ROBUST STABILIZATION OF INTERVAL PLANTS

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Abstract

A simple necessary condition and su±cient condition for robust Hurwitz stability of interval polynomial is obtained using the results of Nie [13] for ¯xed polynomials. These conditions provide a simple and quick test to detect many unstable systems. Furthermore, these conditions are used to derive a robust controller for interval plants which involves less computational complexity compared to extreme point results due to Barmish [2]. The proposed method is illustrated by a numerical example.

1 Introduction

The stability of a linear time invariant continuous time feedback control system is characterized by root location of its characteristic polynomial. For stability the characteristic polynomial must be Hurwitz i.e. all of its roots should lie in the open left half of the complex plane. Robust stability analysis with uncertain parameters has been very important research topic. Since control systems operate under large uncertainties it is important to study stability robustness in the presence of uncertainty. The uncertainty present in the control system causes degradation of system performance and destabilization. An important approach to this subject is via expressing the characteristic polynomial by an interval polynomial, i.e. a polynomial whose coe±cient each vary independently in a prescribed interval. The stability analysis of polynomials subjected to parameter uncertainty have received considerable attention after the celebrated theorem of Kharitonov [11], which assures robust stability under the condition that four specially constructed \extreme polynomials", called Kharitonov polynomials are Hurwitz. The problem of robust stability of interval polynomial is also dealt in [1, 4, 5, 6, 7, 8]. In this paper, we propose a necessary condition and su±cient condition for interval polynomials using the results of Nie [13] for ¯xed polynomials. we also consider linear interval plant family in which the uncertainty in the plant is manifested via a priori interval bounds for each numerator and denominator polynomial coe±cient. Several results have appeared in the literature which aims at reducing test of Hurwitz stability of entire family to a small subset of entire family. In this regard few extreme point results are available in the literature. These includes work due to Ghosh [9], where he has shown that a pure gain compensator $C(s) = K$ stabilizes entire interval plant family if and only if it stabilizes a distinguished set of eight of the extreme plants. Hollot and Yang [10] considered the same setup as Ghosh but allow the controller to be ¯rst order. They prove that to robustly stabilize the entire family, it is necessary and su±cient to stabilize the set of extreme plants which are obtained by taking all possible combinations of extreme values of the plant numerator coe±cients with extreme values of the plant denominator coe±cients. If the plant numerator has degree m and the plant denominator is monic with degree n, the number of extreme plants can be as high as $N_{ext} = 2^{m+n+1}$. In [2], Barmish proved that, it is necessary and su±cient to stabilize only sixteen of the extreme plants. A complete survey of these extreme point is given in [3]. In this paper, we propose a method which involves less computational complexity compared to extreme point results given by Barmish [2] using the result of Nie [13] for ¯xed polynomials. The paper is organized as: Section 2 gives a necessary condition and su±cient condition for robust Hurwitz stability of interval polynomial. In Section 3 a design procedure for robust stabilization of interval plants is presented. In Section 4, a numerical example illustrates the method developed in Section 3. Finally conclusion is given in Section 5.

2 A Necessary and sufficient condition for robust stability of interval polynomial

Consider the set of real polynomials of degree n of the form

$$\delta(s) = \delta_0 + \delta_1 s + \delta_2 s^2 + \delta_3 s^3 + \delta_4 s^4 + \cdots + \delta_n s^n \quad (1)$$

where the coefficients lie within given ranges,

$$\delta_0 \in [x_0, y_0],\ \delta_1 \in [x_1, y_1],\ \cdots, \delta_n \in [x_n, y_n].$$

We assume that the degree remains invariant over the family, so that $0 \notin [x_n, y_n]$. Such a set of polynomials is called a real interval family and is referred as an interval polynomial. The set of polynomials given by (1) is stable if and only if each and every element of the set is a Hurwitz polynomial. We will propose a necessary condition and sufficient condition for the robust stability of interval polynomial (1) using the algebraic stability criterion for fixed polynomials due to Nie [13].
Consider the following fixed polynomial of the form,

$$\Delta(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n \quad (2)$$

Lemma 2.1 A necessary condition that the roots of $\Delta(s)$ lie in the left half plane is

$$a_i > 0; \quad i = 0, 1, 2, \cdots, n$$

$$a_i a_{i+1} > a_{i-1} a_{i+2}; \quad i = 1, 2, \cdots n-2. \quad (3)$$

Lemma 2.2 A sufficient condition that the roots of $\Delta(s)$ lie in the left half plane is

$$a_i > 0; \quad i = 0, 1, 2, \cdots, n$$

$$0.4655 a_i a_{i+1} > a_{i-1} a_{i+2}; \quad i = 1, 2, \cdots n-2. \quad (4)$$

The above results for fixed polynomial due to Nie [13] can be extended to derive a necessary and a sufficient condition for stability of interval polynomial and are stated in the following theorems.

Theorem 2.1 The interval polynomial $\delta(s)$ defined in (1) is Hurwitz for all $\delta_i \in [x_i, y_i]$ where $i = 0, 1, 2, \cdots, n$ if the following necessary conditions are satisfied

$$y_i \geq x_i > 0 \quad i = 0, 1, 2, \cdots, n$$

$$x_i x_{i+1} > y_{i-1} y_{i+2} \quad i = 1, 2, \cdots n-2. \quad (5)$$

Proof: Consider any arbitrary polynomial $\delta_k(s)$, which is a member of the set of real polynomials in equation (1). This polynomial will have fixed coefficients. Then according to Lemma 2.1, this polynomial satisfies the necessary condition for the roots to lie in the left half plane if condition in equation (3)is satisfied. For this arbitrary polynomial, the necessary condition given in equation (5) satisfies the condition given in equation (3). This is true for any arbitrary polynomial in equation (1). This completes the proof.

Theorem 2.2 The interval polynomial $\delta(s)$ defined in (1) is Hurwitz for all $\delta_i \in [x_i, y_i]$ where $i = 0, 1, 2, \cdots, n$ if the following sufficient conditions are satisfied

$$y_i \geq x_i > 0 \quad i = 0, 1, 2, \cdots, n$$

$$0.4655 x_i x_{i+1} > y_{i-1} y_{i+2} \quad i = 1, 2, \cdots n-2. \quad (6)$$

Proof: Consider any arbitrary polynomial $\delta_k(s)$, which is member of the set of real polynomials in equation (1). This polynomial will have fixed coefficients. Then according to Lemma 2.2, this polynomial satisfies the sufficient condition for the roots to lie in the left half plane if condition in equation (4)is satisfied. For this arbitrary polynomial, the sufficient condition given in equation (6) satisfies the condition given in equation (4). This is true for any arbitrary polynomial in equation (1). This completes the proof.

3 Design procedure for robust stabilization of interval plants

Consider a strictly proper interval plant family consisting all plants of the form,

$$G(s, p, q) = \frac{N(s, p)}{D(s, q)} \quad (7)$$

where the numerator and denominator polynomials are of the form

$$N(s, p) = p_0 + p_1 s + p_2 s^2 + \cdots + p_{m-1} s^{m-1} + p_m s^m \quad (8)$$

$$D(s, q) = q_0 + q_1 s + q_2 s^2 + \cdots + q_{n-1} s^{n-1} + q_n s^n \quad (9)$$

and where vectors p and q lie in given rectangles P and Q, respectively, i.e.,

$$p \in P = \left\{ p : p_i^- \leq p_i \leq p_i^+ \ \text{ for } i = 0, 1, \cdots, m \right\} \quad (10)$$

and

$$q \in Q = \left\{ q : q_i^- \leq q_i \leq q_i^+ \ \text{ for } i = 0, 1, \cdots, n \right\} \quad (11)$$

where $q_n = [1, 1]$ and the bounds p_i^-, p_i^+, q_i^-, and q_i^+ are specified a priori.
To stabilize the interval plant family, we consider a proper first-order compensator of the form (any form for the controller can be considered),

$$C(s) = K_1 + \frac{K_2}{s} = \frac{N_c(s)}{D_c(s)} \tag{12}$$

We say that this compensator $C(s)$ robustly stabilizes the interval plant family if, for all $p \in P$ and all $q \in Q$, the resulting closed loop polynomial

$$\Delta(s, p, q) = N_c(s)N(s, p) + D_c(s)D(s, q) \tag{13}$$

has all its roots in the strict left half plane; that is $\Delta(s, p, q)$ is Hurwitz. This is being the case, $C(s)$ is said to be a robust stabilizer and the closed loop system is said to be robustly stable. Let the closed loop interval polynomial be in the form

$$\Delta(s, p, q) = [x_0, y_0] + [x_1, y_1]s + \cdots + [x_n, y_n]s^n + [1, 1]s^{n+1} \tag{14}$$

The stability conditions in (5) and (6) can be applied to closed loop characteristic polynomial in (14), which leads to inequalities in terms of compensator parameters. These inequalities can be solved to obtain compensator parameters. Even though the method in [2] gives a necessary and sufficient condition for robust stabilization using only sixteen extreme plants, the method still involves much computational complexity since it is required to construct sixteen Routh table and solve the constraints (obtained by enforcing positivity in the first column of the Routh table) for stability. Although the proposed method is based on necessary condition and sufficient condition it involves less computational complexity and provides a simple way to obtain a robust controller. The following numerical example illustrates the design procedure.

4 Numerical example

We consider the model given in [2] for an experimental oblique wing aircraft. In the absence of uncertainty, the aircraft transfer function is

$$G(s) = \frac{64s + 128}{s^4 + 3.7s^3 + 65.6s^2 + 32s}$$

Now, for robust stabilization purposes, we replace $G(s)$ by the interval plant family given by

$$G(s, p, q) = \frac{N(s, p)}{D(s, q)}$$

where

$$N(s, p) = [54, 74]s + [90, 166]$$

and

$$D(s, q) = [1, 1]s^4 + [2.8, 4.6]s^3 + [50.4, 80.8]s^2 + [30.1, 33.9]s + [-0.1, 0.1]$$

Consider a PI controller of the form

$$K_1 + \frac{K_2}{s} = \frac{N_c(s)}{D_c(s)}$$

to robustly stabilize given model of aircraft. The $C(s)$ will stabilize the given model of aircraft if the closed loop interval polynomial in (14) is stable. The closed loop interval polynomial in (14) becomes

$$\begin{aligned} \Delta(s, a, b) = {} & [1, 1]s^5 + [2.8, 4.6]s^4 + [50.4, 80.8]s^3 \\ & + [54K_1 + 30.1, 74K_1 + 33.9]s^2 \\ & + [90K_1 + 54K_2 - 0.1, 166K_1 + 74K_2 + 0.1]s \\ & + [90K_2, 166K_2] = 0 \end{aligned} \tag{15}$$

By applying conditions in (5) and (6) to (15) and solving the resulting inequalities using software package MATLAB [12] we obtain one of the robust stabilizing PI controller as,

$$C(s) = 0.5 + \frac{0.1}{s}$$

5 Conclusion

A simple necessary condition and sufficient condition for robust Hurwitz stability of interval polynomial is obtained using the results of Nie for Hurwitz stability of fixed polynomials. These conditions provide a simple and quick test to detect many unstable systems. Even though the method in [2] gives a necessary and sufficient condition for robust stabilization using only sixteen extreme plants, the method still involves much computational complexity since it is required to construct sixteen Routh table and solve the constraint for stability. Although the proposed method is based on necessary condition and sufficient condition it involves less computational complexity and provides a simple way to obtain a robust controller. The proposed method is illustrated by a numerical example.

References

[1] B. R. Barmish, "A generalization of Kharitonov's four polynomial concept for robust stability problems with linearly dependent coefficient perturbations", IEEE Trans. Automatic Control, Vol. 34, pp. 157-165, 1989.

[2] B. R. Barmish, C. V. Hollot, F. J. Kraus, and R. Tempo, "Extreme point results for robust stabilization of interval plants with first order compensator", IEEE Trans. Automatic Control, Vol. AC-37, pp. 707-714,1992.

[3] B. R. Barmish, and H. I. Kang, "A survey of extreme point results for robustness of control systems", Automatica, Vol. 29, No. 1, pp. 13-35, 1993.

[4] A. C. Bartlett, C. V. Hollot and L. Huang, \Root location of an entire polytope of polynomials: it su±ces to check edges", Mathematics of Control, Signals and Systems, Vol. 1, pp. 61-71,1988.

[5] S. P. Bhattachrya, \Robust stabilization against structured perturbations ", Lect. Notes in Control and Information Sciences, Vol. 99, Springer Verlag, Berlin, 1987.

[6] S. P. Bhattachrya, H. Chapelett, L. H. Keel, \Robust control: The parametric approach", Prentice Hall Inc., NJ. 1995.

[7] P. Dorato(Ed.), \Robust Control", IEEE Press, NY, 1987.

[8] P. Dorato, and R. K. Yedavalli(Eds.), \Recent advances in robust control", IEEE Press, NY, 1990.

[9] B. K. Ghosh, \Some new results on the simultaneous stabilization of a family of single input , single output systems ", Systems and Control Letters, Vol. 06, pp.39-45,1985.

[10] C. V. Hollot and F. Fang, \Robust stabilization of interval plants using lead or lag compensators", Systems and Control Letters, Vol. 14, pp. 9-12, 1990.

[11] V. L. Kharitanov, \Asymptotic stability of an equilibrium position of a family of systems of linear di®erential equations ",di®erentsial'nye Uravneniya, Vol. 14, No.11, pp. 2086-2088, 1978: and English translation, Di®erential Equations, Vol. 14, pp. 1483-1485, 1979.

[12] Mathworks Inc., \Matlab: Users Guide", 1999.

[13] Y. Y. Nie, \A new class of criterion for the stability of the polynomials", Acta Mechnica Sinica, pp. 110-116, 1976.