ROBUST STABILITY OF TIME-DELAY CONTINUOUS-TIME SYSTEMS IN POLYTOPIC DOMAINS

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Abstract

Most of the existent linear matrix inequality based conditions for robust stability of time-delay systems in polytopic domains are expressed in terms of constant Lyapunov-Krasovskii functions. This note presents a simple way to extend these conditions in order to construct parameter-dependent functions that provide less conservative results, in both delay-independent and delay-dependent situations.

1 Introduction

During the last decades several works have dealt with the problem of stability of time-delay systems [9], [14], [18], [23]. One of the most popular techniques for the stability analysis of this kind of linear systems is undoubtedly the one based on Lyapunov-Krasovskii functionals [17], [32].

Since the numerical efficiency of these conditions for stability is a major concern, most of them have been rewritten as linear matrix inequalities (LMIs) which can nowadays be solved by polynomial time interior point algorithms [2], [11]. Several LMI conditions assuring robust stability appeared, in both delay-independent (i.e. the stability does not depend on the size of the time delay) [4], [21], [31], [34] and delay-dependent situations [3], [12], [19], [20].

For stabilizability purposes, including $\mathcal{H}_\infty$ or $\mathcal{H}_2$ norm optimization criteria, several results were developed as extensions of the stability analysis based on Lyapunov-Krasovskii functions. As a natural consequence, for the uncertain linear systems with time-delay, quadratic stability and quadratic stabilizability concepts [1] were used to accomplish with robust stability analysis, robust control and robust filter design [6], [7], [15], [16], [19], [22], [24], [26]. For uncertain systems in polytopic domains, a simple evaluation of the feasibility of a set of LMIs defined at the vertices of the polytope provides sufficient conditions for the existence of a robust feedback gain or a full order linear filter. However, the analysis of stability based on constant Lyapunov functions can sometimes provide very conservative results. Some recent works introduced the analysis of robust stability and other closed-loop properties by means of parameter dependent Lyapunov functions [8], [10], [13].

Very recently, systematic ways to test for the existence of parameter dependent Lyapunov functions have appeared, as in [5], [28], where an augmented LMI formulation with extra matrix variables yields sufficient conditions for the robust stability of a polytope of matrices. Another simple and efficient way to construct such Lyapunov matrices can be found in [29], [30].

This note exploits the methodology first introduced in [29], [30] to provide less conservative sufficient conditions for the robust stability of time-delay systems with uncertain parameters in polytopic type domains. The key idea is to use homogeneity properties of the LMIs and simple algebraic manipulations to derive sufficient conditions for the negative definiteness of the Lyapunov-Krasovskii functional time-derivative associated to the time-delayed system. As a result, a feasibility LMI test formulated at the vertices of the uncertainty polytope provides parameter dependent matrices for that functional. To illustrate the technique proposed, some existent LMI conditions for the stability of time-delay systems are here extended to cope with robust stability in both delay-dependent and delay-independent cases. The conditions proposed encompass previous results based on quadratic stability and are illustrated by means of some examples.

Notations. $\Re^+$ is the set of nonnegative real numbers. I denotes the identity matrix of appropriate dimensions. $\mathcal{C}_\tau = \mathcal{C}([-\tau, 0], \Re^n)$ denotes the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\Re^n$ with the topology of uniform convergence. $\| \cdot \|$ refers to either the Euclidean vector norm or the induced matrix 2-norm. $\| \phi \|_c = \sup_{-\tau \le t \le 0} \| \phi(t) \|$ stands for the norm of a function $\phi \in \mathcal{C}_\tau$. When the delay is finite then "sup" can be replaced by "max". $\mathcal{C}_\tau^v$ is the set defined by $\mathcal{C}_\tau^v = \{\phi \in \mathcal{C}_\tau \; ; \; \| \phi \|_c < v, \; v > 0\}$. The symbol $\star$ stands for symmetric blocks in the LMIs.

2 Preliminaries

Consider a continuous-time linear system given by

$$\dot{x}(t) = Ax(t) + A_\tau x(t - \tau) \tag{1}$$

with the initial conditions

$$x(t_0 + \theta) = \phi(\theta), \forall \theta \in [-\tau, 0], \; t_0, \phi \in \Re_+ \times \mathcal{C}_\tau^v \tag{2}$$

where $x \in \mathbb{R}^n$ is the state, $\tau > 0$ is a constant time-delay[1]. The matrices A and A_τ are not precisely known, but belong to a polytope type uncertain domain $\mathcal{D}$ given by

$$\mathcal{D} = \Big\{ (A, A_\tau)(\xi) \; : \; (A, A_\tau)(\xi) = \sum_{j=1}^{N} \xi_j (A, A_\tau)_j \; ; \quad \sum_{j=1}^{N} \xi_j = 1 \; ; \; \xi_j \geq 0 \Big\} \tag{3}$$

in such a way that any matrix pair inside the domain $\mathcal{D}$ can be written as a convex combination of the vertices $(A, A_\tau)_j$ of the uncertainty polytope. In the LMIs that follow, A_j and $A_{\tau j}$ appearing separately are related to the vertex $(A, A_\tau)_j$ of $\mathcal{D}$, $j = 1, \ldots, N$.

The objective we pursue in this paper consists in providing less conservative sufficient conditions for the robust stability of system (1)-(3) in both delay-dependent and delay-independent contexts. For this, we will use some quadratic Lyapunov functionals depending on the parameter ξ.

3 Delay-dependent conditions

In the literature, there are several LMI conditions assuring delay-dependent robust stability for linear systems described as in (1)-(3). The following result is from [6].

Lemma 1 *Consider the state delayed uncertain linear system (1) with $(A, A_\tau)(\xi) \in \mathcal{D}$ and a scalar $\bar{\tau} > 0$ given. This system is stable for any constant time delay τ such that $0 \leq \tau \leq \bar{\tau}$ if there exist symmetric positive-definite matrices X, X_1 and X_2 with appropriate dimensions satisfying*

$$\begin{bmatrix} \Pi_{11} & X_1 A'_{\tau j} & X_2 A'_j & \bar{\tau} X A'_j & \bar{\tau} X A'_{\tau j} \\ \star & -X_1 & 0 & 0 & 0 \\ \star & \star & -X_2 & 0 & 0 \\ \star & \star & \star & -X_1 & 0 \\ \star & \star & \star & \star & -X_2 \end{bmatrix} < 0 \tag{4}$$

$$\Pi_{11} \triangleq X(A_j + A_{\tau j})' + (A_j + A_{\tau j})X$$

$$\forall \, j = 1, \cdots, N$$

Note that the above result exploits the convexity of the set of LMIs defined at the vertices of $\mathcal{D}$ and that matrices X, X_1 and X_2 are constant for all $A, A_\tau \in \mathcal{D}$. A line search on $\bar{\tau}$ can be easily implemented in order to provide the maximum value for the time-delay such that the uncertain system (1)-(3) maintains its stability. When $\bar{\tau} = 0$, one recovers the classical quadratic stability result for the uncertain system $\dot{x}(t) = (A + A_\tau)x(t)$ expressed in the block $(1,1)$ of (4).

[1] For simplicity, only the case of a single delay is presented. The extension of the results to handle multiple time delays is straightforward.

The following theorem exploits the homogeneity of the above set of LMIs in order to provide less conservative stability conditions which have as a particular case the quadratic stability of system (1)-(3) with respect to uncertainties and time-delay.

Theorem 1 *Consider the state delayed uncertain linear system (1) with $A, A_\tau \in \mathcal{D}$ and a scalar $\bar{\tau} > 0$ given. This system is stable for any constant time delay τ such that $0 \leq \tau \leq \bar{\tau}$ if there exist symmetric positive-definite matrices W_j, X_j and Y_j, $j = 1, \ldots, N$ with appropriate dimensions satisfying*

$$\mathfrak{M}_j \triangleq \begin{bmatrix} \Theta_{11} & X_j A'_{\tau j} & Y_j A'_j & \bar{\tau} W_j A'_j & \bar{\tau} W_j A'_{\tau j} \\ \star & -X_j & 0 & 0 & 0 \\ \star & \star & -Y_j & 0 & 0 \\ \star & \star & \star & -X_j & 0 \\ \star & \star & \star & \star & -Y_j \end{bmatrix} < -\mathrm{I} \tag{5}$$

$$\mathfrak{M}_{jk} \triangleq \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} & \mathcal{M}_{14} & \mathcal{M}_{15} \\ \star & -X_j - X_k & 0 & 0 & 0 \\ \star & \star & -Y_j - Y_k & 0 & 0 \\ \star & \star & \star & -X_j - X_k & 0 \\ \star & \star & \star & \star & -Y_j - Y_k \end{bmatrix} < \frac{2}{N-1}\mathrm{I}$$

$$j = 1, \ldots, N-1 \; ; \; k = j+1, \ldots, N \tag{6}$$

with

$$\Theta_{11} \triangleq W_j(A_j + A_{\tau j})' + (A_j + A_{\tau j})W_j$$

$$\mathcal{M}_{11} \triangleq W_j(A_k + A_{\tau k})' + (A_k + A_{\tau k})W_j + W_k(A_j + A_{\tau j})' + (A_j + A_{\tau j})W_k$$

$$\mathcal{M}_{12} \triangleq X_j A'_{\tau k} + X_k A'_{\tau j} \; ; \; \mathcal{M}_{13} \triangleq Y_j A'_k + Y_k A'_j$$

$$\mathcal{M}_{14} \triangleq \bar{\tau}(W_j A'_k + W_k A'_j) \; ; \; \mathcal{M}_{15} \triangleq \bar{\tau}(W_j A'_{\tau k} + W_k A'_{\tau j})$$

Proof: The aim is to prove that $W(\xi)$, $X(\xi)$ and $Y(\xi)$ are positive definite parameter dependent Lyapunov matrices given by

$$W(\xi) = \sum_{j=1}^{N} \xi_j W_j \; ; \; X(\xi) = \sum_{j=1}^{N} \xi_j X_j \; ; \; Y(\xi) = \sum_{j=1}^{N} \xi_j Y_j$$

$$\sum_{j=1}^{N} \xi_j = 1 \; ; \; \xi_j \geq 0$$

such that

$$\mathfrak{M}(\xi) \triangleq \begin{bmatrix} \Upsilon_{11}(\xi) & \Upsilon_{12}(\xi) & \Upsilon_{13}(\xi) \\ \star & -X(\xi) & 0 \\ \star & \star & -Y(\xi) \\ \star & \star & \star \\ \star & \star & \star \end{bmatrix}$$

$$\begin{matrix} \Upsilon_{14}(\xi) & \Upsilon_{15}(\xi) \\ 0 & 0 \\ 0 & 0 \\ -X(\xi) & 0 \\ \star & -Y(\xi) \end{matrix} \left. \vphantom{\begin{matrix} a \\ a \\ a \\ a \\ a \end{matrix}} \right] < 0 \tag{7}$$

$$\Upsilon_{11}(\xi) \triangleq W(\xi)(A(\xi) + A_\tau(\xi))' + (A(\xi) + A_\tau(\xi))W(\xi)$$
$$\Upsilon_{12}(\xi) \triangleq X(\xi)A_\tau(\xi)' \;\; ; \;\; \Upsilon_{13}(\xi) \triangleq Y(\xi)A(\xi)'$$
$$\Upsilon_{14}(\xi) \triangleq \bar{\tau}W(\xi)A(\xi)' \;\; ; \;\; \Upsilon_{15}(\xi) \triangleq \bar{\tau}W(\xi)A_\tau(\xi)'$$

holds for all $(A, A_\tau)(\xi) \in \mathcal{D}$. The left-hand side of equation (7) can be rewritten as

$$\mathfrak{M}(\xi) = \sum_{j=1}^{N} \xi_j^2 \, \mathfrak{M}_j + \sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \xi_j \xi_k \, \mathfrak{M}_{jk}$$

Imposing conditions (5)-(6) and taking into account that $\xi_j \xi_k$ is always nonnegative, one gets

$$\mathfrak{M}(\xi) < -\left(\sum_{j=1}^{N} \xi_j^2 - \frac{2}{N-1} \sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \xi_j \xi_k \right) \mathrm{I} \leq 0 \tag{8}$$

since

$$(N-1)\sum_{j=1}^{N} \xi_j^2 - 2\sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \xi_j \xi_k = \sum_{j=1}^{N-1} \sum_{k=j+1}^{N} (\xi_j - \xi_k)^2 \geq 0 \tag{9}$$

This proves that (7) holds for all $(A, A_\tau)(\xi) \in \mathcal{D}$. The stability with respect to any time-delay τ such that $0 \leq \tau \leq \bar{\tau}$ follows from [6]. □

Note that if the constraints $W = W_j$, $X = X_j$ and $Y = Y_j$, $j = 1, \ldots, N$ are imposed, one recovers the quadratic stability conditions of time-delay systems as in (4). In this case, the constraints (6) are redundant and do not need to be taken into account. The main idea of the above theorem is to exploit the homogeneity of the left-hand side of (5) (the robust stability of the vertices is indeed a necessary condition for the stability of the entire polytope), that is, if there is a feasible solution (W_j, X_j, Y_j) such that the LMIs hold, then $(\rho W_j, \rho X_j, \rho Y_j)$ is also a feasible solution for all $\rho > 0$. This fact allows to impose $-I$ as the right-hand side of (5) without loss of generality. Moreover, with the above conditions, stable systems which are not quadratically stable can be tested with respect to their robust stability against time-delays (until now, the conditions to verify the robust stability for state-delayed uncertain systems were based on quadratic stability assumptions).

Using similar ideas, it is possible to obtain less conservative LMI conditions for state delayed uncertain systems in convex bounded domains from other existing conditions in the literature. For instance, consider the LMIs presented in [25] which are reproduced in the next lemma.

Lemma 2 *Consider the state delayed uncertain linear system (1) with $(A, A_\tau)(\xi) \in \mathcal{D}$ and a scalar $\bar{\tau} > 0$ given. This system is stable for any constant time delay τ such that $0 \leq \tau \leq \bar{\tau}$ if there exist symmetric positive-definite matrices P, Q, X and Z and matrix Y with appropriate dimensions satisfying*

$$\begin{bmatrix} \Gamma_{11} & -Y + PA_{\tau j} & \bar{\tau}A_j'Z \\ \star & -Q & \bar{\tau}A_{\tau j}'Z \\ \star & \star & -\bar{\tau}Z \end{bmatrix} < 0 \;\; ; \;\; j = 1, \ldots, N \tag{10}$$

$$\begin{bmatrix} X & Y \\ Y' & Z \end{bmatrix} \geq 0 \tag{11}$$

with

$$\Gamma_{11} \triangleq A_j'P + PA_j + \bar{\tau}X + Y + Y' + Q$$

Proof: See [25]. □

The extension of the above conditions following the lines of Theorem 1 is given below.

Theorem 2 *Consider the state delayed uncertain linear system (1) with $(A, A_\tau)(\xi) \in \mathcal{D}$ and a scalar $\bar{\tau} > 0$ given. This system is stable for any constant time delay τ such that $0 \leq \tau \leq \bar{\tau}$ if there exist symmetric positive-definite matrices P_j, Q_j, X_j and Z_j and matrix Y_j, $j = 1, \ldots, N$ with appropriate dimensions satisfying*

$$\begin{bmatrix} \mathcal{R}_{11} & -Y_j + P_jA_{\tau j} & \bar{\tau}A_j'Z_j \\ \star & -Q_j & \bar{\tau}A_{\tau j}'Z_j \\ \star & \star & -\bar{\tau}Z_j \end{bmatrix} < -\mathrm{I}$$
$$j = 1, \ldots, N \tag{12}$$

$$\begin{bmatrix} X_j & Y_j \\ Y_j' & Z_j \end{bmatrix} \geq 0 \;\; ; \;\; j = 1, \ldots, N \tag{13}$$

and

$$\begin{bmatrix} \mathcal{N}_{11} & \mathcal{N}_{12} & \mathcal{N}_{13} \\ \star & -Q_j - Q_k & \bar{\tau}(A_{\tau j}'Z_k + A_{\tau k}'Z_j) \\ \star & \star & -\bar{\tau}(Z_j + Z_k) \end{bmatrix} < \frac{2}{N-1}\mathrm{I}$$
$$j = 1, \ldots, N-1 \; ; \; k = j+1, \ldots, N \tag{14}$$

with

$$\mathcal{R}_{11} \triangleq A_j'P_j + P_jA_j + \bar{\tau}X_j + Y_j + Y_j' + Q_j$$
$$\mathcal{N}_{11} \triangleq A_j'P_k + P_kA_j + \bar{\tau}(X_j + X_k) + Y_j + Y_k + Y_j' + Y_k' + Q_j + Q_k$$
$$\mathcal{N}_{12} \triangleq -Y_j - Y_k + P_jA_{\tau k} + P_kA_{\tau j}$$
$$\mathcal{N}_{13} \triangleq \bar{\tau}(A_j'Z_k + A_k'Z_j)$$

Proof: It follows similar steps as in the proof of Theorem 1. □

Note again that the results of Theorem 2 encompass the quadratic stability based results of Lemma 2, since if there exist $P = P_j$, $Q = Q_j$, $X = X_j$ and $Z = Z_j$ and matrix $Y = Y_j$ satisfying (10)-(11), then (12)-(13) also holds and (14) becomes redundant.

Following similar manipulation, many other LMI based conditions for the robust stability of uncertain systems could be improved. When discrete-time systems are under investigation, similar manipulation as in the lines depicted in [29] can be followed.

4 Delay-independent conditions

Delay-independent stability conditions for system (1)-(3) can be encountered in many references in the literature [27]. A sufficient condition formulated in terms of LMIs is reproduced below.

Lemma 3 *Consider the state delayed uncertain linear system (1) with $(A, A_\tau)(\xi) \in \mathcal{D}$. This system is stable for any constant finite time delay τ if there exist symmetric positive-definite matrices P and S with appropriate dimensions satisfying*

$$\begin{bmatrix} A_j'P + PA_j + S & A_{\tau j}'P \\ \star & -S \end{bmatrix} < 0 \ ; \ j = 1, \ldots, N \qquad (15)$$

Note that the above condition does not depend on the size of the time delay τ. The extension of the above result to the case where P and S are respectively replaced by the parameter dependent Lyapunov functions

$$P(\xi) = \sum_{j=1}^{N} \xi_j P_j \ ; \ S(\xi) = \sum_{j=1}^{N} S_j \ ; \ \sum_{j=1}^{N} \xi_j = 1 \ ; \ \xi_j \geq 0 \qquad (16)$$

is presented in the following theorem.

Theorem 3 *Consider the state delayed uncertain linear system (1) with $(A, A_\tau)(\xi) \in \mathcal{D}$. This system is stable for any constant finite time delay τ if there exist symmetric positive-definite matrices P_j and S_j, $j = 1, \ldots, N$ with appropriate dimensions satisfying*

$$\begin{bmatrix} A_j'P_j + P_jA_j + S_j & A_{\tau j}'P_j \\ \star & -S_j \end{bmatrix} < -\mathrm{I} \ ; \ j = 1, \ldots, N \qquad (17)$$

$$\begin{bmatrix} \Omega_{11} & A_{\tau j}'P_k + A_{\tau k}'P_j \\ \star & -S_j - S_k \end{bmatrix} < \frac{2}{N-1}\mathrm{I}$$

$$j = 1, \ldots, N-1 \ ; \ k = j+1, \ldots, N \qquad (18)$$

with

$$\Omega_{11} \triangleq A_j'P_k + P_kA_j + A_k'P_j + P_jA_k + S_j + S_k$$

In the affirmative case, $P(\xi)$ and $S(\xi)$ given by (16) are parameter dependent quadratic functions that verify

$$\begin{bmatrix} A(\xi)'P(\xi) + P(\xi)A(\xi) + S(\xi) & A_{\tau j}(\xi)'P(\xi) \\ \star & -S(\xi) \end{bmatrix} < 0 \qquad (19)$$

for all $(A, A_\tau)(\xi) \in \mathcal{D}$.

Proof: To show that conditions (17)-(18) imply (19), just follow similar steps as the ones used in the proof of Theorem 1. From (19), it can be proved using standard manipulations that the augmented state vector $\begin{bmatrix} x(t) & x(t-\tau) \end{bmatrix}'$ decreases asymptotically to the origin independently of the time-delay τ [23]. □

Other extensions could be formulated using the manipulation presented here or similar. However, the extension of the present technique to the case of polytopic uncertain systems with saturating actuators, as studied in [33], requires special attention and has to be investigated more deeply.

5 Illustrative Examples

In this section, some examples illustrate the previous results.

Example 1. A first example has been randomly generated in order to provide a system whose stability is time-dependent. The two-vertices are given by

$$(A, A_\tau)_1 = \left(\begin{bmatrix} -1.3451 & 0.6510 \\ 0.6135 & -0.3007 \end{bmatrix}, \begin{bmatrix} 0.0025 & -0.7350 \\ 0.0859 & -0.0086 \end{bmatrix} \right)$$

$$(A, A_\tau)_2 = \left(\begin{bmatrix} -0.1849 & 0.1202 \\ -0.9822 & 0.1787 \end{bmatrix}, \begin{bmatrix} -0.3219 & 0.1123 \\ 0.4372 & -0.1571 \end{bmatrix} \right)$$

The aim here is not to compare conditions from Lemmas 1 and 2 to show that one is less conservative than the other, but to illustrate that both LMI conditions (4) and (10)-(11) can be considerably improved for the analysis of uncertain systems using the ideas presented in this paper.

From Lemma 1, the maximum value of τ for which the LMIs of (4) admit a feasible solution has found to be $\tau_{Lemma1} = 0.354$. Using the extension provided by the results of Theorem 1, one gets $\tau_{Theorem1} = 0.599$.

If the alternative LMI conditions of Lemma 2 were used, the maximum allowed time delay while robust stability is preserved is $\tau_{Lemma2} = 0.522$, while the improved results of Theorem 2 yield $\tau_{Theorem2} = 1.480$.

As it can be seen, the use of parameter dependent matrices allows the robust stability test to yield less conservative results.

Example 2. As second example, consider the uncertain state delayed linear system given as in (1)-(3) with the following vertex matrices

$$(A, A_\tau)_1 = \left(\begin{bmatrix} -0.6649 & 0.7192 \\ 0.3376 & -0.3726 \end{bmatrix}, \begin{bmatrix} -0.0032 & 0.0034 \\ 0.0016 & -0.0018 \end{bmatrix} \right)$$

$$(A, A_\tau)_2 = \left(\begin{bmatrix} -1.3117 & -0.7129 \\ -0.4754 & -0.2641 \end{bmatrix}, \begin{bmatrix} -0.0063 & -0.0034 \\ -0.0023 & -0.0013 \end{bmatrix} \right)$$

$$(A, A_\tau)_3 = \left(\begin{bmatrix} 0.0218 & 0.0583 \\ -0.5268 & -1.1624 \end{bmatrix}, \begin{bmatrix} 0.0001 & 0.0003 \\ -0.0025 & -0.0056 \end{bmatrix} \right)$$

Without taking into account the vertex $(A, A_\tau)_3$, the uncertain system is verified to be stable independently of the time-delay τ through the conditions of Lemma 3. The same is no longer true when vertex #3 is included in the analysis (the condition of Lemma 3 fails). Nevertheless, a feasible solution is obtained from the LMIs of Theorem 3, assuring for the entire uncertainty domain the robust stability independently of the size of the time-delay τ. Once again, it is clear that the extension provided by Theorem 3 proposes less conservative robust stability analysis results.

6 Conclusion

A very simple LMI-based sufficient condition for robust stability of uncertain time-delay linear systems in convex bounded domains has been presented in this paper. With these conditions, less conservative results than the ones existing in the literature are obtained in both delay-dependent and delay-independent cases. Moreover, the manipulations presented here give the main lines based on which many other LMI-based analysis conditions can be significantly improved.

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