right)^q h^q(\underline{C}, \underline{x}, C_o)}{\Sigma_{q=1}^{q_{max}} \left(\prod_{\rho=1}^{\rho_{max}} \mu_{D_\rho^{j_\rho}}(x_\rho) \right)^q}) \tilde{u}_2 \\
\tilde{r} = & (\frac{\Sigma_{q=1}^{q_{max}} \left(\prod_{\rho=1}^{\rho_{max}} \mu_{D_\rho^{j_\rho}}(x_\rho) \right)^q h^q(\underline{C}, \underline{x}, C_o)}{\Sigma_{q=1}^{q_{max}} \left(\prod_{\rho=1}^{\rho_{max}} \mu_{D_\rho^{j_\rho}}(x_\rho) \right)^q}) \tilde{r}_1 \quad + \\
& (1 - \frac{\Sigma_{q=1}^{q_{max}} \left(\prod_{\rho=1}^{\rho_{max}} \mu_{D_\rho^{j_\rho}}(x_\rho) \right)^q h^q(\underline{C}, \underline{x}, C_o)}{\Sigma_{q=1}^{q_{max}} \left(\prod_{\rho=1}^{\rho_{max}} \mu_{D_\rho^{j_\rho}}(x_\rho) \right)^q}) \tilde{r}_2
\end{aligned}
\right\}
\tag{8}
$$

The local linear TSK models for each fuzzy rule i for i_{max} rules are given by equations 4 and 5 in a state space form. These models were estimated using the subtractive clustering method [7] together with non-casual filtering for bounded normalised errors $\bar{\varepsilon}_{\dot{\mathbf{u}}}, \bar{\varepsilon}_{\dot{\mathbf{r}}}$ as defined in equations 9 in the state variables for the two experimental data sets $\xi = \{1, 2\}$.

$$
\begin{aligned}
\dot{\varepsilon}_{\mathbf{u}\xi} &= 100 \frac{\dot{\mathbf{u}}_\xi - \dot{\tilde{\mathbf{u}}}_\xi}{max(max(|\dot{\mathbf{u}}_\xi|))} \\
\dot{\varepsilon}_{\mathbf{r}\xi} &= 100 \frac{\dot{\mathbf{r}}_\xi - \dot{\tilde{\mathbf{r}}}_\xi}{max(max(|\dot{\mathbf{r}}_\xi|))}
\end{aligned}
\tag{9}
$$

where the integers i = TSK rule number and i_{max} = maximum number of TSK rules. The subtractive clustering method is satisfied when, for given error boundaries in the state variable matrices $\bar{\mathbf{e}}_{\dot{\mathbf{u}}}$ and $\bar{\mathbf{e}}_{\dot{\mathbf{r}}}$ over the experimental data sets, the inequalities in 10 are simultaneously satisfied.

$$
\begin{aligned}
max(|\varepsilon_{\dot{\mathbf{u}}}|)_\xi &\leq \bar{e}_{\dot{u}} \\
max(|\varepsilon_{\dot{\mathbf{r}}}|)_\xi &\leq \bar{e}_{\dot{r}}
\end{aligned}
\tag{10}
$$

The flowchart in figure 5 shows the mobile robot

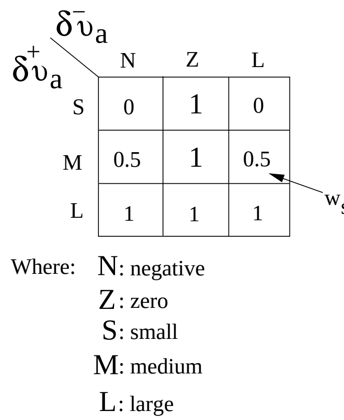

Figure 3: *Tagaki Sugeno Fuzzy Logic Inference Engine.*

mathematical model generation algorithm. The algorithm requires the experimental data and the error bounds $\bar{\mathbf{e}}_{\dot{\mathbf{u}}}$ and $\bar{\mathbf{e}}_{\dot{\mathbf{r}}}$. From the experimental mobile robot input-output data and a random recursively generated sequence for the radii $r_{sc} = [r_1, r_2, r_3, r_4, r_5, r_6]$ the algorithm uses the subtractive clustering method to estimate the TSK model coefficients.

The clusters generated represent natural groupings of data from a large set of experimental data. The subtractive clustering method considers each data point as a potential cluster. As a result the number of grid points to be evaluated are the number of data points. According to [7] the subtractive clustering algorithm reduces the maximum number of clusters simply by selecting the data points with the highest potential. The algorithm suggested in [7] produces an optimum number of rules (TSK models) which are different for the same experimental data when the initial radii are different. In this paper the method in [7] is improved by including a loop which randomly varies the ranges of influence (radii) recursively till a prespecified error bound criterion is met.

For the mobile robot application the fuzzy inference rules are generated and the normalised error matrices $\bar{\mathbf{e}}_{\dot{\mathbf{u}}}$ and $\bar{\mathbf{e}}_{\dot{\mathbf{r}}}$ are compared to the required error bounds. When the normalised error matrices over the specific data sets $\xi = \{1, 2\}$ are less than the required error bounds, then the subtractive clustering fuzzy inference rule base is accepted [7]. If not, then the algorithm repeats until the conditions in 10 are met for both state variables.

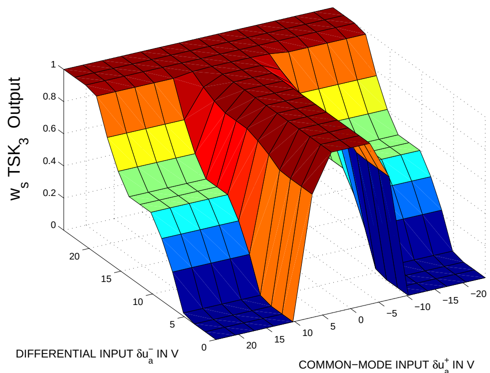

Figure 4: *Tagaki Sugeno Fuzzy Logic Output Surface.*

3 Experimental and Fuzzy-TSK Estimated Data

The suggested approach shown in this paper was implemented for data obtained from a real small scale mobile robot PROMETHEUS. The skid steer wheeled mobile robot was fully instrumented including the yaw rate and accelerator sensors. In particular the velocity signal was relatively smooth as this was obtained by the integration of the acceleration. However, the yaw rate signal was relatively more noisy and the recursive subtractive filtering approach required more samples in order to obtain a smoother signal for the estimation process. The demanded motor voltages were measured with an RC filter since the power electronics operated using an H-bridge topology with a variable duty cycle. A non-causal filtering approach was used here recursively over the entire data set time horizon in combination with the subtractive clustering method. The latter resulted in a reduced number of TSK fuzzy models which satisfy the pre-specified error bound criterion. The results shown in figures 6 and 7 indicate the effectiveness of the subtractive filtering approach for the HRT mode of operation, which retains very good tracking properties. In figure 6(b), in addition to the experimental data and the fuzzy logic TSK data the filtered data, are also shown and these form the basis of the estimation process while retaining the original signal trend. Figures 8 and 9 show the effectiveness for the LRT mobile robot mode of operation. During this mode the longitudinal velocity is kept relatively low in relation to an increasing yaw rate while the tracking error is kept to an acceptable value. The algorithm was run with an error of less than 20% and the results in figures 7 and 9 demonstrate this

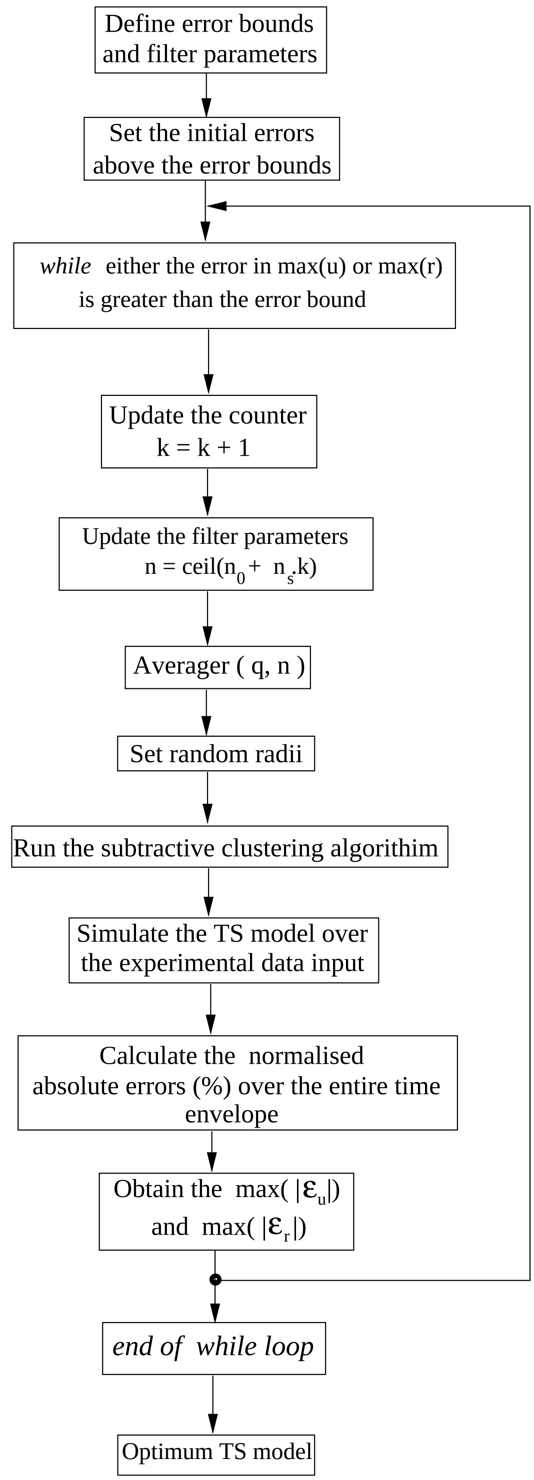

Figure 5: *Recursive Subtractive Clustering Filtering Algorithm.*

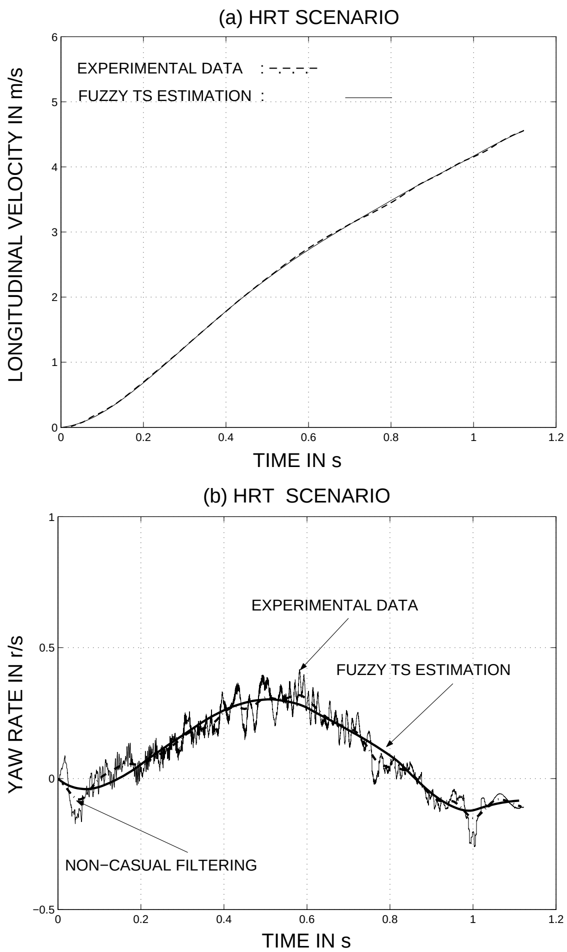

Figure 6: *HRT Scenario: Comparisons of Experimental Data and TSK Model Results.*

for the entire time for both data sets. Finally, the HRT and LRT TSK fuzzy logic designs were blended using an additional TSK system. The resulting suboptimum design was simulated for a combined (LRT and HRT) turn, as shown in figure 10.

4 Conclusions

A small scale differentially steered mobile robot PROMETHEUS was used for the data capturing of two steering scenarios: (a) HRT mode and (b) LRT mode. These data were independently used to estimate using the subtractive clustering method, the 1-st order multivariate fuzzy-TSK model parameters for both cases (a) and (b). A non-casual filtering approach was also used part of the parameter estimation process in order to reduce the number of TSK models. Finally a suitable blending function was used to blend the two modes of the mobile robot operation. The experimental and estimated data were

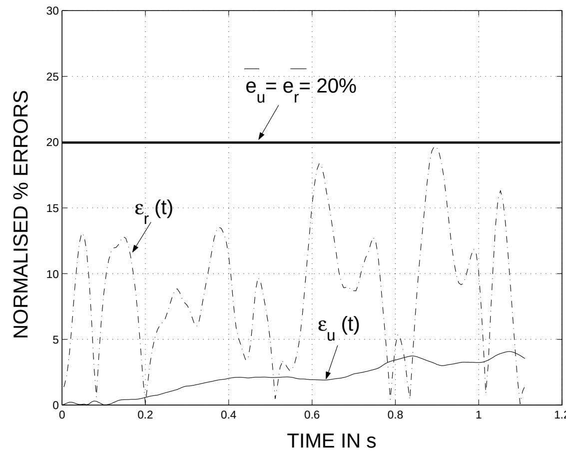

Figure 7: *HRT Scenario: Normalised Errors.*

presented in order to show the effectiveness of the proposed approach.

Acknowledgements

The authors wish to thank Mr. Chris Ransom and Mr. Colin Offer for their contributions during the prototype development phase and Cranfield University for financial support.

References

[1] A. Ollero, A. Garca-Cerezo, "Design of fuzzy logic control systems. Application to robotics". Plenary Session. Proceedings of the European Workshop on Industrial Fuzzy Control and Applications. Tarrasa (Spain), April, 1993

[2] J. S. Shamma, M. Athans, “Guaranteed Properties of Gain Scheduled Control for Linear Parameter-Varying Plants”, Automatica, vol. 27, no. 3, pages 559-564, 1991.

[3] R. Babuska, "Fuzzy Modelling- a control engineering perspective", IEEE Proceedings in Fuzzy Systems,Vol 4, 1995.

[4] U. Kaymak, R. Babuska, "Compatible Cluster Merging For Fuzzy Modelling", IEEE Proceedings in Fuzzy Systems,Vol 2, 1995.

[5] A. Sonbol, M. Sami, "A New Approach for Designing TSK Fuzzy Systems From Input-Output Data", Proceedings American Control Conference, Anchorage, Alaska, May 2002.

[6] A. F. Gomez-Skarmeta, M. Delado, M.A. Vila, "Fuzzy Model Identification Based on Cluster Estimation", Journal of Intelligent and Fuzzy Systems, Vol. 2, No. 3, Sept. 1994.

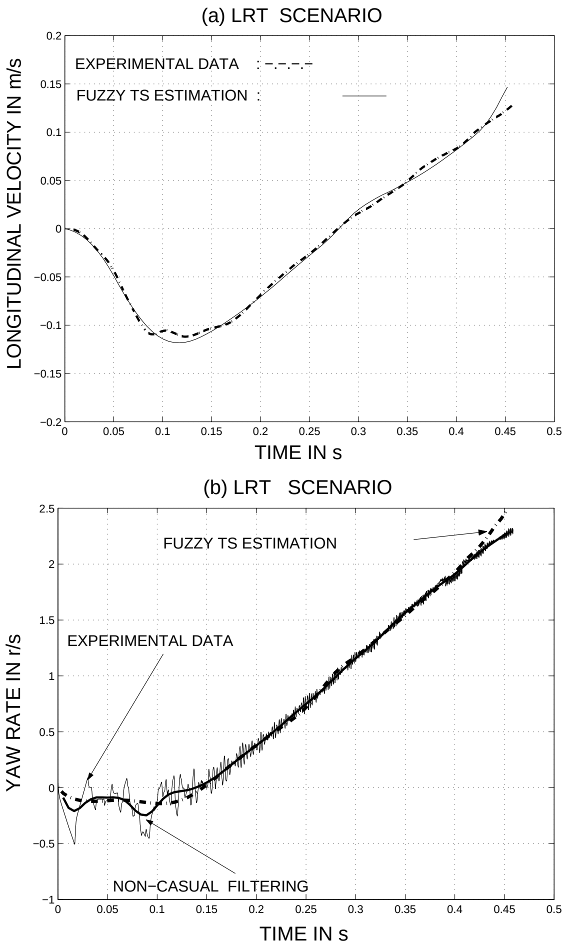

Figure 8: *LRT Scenario: Comparisons of Experimental Data and TSK Model Results.*

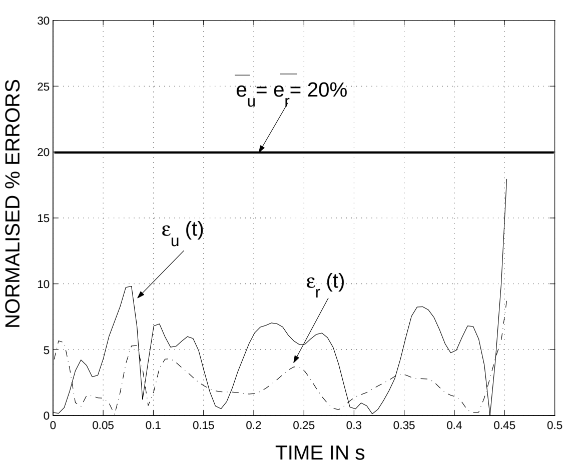

Figure 9: *LRT Scenario: Normalised Errors.*

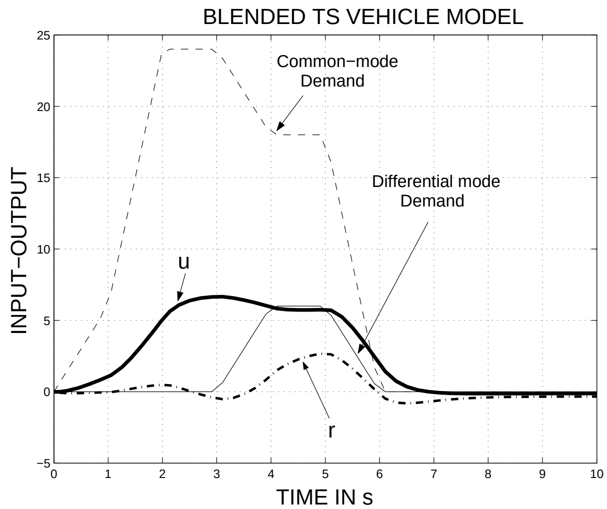

Figure 10: *Blended HRT and LRT Fuzzy Logic TSK Models.*

[7] S. Chiu, "Fuzzy Model Identification Based on Cluster Estimation", Journal of Intelligent and Fuzzy Systems, Vol. 2, No. 3, Sept. 1994.

[8] C E Barbier, B Nogarede and H L Meyer, "Global Control Strategy Optimisation of an Asynchronous Drive System for an Electric Vehicle", Pergamon Journal, Control Eng. Practice, Vol. 4. No. 8, pp.1053-1066, 1996.

[9] A. Tsourdos, R. Żbikowski, B.A. White,"Robust autopilot for a quasi-linear parameter-varying missile model", AIAA Journal of Guidance, Control and Dynamics , vol. 24 no.2. pages 287-295 March-April 2001.

[10] J.T. Economou, R.E. Colyer, "Modelling of Skid Steering and Fuzzy Logic Vehicle Ground Interaction",Proceedings of the American Control Conference, Chicago, Illinois, June 2000.