GLOBAL STABILIZING CONTROL DESIGN FOR THE PVTOL AIRCRAFT USING SATURATION FUNCTIONS ON THE INPUTS

I. Fantoni, R. Lozano, A. Palomino

Heudiasyc, UMR CNRS 6599
UTC, BP 20529
60205 Compiègne, France
e-mail:{ifantoni/rlozano/apalomino}@hds.utc.fr

Keywords: Aircraft control, Non-linear control systems, Global stabilization, Linear saturation functions, Bounded inputs.

Abstract

We present a global stabilization algorithm for the Planar Vertical Takeoff and Landing (PVTOL) aircraft, with bounded inputs. We first stabilize the altitude of the aircraft and then take care of the horizontal position and the roll angle. The control strategy is based on the use of nonlinear combinations of linear saturation functions bounding the thrust input and the rolling moment to arbitrary saturation limits. We provide global convergence of the state to the origin. Note that the methodology that we present here, is similar to previous works that we already proposed for such a system. The interest of this alternative control strategy relies on the fact that the altitude of the aircraft is first stabilized, which is more reliable for implementations on real experiments.

1 Introduction

The existing design methodologies for the flight control of the Planar Vertical Takeoff and Landing (PVTOL) aircraft model are numerous. This particular system is, indeed, a simplified aircraft model with a minimal number of states and inputs but retains the main features that must be considered when designing control laws for a real aircraft. Since, the system possesses special properties such as, for instance, unstable zero dynamics [5], several methodologies for controlling such a system have been proposed.

Hauser et al. [5] in 1992 applied an approximate I-O linearization procedure which results in bounded tracking and asymptotic stability for the V/STOL aircraft. In 1996, Andrew R. Teel [14] illustrated his central result of nonlinear small gain theorem using the example of the PVTOL aircraft with input corruption. His theorem provided a formalism for analyzing the behavior of control systems with saturation. He established a stabilization algorithm for nonlinear systems in so-called feedforward form which includes the PVTOL aircraft. The same year, Martin et al. [8] presented an extension of the result proposed by Hauser et al. [5]. Their idea was to find a flat output for the system and to split the output tracking problem in two steps. Firstly, they designed a state tracker based on exact linearization by using the flat output and secondly, they designed a trajectory generator to feed the state tracker. They thus controlled the tracking output through the flat output. In contrast to the approximate-linearization based control method proposed by Hauser et al., their control scheme provided output tracking of non-minimum phase flat systems. They have also taken into account in the design the coupling between the rolling moment and the lateral acceleration of the aircraft (i.e. $\varepsilon \neq 0$). Sepulchre et al. [11] applied a linear high gain approximation of backstepping to the approximated model neglecting the coupling. In 1999, Lin et al. [6] studied robust hovering control of the PVTOL using nonlinear state feedback based on optimal control. Reza Olfati-Saber [9] proposed a configuration stabilization for the VTOL aircraft with a strong input coupling using a smooth static state feedback. M. Saeki et al. [10] offered a new design method which makes use of the center of oscillation and a two-step linearization. In fact, they designed a controller by applying a linear high gain approximation of backstepping to the model. A paper on an internal-model based approach for the autonomous vertical landing on an oscillating platform has been proposed by Marconi et al. [7]. They presented an error-feedback dynamic regulator that is robust with respect to uncertainties of the model parameters and they provided global convergence to the zero-error manifold.

Recently, in [3, 4, 15], we developed new control strategies which coped with (arbitrarily) bounded inputs and which provided global convergence to the origin. In the proposed paper, we present a global stabilizing algorithm for the control of the PVTOL aircraft. The proposed methodology is an alternative of our previous approaches, where we also use saturation functions in the control design. Here, the main contribution is to stabilize first the altitude of the aircraft, which is more reliable for the stabilization purposes of aircraft. The approach takes into account the idea of a possible implementation of such methodology on a real experiment. Moreover, the methodology requires relatively less calculations than the previous one. The paper is organized as follows. In section 2, we recall the equations of motion for the PVTOL aircraft and state the control objective. In section 3, the proposed approach is presented. Simulations are shown in section 4 and conclusions are finally given in section 5.

2 The PVTOL aircraft model

The PVTOL aircraft dynamics, depicted on Figure 1, are mod-

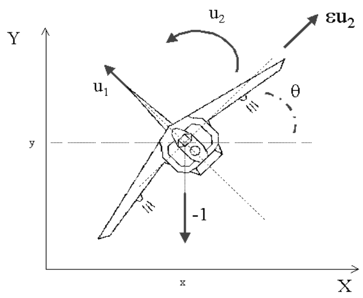

Figure 1: The PVTOL aircraft (front view)

elled by the following equations [5]

$$\begin{aligned} \ddot{x} &= -u_1 \sin\theta + \varepsilon u_2 \cos\theta \\ \ddot{y} &= u_1 \cos\theta + \varepsilon u_2 \sin\theta - 1 \\ \ddot{\theta} &= u_2 \end{aligned} \tag{1}$$

where x, y denote the center of mass horizontal and vertical position and θ is the roll angle of the aircraft with the horizon. The control inputs u_1 and u_2 are respectively the thrust (directed out the bottom of the aircraft) and the angular acceleration (rolling moment). The constant "-1" is the normalized gravitational acceleration. The parameter ε is a (small) coefficient characterizing the coupling between the rolling moment and the lateral acceleration of the aircraft. Its value is in general so small, that $\varepsilon = 0$ can be supposed in (1) (see for instance [5, §2.4]). Furthermore, several authors have shown that by an appropriate coordinate transformation, we can obtain a representation of the system without the term due to ($\varepsilon \neq 0$) [9, 10, 12]. Consequently, in this study we choose to consider the PVTOL aircraft dynamics with $\varepsilon = 0$, i.e

$$\ddot{x} = -u_1 \sin\theta \tag{2}$$

$$\ddot{y} = u_1 \cos\theta - 1 \tag{3}$$

$$\ddot{\theta} = u_2 \tag{4}$$

whatever this means that ε has been neglected. Our control objective is to stabilize the PVTOL aircraft to the origin with bounded inputs, i.e. $|u_1| \leq U_1$ and $|u_2| \leq U_2$ for some positive constants U_1 and U_2.

3 Global stabilizing control law

In the proposed control strategy, we first stabilize the altitude of the aircraft, by using the control input u_1. The controller is obtained by defining the following desired behavior for the altitude y. Let us, therefore, define the variable r_2 as follows

$$\ddot{y} = r_2(y, \dot{y}) \triangleq -\sigma_{22}(\dot{y} + \sigma_{21}(y + \dot{y})) \tag{5}$$

where the functions $\sigma_{ij}(\cdot)$ are twice differentiable linear saturations. Furthermore, we define the desired behavior for the position x and then, the variable r_1:

$$\ddot{x} = r_1(x, \dot{x}) \triangleq -\frac{1}{k}\sigma_{12}(\dot{x} + \sigma_{11}(\frac{1}{k}x + \dot{x})) \tag{6}$$

where k is a positive constant greater than unity. The definition of linear saturation functions $\sigma_{ij}(\cdot)$ is described below.

Definition 3.1 *Given positive constants L and M, with $L \leq M$, a function $\sigma : \mathbb{R} \to \mathbb{R}$ is said to be a* ***linear saturation*** *for (L, M) if it is a continuous, nondecreasing function satisfying*

(a) $\sigma(s) = s$ when $|s| \leq L$
(b) $|\sigma(s)| \leq M$ for all $s \in \mathbb{R}$

A second definition in terms of linear saturation functions is also given and will be extensively used throughout the paper.

Definition 3.2 *Given positive constants L, M, N with $L \leq \min\{M, N\}$, a function $\sigma : \mathbb{R} \to \mathbb{R}$ is said to be a* ***2-level linear saturation*** *for (L, M, N) if it is a continuous, nondecreasing function satisfying*

(a) $\sigma(s) = s$ for all $s \in [-L, L]$
(b) $-M < \sigma(s) < M$ for all $s \in (-N, N)$
(c) $\sigma(s) = -M$ for all $s \leq -N$
(d) $\sigma(s) = M$ for all $s \geq N$

By selecting

$$u_1 = \frac{1}{\cos(\sigma_{B_\theta}(\theta))}(1 + r_2) \tag{7}$$

with $B_\theta < \frac{\pi}{2}$ (this will be proved in the stability analysis). Then from (3) and when $|\theta| < B_\theta$, $\ddot{y} \to r_2$. In the stability analysis, we will prove the existence of a time $t_2 \geq 0$, such that $|\tan\theta(t)| \leq \tan B_\theta \triangleq T_\theta$, $\forall t \geq t_2$ for some initial-condition-independent positive constant T_θ, i.e. $\tan\theta(t)$ is bounded. Therefore, $y \to 0$, which will be proved in the stability analysis and results from [13] and then, $r_2 \to 0$. The reduced system becomes

$$\ddot{x} = -\tan\theta(1 + r_2) \tag{8}$$

$$\ddot{\theta} = u_2 \tag{9}$$

At this stage, the only remaining input is the angular acceleration u_2. The idea is to choose u_2, such that $\lim_{t\to\infty} \ddot{x}(t) = r_1$, by means of the definition of a desired roll angle θ_d and using $r_2 \to 0$. Therefore, we define the following desired roll angle

$$\theta_d = \arctan(-r_1) \tag{10}$$

The objective is to determine an appropriate u_2 that makes θ follow the desired motion expressed in (10). Consequently and using $r_2 \to 0$, we will see in the stability analysis that $\lim_{t\to\infty} \ddot{x}(t) = r_1$. Nevertheless, the second-order dynamics (4) do not permit u_2 to give directly any desired form to θ. The idea is, then, to achieve $\theta(t) \to \theta_d(t)$ as $t \to \infty$. r_1 and r_2 could have been simply selected as linear stabilizing state feedbacks, i.e. $r_1 = -k_{11}x - k_{12}\dot{x}$ and $r_2 = -k_{21}y - k_{22}\dot{y}$, with $k_{ij} > 0, \forall i, j = 1, 2$, as is actually proposed in [10] and [11]. Moreover, a similar tracking version could be considered for u_2 in (4), i.e. $u_2 = \ddot{\theta}_d - k_{31}(\theta - \theta_d) - k_{32}(\dot{\theta} - \dot{\theta}_d)$, with $k_{3i} > 0$, $\forall i = 1, 2$, as exposed in [10]. Nevertheless, such approaches do not seem to be appropriate whenever u_1 and u_2 are (physically) bounded inputs. This constitutes the interest of the present study: to provide a solution to our control problem whenever u_1 and u_2 are furnished by actuators with (output) saturation limits (which is a realistic case). In other words, we take $|u_1| \leq U_1$ and $|u_2| \leq U_2$ for some finite positive (constants) U_1 and U_2. Notice, from (3), that $U_1 > 1$ is a necessary condition for the PVTOL to be stabilizable at any desired position. Indeed, any static condition implies that the aircraft weight be compensated. In such scenario, the selected functions r_1, r_2, and u_2 are based on the stabilization approach proposed in [13]. Therefore, they are defined in terms of *linear saturation* [13, Def. 1] functions.

Let us now state the main result of our paper.

Theorem 3.1 *Consider the PVTOL dynamics (2)–(4) with input saturation bounds $U_1 > 1$ and $U_2 > 0$. Let us define*

$$u_1 = \frac{1}{\cos(\sigma_{B_\theta}(\theta))}(1 + r_2) \tag{11}$$

with

$$r_1 = -\frac{1}{k}\sigma_{12}(\dot{x} + \sigma_{11}(\frac{1}{k}x + \dot{x})) \tag{12}$$

$$r_2 = -\sigma_{22}(\dot{y} + \sigma_{21}(y + \dot{y})) \tag{13}$$

where k is a positive constant greater than unity and the functions $\sigma_{ij}(\cdot)$ are twice differentiable linear saturations for given (L_{ij}, M_{ij}, N_{ij}) such that $M_{i1} < \frac{L_{i2}}{2}$ $\forall i = 1, 2$ and

$$\begin{aligned} u_2 \;=\;& \sigma_{41}(\ddot{\theta}_d) - \sigma_{32}(\dot{\theta} - \sigma_{42}(\dot{\theta}_d) \\ &+ \sigma_{31}(\dot{\theta} - \sigma_{43}(\dot{\theta}_d) + \theta - \theta_d)) \end{aligned} \tag{14}$$

where

$$\theta_d \triangleq \arctan(-r_1) \tag{15}$$

and the functions $\sigma_{mn}(\cdot)$ are linear saturations for given (L_{mn}, M_{mn}) such that the following conditions are satisfied

(a) $M_{41} + M_{32} \leq U_2$
(b) $M_{41} + M_{42} < M_{31}$
(c) $M_{41} + 2M_{42} + 2M_{31} < L_{32}$
(d) $M_{41} + M_{42} + 2M_{43} + 2\arctan\left(\frac{M_{12}}{k}\right) < L_{31}$
(e) $2M_{41} + 2M_{42} + M_{31} + M_{43} + B_{\theta_d} < \frac{\pi}{2}$

Then, provided k is sufficiently large,

(1) global asymptotic stabilization of the closed-loop system dynamics (2)–(4),(11)–(14) towards $(x, \dot{x}, y, \dot{y}, \theta, \dot{\theta}) = (0, 0, 0, 0, 0, 0)$ is achieved, with

(2) $|u_1(t)| \leq \frac{1}{\cos(B_\theta)}(1 + M_{22}) \leq U_1$ *and*
$|u_2(t)| \leq M_{41} + M_{32} \leq U_2$, $\forall t \geq 0$,

Proof. Property *(2)* of the statement is a direct consequence of the definitions of u_1, u_2, r_1, and r_2. Its proof is consequently straightforward. The proof of property *(1)* is divided in two parts. The first part shows that the bounds on θ_d, $\dot{\theta}_d$, and $\ddot{\theta}_d$ are directly influenced by the parameter k. The second part details the closed-loop stability analysis.

First part

Let us begin by noting, from the strict increasing property of $\arctan(\cdot)$ and the definition of r_1 in (12) that $|\theta_d(t)| \leq \arctan\left(\frac{M_{12}}{k}\right) \triangleq B_{\theta_d}$, $\forall t \geq 0$, which shows the direct influence of k on B_{θ_d}. From the definition of θ_d (15), one can easily verify that

$$\dot{\theta}_d = \frac{-\dot{r}_1}{1 + r_1^2} \tag{16}$$

$$\ddot{\theta}_d = \frac{-\ddot{r}_1}{1 + r_1^2} + \frac{2\dot{r}_1^2 r_1}{(1 + r_1^2)^2} \tag{17}$$

with

$$\begin{aligned} \dot{r}_1 =& -\frac{1}{k}\sigma'_{12}(.)\left[\ddot{x} + \sigma'_{11}(\frac{1}{k}x + \dot{x})\left(\frac{1}{k}\dot{x} + \ddot{x}\right)\right] \\ \ddot{r}_1 =& -\frac{1}{k}\sigma''_{12}(.)\left[\ddot{x} + \sigma'_{11}(\frac{1}{k}x + \dot{x})\left(\frac{1}{k}\dot{x} + \ddot{x}\right)\right]^2 \\ & -\frac{1}{k}\sigma'_{12}(.)\left[x^{(3)} + \sigma''_{11}(\frac{1}{k}x + \dot{x})\left(\frac{1}{k}\dot{x} + \ddot{x}\right)^2\right. \\ & \left. + \sigma'_{11}(\frac{1}{k}x + \dot{x})\left(\frac{1}{k}\ddot{x} + x^{(3)}\right)\right] \end{aligned}$$

and

$$\begin{aligned} \ddot{x} =& -u_1 \sin\theta \\ x^{(3)} =& -u_1\dot{\theta}\cos\theta - \dot{u}_1 \sin\theta \end{aligned} \tag{18}$$

Using (11), let us calculate the time derivative of u_1, when $|\theta(t)| < B_\theta$

$$\dot{u}_1 = \frac{\sin(\theta)\dot{\theta}}{\cos^2(\theta)}(1 + r_2) + \frac{\dot{r}_2}{\cos\theta} \tag{19}$$

Let us also calculate $\dot{r}_2$ which will be used in the sequel.

$$\dot{r}_2 = -\sigma'_{22}(\dot{y} + \sigma_{21}(y + \dot{y}))\left[\ddot{y} + \sigma'_{21}(y + \dot{y})\left(\dot{y} + \ddot{y}\right)\right] \tag{20}$$

with

$$\ddot{y} = u_1 \cos\theta - 1 \tag{21}$$

Let us note that twice differentiability of $\sigma_{ij}(s)$ ($i = 1, 2$, $j = 1, 2$) on $I\!R$ ensures boundedness of $\sigma'_{ij}(s)$ and $\sigma''_{ij}(s)$ on $[-N_{ij}, N_{ij}]$ (see [1, Theo. 4.27]), i.e. there exist positive (real) constants A_{ij} and B_{ij} ($i = 1, 2$, $j = 1, 2$) such that $\left|\sigma'_{ij}(s)\right| \leq A_{ij}$ and $\left|\sigma''_{ij}(s)\right| \leq B_{ij}$, $\forall s \in [-N_{ij}, N_{ij}]$. Taking into account the functions $\sigma_{ij}(s)$ chosen in the present paper (see Appendix A), we will simplify the calculations and consider that $\left|\sigma'_{ij}(s)\right| \leq 1$ and $\left|\sigma''_{ij}(s)\right| \leq 1$, $\forall s \in [-N_{ij}, N_{ij}]$. On the other hand, $\sigma'_{ij}(s) = \sigma''_{ij}(s) = 0$ when $|s| \geq N_{ij}$. Consequently, for any nonnegative scalar p, $\left|s^p\sigma'_{ij}(s)\right| \leq N_{ij}^p$ and $\left|s^p\sigma''_{ij}(s)\right| \leq N_{ij}^p$, $\forall s \in I\!R$, $\forall i, j = 1, 2$. Therefore, recalling that $\frac{1}{k}$ is less than unity, we obtain

$$\begin{aligned} |\dot{r}_1(t)| \;\leq&\; \tfrac{1}{k}\left[2\tan\theta(1 + M_{22}) + N_{12} + M_{11}\right] \\ |\dot{r}_2(t)| \;\leq&\; 2M_{22} + N_{22} + M_{21} \triangleq B_{\dot{r}_2} \end{aligned}$$

$\forall t \geq 0$. Using $\frac{1}{1+r_1^2} \leq 1$ and assuming the existence of a time $t_1 \geq 0$ such that $|\tan\theta(t)| \leq T_\theta$, $\forall t \geq t_1$, for some initial-condition-independent positive constant T_θ, i.e. $\tan\theta$ is bounded[1], it results from the above that

$$\left|\dot{\theta}_d(t)\right| \leq |\dot{r}_1(t)| \leq \frac{1}{k}\left[2T_\theta(1+M_{22}) + N_{12} + M_{11}\right] \triangleq \frac{B_{\dot{r}_1}}{k} \tag{22}$$

$\forall t \geq t_1$ (see (16)), showing the boundedness of $\dot{\theta}_d$, but also the direct influence of k on its bound. Furthermore, assuming the existence of a time $t_2 \geq t_1 \geq 0$ such that $|\dot{\theta}(t)| \leq B_{\dot{\theta}}$, $\forall t \geq t_2$, for some initial-condition-independent positive constant $B_{\dot{\theta}}$[2] and using (11), (19) and (18), we have

$$\begin{aligned} \ddot{x} &\leq T_\theta(1+M_{22}) \\ x^{(3)} &\leq (1+T_\theta^2)B_{\dot{\theta}}(1+M_{22}) + T_\theta B_{\dot{r}_2} \end{aligned} \tag{23}$$

and

$$\begin{aligned} |\ddot{r}_1(t)| \leq \frac{1}{k}\Big[(B_{\dot{r}_1})^2 &+ 2\left((1+T_\theta^2)B_{\dot{\theta}}(1+M_{22})\right. \\ &\left.+T_\theta B_{\dot{r}_2}\right) + \left[T_\theta(1+M_{22}) + N_{12} + M_{11}\right]^2 \\ &+T_\theta(1+M_{22})\Big] \triangleq \frac{B_{\ddot{r}_1}}{k} \end{aligned} \tag{24}$$

$\forall t \geq t_2$. Therefore,

$$\left|\ddot{\theta}_d(t)\right| \leq |\ddot{r}_1| + 2|\dot{r}_1|^2 r_1 \tag{25}$$

$\forall t \geq t_2$, then

$$|\ddot{\theta}_d(t)| \quad \leq \quad \frac{1}{k}(B_{\ddot{r}_1} + 2(B_{\dot{r}_1})^2 M_{12}) \tag{26}$$

(see (17)), which shows that the bound of $\ddot{\theta}_d(t \geq t_2)$ is directly influenced by k too.

Second part: closed-loop stability analysis

Firstly, as mentioned in the previous part of the proof, we will show the existence of a time $t_1 \geq 0$ such that $|\dot{\theta}(t)| \leq B_{\dot{\theta}}$, $\forall t \geq t_1$, and a time $t_2 \geq 0$, $\forall t \geq t_2$ such that $|\tan\theta(t)| \leq T_\theta$, for some initial-condition-independent positive constants $B_{\dot{\theta}}$ and T_θ. For that reason, let us introduce the following positive function $V_1 = \dot{\theta}^2$. Differentiating V_1 with respect to time, we obtain

$$\begin{aligned} \dot{V}_1 \quad = \quad & -2\dot{\theta}(-\sigma_{41}(\ddot{\theta}_d) + \sigma_{32}(\dot{\theta} - \sigma_{42}(\dot{\theta}_d) \\ & +\sigma_{31}(\dot{\theta} - \sigma_{43}(\dot{\theta}_d) + \theta - \theta_d))) \end{aligned} \tag{27}$$

Observe that condition (b) implies $M_{41} < M_{31}$ and from Definition (3.2), since $M_{31} < M_{32}$, then $M_{41} < M_{32}$. Then, if $|\dot{\theta}| > M_{41}+M_{42}+M_{31}$ it follows that $\dot{V}_1 < 0$. Therefore, there exists a time $t_1 \geq 0$ such that $|\dot{\theta}(t)| \leq M_{41}+M_{42}+M_{31} \triangleq B_{\dot{\theta}}$, $\forall t \geq t_1$.

By using condition (c) it then follows that the argument of σ_{32} in (27) is in its linear part, for $t \geq t_1$. Hence, for $t \geq t_1$, $\ddot{\theta} = \sigma_{41}(\ddot{\theta}_d) - \dot{\theta} + \sigma_{42}(\dot{\theta}_d) - \sigma_{31}(\dot{\theta} - \sigma_{43}(\dot{\theta}_d) + \theta - \theta_d)$. Let us now introduce a new variable: $z = \theta + \dot{\theta}$. We propose the following positive function $V_2 = z^2$. Its time derivative becomes, for $t \geq t_1$

$$\dot{V}_2 = -2z(-\sigma_{41}(\ddot{\theta}_d) - \sigma_{42}(\dot{\theta}_d) + \sigma_{31}(z - \sigma_{43}(\dot{\theta}_d) - \theta_d)) \tag{28}$$

From condition (b), it follows that if $|z| > M_{41}+M_{42}+M_{43}+B_{\theta_d}$ then $\dot{V}_2 < 0$. Hence, there exists a time $t_2 \geq t_1$ such that $|z(t)| \leq M_{41}+M_{42}+M_{43}+B_{\theta_d}$, $\forall t \geq t_2$. Since

$$|\theta| - |\dot{\theta}| \leq |\theta + \dot{\theta}| \leq M_{41} + M_{42} + M_{43} + B_{\theta_d} \tag{29}$$

$\forall t \geq t_2$ and since $|\dot{\theta}(t)| \leq M_{41}+M_{42}+M_{31} \triangleq B_{\dot{\theta}}$, $\forall t \geq t_1$, then $|\theta(t)| \leq 2M_{41}+2M_{42}+M_{31}+M_{43}+B_{\theta_d} \triangleq B_\theta$ $\forall t \geq t_2$ and then $|\tan\theta(t)| \leq \tan(B_\theta) \triangleq T_\theta$, $\forall t \geq t_2$. Therefore, $\forall t \geq t_2$, $\tan\theta(t)$ is bounded. By using condition (e), it yields $\theta(t) \leq B_\theta < \frac{\pi}{2}$, $\forall t \geq t_2$ and the argument of σ_{B_θ} in (11), is in its linear part, for $t \geq t_2$. Now, by using condition (d), it follows, as above, that the argument of σ_{31} in (28) is in its linear part, for $t \geq t_2$. We then have, for $t \geq t_2$

$$\ddot{\theta} = \sigma_{41}(\ddot{\theta}_d) - (2\dot{\theta} - \sigma_{42}(\dot{\theta}_d) - \sigma_{43}(\dot{\theta}_d)) - (\theta - \theta_d) \tag{30}$$

From the first part of the proof, we see that a sufficiently large k can be chosen such that $B_{\ddot{\theta}_d} < L_{41}$ and $B_{\dot{\theta}_d} < \min\{L_{42}, L_{43}\}$. Therefore, the arguments of the saturations σ_{41}, σ_{42} and σ_{43} in (30) are respectively in their linear parts and we finally obtain that θ follows the desired trajectory θ_d as $t \to \infty$. The $(y, \dot{y})$ subsystem is given by

$$\ddot{y} = -\sigma_{22}(\dot{y} + \sigma_{21}(y + \dot{y})) \tag{31}$$

where global stability is obtained from Theorem 2.1 in [13]. Then $y \to 0$ and $r_2 \to 0$. Therefore, the $(x, \dot{x})$ subsystem, in the limit when $\theta = \theta_d$, can be expressed as

$$\ddot{x} = -\frac{1}{k}\sigma_{12}(\dot{x} + \sigma_{11}(\frac{1}{k}x + \dot{x})) \tag{32}$$

Finally, global stability of system (32) follows from Theorem 2.1 in [13]. □

4 Simulation results

In this section, we present some simulation results using MATLAB and SIMULINK in order to observe the performance of the proposed control law. In particular, we started the PVTOL aircraft at the following initial conditions, i.e. $(x(0), \dot{x}(0), y(0), \dot{y}(0), \theta(0), \dot{\theta}(0)) = (50, 0, 50, 0, \frac{3\pi}{5}, 0)$. Note that the aircraft initial roll angle exceeds $\frac{\pi}{2}$. In fact, in contrast to some approaches which require that the roll angle be restricted to $(-\frac{\pi}{2}, \frac{\pi}{2})$ (see for instance [3, 5, 9]), the proposed method allows $\theta(t) \in I\!R$, $\forall t \geq 0$. We have chosen a thrust saturation bound $U_1 = 10$ and a rolling moment saturation bound $U_2 = 5$. The linear saturation functions are described in Appendix A; the following parameters

[1] In the second part of the proof, such an assumption will be proved to be satisfied with $T_\theta = \tan(B_\theta)$ and $B_\theta = 2M_{41}+2M_{42}+M_{31}+M_{43}+B_{\theta_d}$.

[2] Such an assumption will be proved to be satisfied with $B_{\dot{\theta}} = M_{41} + M_{42} + M_{31}$ in the second part of the proof.

were taken: $M_{22} = 1$, $L_{22} = 0.9$, $M_{21} = 0.4$, $L_{21} = 0.3$, $M_{12} = 0.8$, $L_{12} = 0.7$, $M_{11} = 0.3$, $L_{11} = 0.2$, $M_{32} = 4$, $L_{32} = 3.2$, $M_{31} = 0.808$, $L_{31} = 0.436$, $M_{41} = 0.032$, $L_{41} = 0.026$, $M_{42} = 0.032$, $L_{42} = 0.026$, $M_{43} = 0.047$, and $L_{43} = 0.037$. The results are shown on Figures 2 and 3. Observe that the inputs u_1 and u_2 remain within their saturation bounds. On the other hand, the time response of x is large compared to the one of y. This is due to the restrictions on the choices of parameters and especially the parameter k. Indeed, our algorithm requires to choose a suitably large parameter k, in order to have the appropriate bounds for $\ddot{\theta}_d$ and $\dot{\theta}_d$ (see the second part of the stability analysis). In the present case, we have chosen $k = 6.84$ to ensure that condition (d) (i.e. $M_{41} + M_{42} + 2M_{43} + 2\arctan\left(\frac{M_{12}}{k}\right) < L_{31}$) is satisfied. The parameters slow down directly the performance of the horizontal motion through r_1. We also provide simulations with the same control law and the same parameters as above, but with different initial conditions, in order to look at the performance of the proposed control law. Figures 4 and 5 show the results for the following initial conditions: $(x(0), \dot{x}(0), y(0), \dot{y}(0), \theta(0), \dot{\theta}(0)) = (10, 0, 10, 0, 0, 0)$. Notice that the altitude of the aircraft is first stabilized with a good performance. Then, the horizontal motion converges to zero with satisfaction, taking into account that u_1 and u_2 are small and bounded.

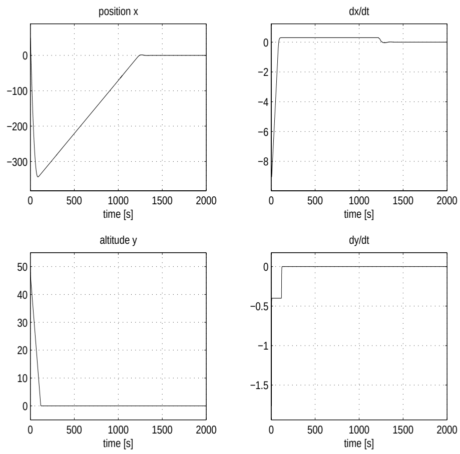

Figure 2: System states with initial states $(50, 0, 50, 0, \frac{3\pi}{5}, 0)$

5 Conclusions

Based on the results presented in [13] a global stabilization control scheme is proposed for the PVTOL aircraft. Compared to the previous works, the exposed solution stabilizes first the altitude of the aircraft and takes into account input saturation bounds. Simulations support the theoretical results and some real experiments have already been performed on a real prototype of the PVTOL aircraft by using vision. The experimental platform will be described in a future paper.

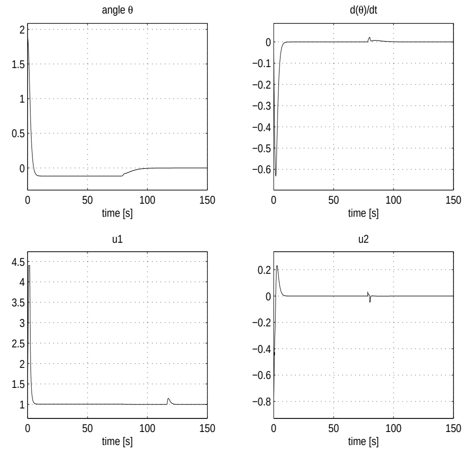

Figure 3: System states with initial states $(50, 0, 50, 0, \frac{3\pi}{5}, 0)$ and control inputs, for $t \in [0, 150]$

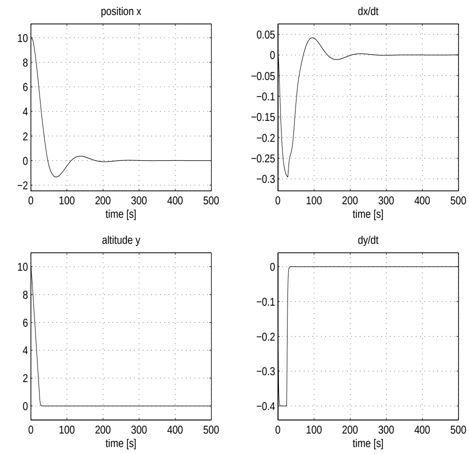

Figure 4: System states with initial states $(10, 0, 10, 0, 0, 0)$

Appendix A

A family of twice differentiable 2-level linear saturations is presented below. All the linear saturations (including those involved in u_2) used to get the simulation results of Section 4 were actually defined this way.

$$\sigma(s) = \begin{cases} -M & \text{if } s \leq -N \\ -P(-s) & \text{if } s \in (-N, -L) \\ s & \text{if } s \in [-L, L] \\ P(s) & \text{if } s \in (L, N) \\ M & \text{if } s \geq N \end{cases} \qquad (33)$$

where $N = 2M - L$, $M > L$, and

$$P(s) = \frac{(s-M)^4}{16(M-L)^3} - \frac{3(s-M)^2}{8(M-L)} + \frac{(s-M)}{2} + \frac{13M+3L}{16} \qquad (34)$$

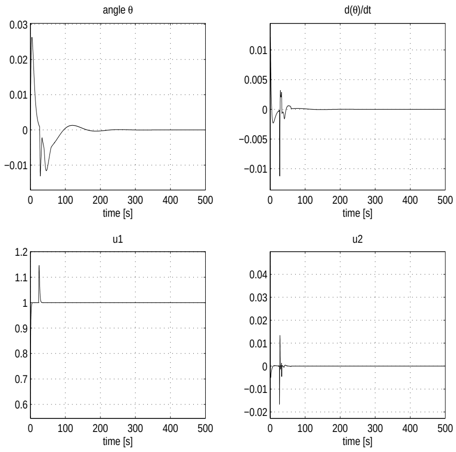

Figure 5: System states with initial states $(10, 0, 10, 0, 0, 0)$ and control inputs

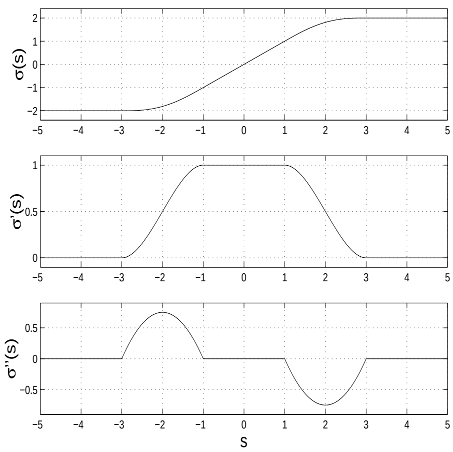

Figure 6: Twice differentiable 2-level linear saturation, as defined by (33)–(34), for $(L, M, N) = (1, 2, 3)$, and its first and second derivatives with respect to its argument.

The form of (33)–(34) taking $L = 1$ and $M = 2$ ($\Rightarrow N = 3$), and those of its first and second derivatives with respect to its argument, are shown on figure 6.

References

[1] T. M. Apostol. *Mathematical Analysis*. Reading, MA, second edition, 1974.

[2] I. Fantoni and R. Lozano. *Non linear control for underactuated mechanical systems*. Springer-Verlag, Communications and Control Engineering Series, 2002.

[3] I. Fantoni, R. Lozano, and P. Castillo. A simple stabilization algorithm for the PVTOL aircraft. In *15th IFAC World Congress*, Barcelona, Spain, 2002.

[4] I. Fantoni, A. Zavala, and R. Lozano. Global stabilization of a pvtol aircraft with bounded thrust. In *IEEE Conference on Decision and Control, CDC2002*, Las vegas, USA, December 2002.

[5] J. Hauser, S. Sastry, and G. Meyer. Nonlinear control design for slightly nonminimum phase systems: Application to V/STOL aircraft. *Automatica*, 28(4):665–679, 1992.

[6] F. Lin, W. Zhang, and R. D. Brandt. Robust hovering control of a PVTOL aircraft. *IEEE Transactions on Control Systems Technology*, 7(3):343–351, 1999.

[7] L. Marconi, A. Isidori, and A. Serrani. Autonomous vertical landing on an oscillating platform: an internal-model based approach. *Automatica*, 38:21–32, 2002.

[8] P. Martin, S. Devasia, and Brad Paden. A different look at output tracking: Control of a VTOL aircraft. *Automatica*, 32(1):101–107, 1996.

[9] R. Olfati-Saber. Global configuration stabilization for the VTOL aircraft with strong input coupling. In *Proceedings of the 39^{th} IEEE Conf. on Decision and Control*, Sydney, Australia, 2000.

[10] M. Saeki and Y. Sakaue. Flight control design for a nonlinear non-minimum phase VTOL aircraft via two-step linearization. In *Proceedings of the 40^{th} Conference on Decision and Control CDC'01*, 2001.

[11] R. Sepulchre, M. Janković, and P. Kokotović. *Constructive Nonlinear Control*. Springer-Verlag London, 1997.

[12] P. Setlur, Y. Fang D. Dawson, and B. Costic. Nonlinear tracking control of the VTOL aircraft. In *Proceedings of the 40^{th} Conference on Decision and Control CDC'01*, 2001.

[13] A. R. Teel. Global stabilization and restricted tracking for multiple integrators with bounded controls. *Systems & Control Letters*, 18:165–171, 1992.

[14] A. R. Teel. A nonlinear small gain theorem for the analysis of control systems with saturation. *IEEE Transactions on Automatic Control*, 41(9):1256–1270, 1996.

[15] A. Zavala, I. Fantoni, and R. Lozano. Global stabilization of a pvtol aircraft with bounded inputs. In *IFAC Latin-American Conference on Automatic Control CLCA*, Guadalajara, Mexico, December 2002.