INFLUENCE OF TIME-DELAY MISMATCH ON ROBUSTNESS AND PERFORMANCE

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Abstract

The effect of the uncertainty in time-delay on the performance and robustness of a two-degree-of-freedom control system is discussed. The result shows that this delay mismatch strongly influences the reachable closed-loop performance and vice-versa. Acceptable robustness and robustness sensitivity can only be reached for less demanding performance requirements.

1 Introduction

Most of the discrete-time identification, control even adaptive algorithms assume the apriori knowledge of the process time-delay. This apriori knowledge is sometimes very uncertain and the uncertainty can result from a lack of precision in mathematical modeling of the plant and/or changes in the plant parameters with time. It would be desirable to know how the time-delay mismatch influences the basic robustness and performance behaviors of the closed-loop control.

An important area of research in control theory is the design of feedback controllers for systems which have significant uncertainties in the plant and the explicit incorporation of model uncertainties in the design of high performance control systems. This leads to methods for designing robust stability and performance. In the past decade the dialectics of performance versus robustness became one of the major control paradigm.

Some controller design methodology, mostly for discrete-time systems, include the time-delay of the plant also into the parameters. Unfortunately relatively few papers [1, 2, 9, 10], can be found dealing with the influence of the accuracy of the apriori knowledge or estimate of the time-delay, which is sometimes called the time-delay mismatch problem. Our paper investigates the influence of the time-delay mismatch on the robustness and performance.

The *generic two-degree of freedom* (*GTDOF*) system [3] is used here which is based on the *Youla-parametrization* [8] providing all realizable stabilizing regulators (*ARS*) for open-loop stable plants and capable to handle the plant time-delay.

A *GTDOF* control system is shown in Fig. 1, where y_r, u, y and w are the reference, process input, output and disturbance signals, respectively. The optimal *ARS* regulator of the *GTDOF* scheme [4] is given by

$$R_o = \frac{P_w K_w}{1 - P_w K_w S} = \frac{Q_o}{1 - Q_o S} = \frac{P_w G_w S_+^{-1}}{1 - P_w G_w S_- z^{-d}} \qquad (1)$$

where

$$Q_o = Q_w = P_w K_w = P_w G_w S_+^{-1} \qquad (2)$$

is the associated *Y-parameter* [6] furthermore

$$Q_r = P_r K_r = P_r G_r S_+^{-1} \; ; \; K_w = G_w S_+^{-1} \; ; \; K_r = G_r S_+^{-1} \qquad (3)$$

assuming that the process is factorable as

$$S = S_+ \overline{S}_- = S_+ S_- z^{-d} \qquad (4)$$

where S_+ means the inverse stable (*IS*) and S_- the inverse unstable (*IU*) factors, respectively. z^{-d} corresponds to the discrete time-delay, where d is the integer multiple of the sampling time. Here P_r and P_w are assumed stable and proper transfer functions (reference models). An interesting result was [5] that the optimization of the *GTDOF* scheme can be performed in $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norm spaces by the proper selection of the serial G_r and G_w embedded filters.

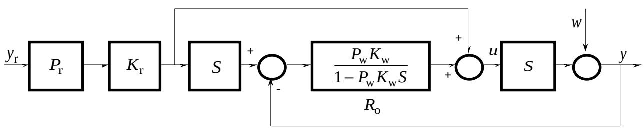

Figure 1: The *generic TDOF* (*GTDOF*) control system

Be M the model of the process. Assume that the process and its model are factorizable as

$$S = S_+ S_- = S_+ \overline{S}_- z^{-d} = \frac{B}{A} z^{-d}$$

and

$$M = M_+ M_- = M_+ \overline{M}_- z^{-d_m} = \frac{\hat{B}}{\hat{A}} z^{-d_m} \qquad (5)$$

where S_+ and M_+ mean the inverse stable (*IS*), $\overline{S}_-$ and $\overline{M}_-$ the inverse unstable (*IU*) factors, respectively. z^{-d} and z^{-d_m} correspond to discrete time delays, where d and d_m are the integer multiple of the sampling time, usually $d = d_m$ is assumed. (To get a unique factorization it is reasonable to ensure that $\overline{S}_-$ and $\overline{M}_-$ are monic, i.e., $\overline{S}_-(1) = \overline{M}_-(1) = 1$, having unity gain.) It is important that the inverse of the term z^{-d} is not realizable, because it would mean an ideal predictor z^d. These assumptions mean that $S_- = \overline{S}_- z^{-d}$ and $M_- = \overline{M}_- z^{-d_m}$ are uncancelable invariant factors for any design procedure. Introduce the additive

$$\Delta = S - M \quad ; \quad \Delta_+ = S_+ - M_+ \quad ; \quad \Delta_- = S_- - M_- \qquad (6)$$

and relative

$$\ell = \frac{\Delta}{M} = \frac{S - M}{M} \quad ; \quad \ell_+ = \frac{\Delta_+}{M_+} \quad ; \quad \ell_- = \frac{\Delta_-}{M_-} \qquad (7)$$

model errors. It is easy to show that the characteristic equation using the *ARS* regulator is (for $d = d_m = 0$)

$$M_+ M_- = 0 \qquad (8)$$

if a $Q = \tilde{Q}(M_+ M_-)^{-1}$ parameter is applied, i.e., if someone tries to cancel both factors. This means that the zeros of the *IU* factor will appear in the characteristic equation and cause unstability. This is why these zeros (and the time delay itself) are called invariant uncancelable factors.

The well known robust stability condition $\|Z\,\ell\|_\infty < 1$ for the *ARS* regulator gives $\|Q\,M\,\ell\|_\infty < 1$, i.e.,

$$\|Q\,M\|_\infty < \frac{1}{\|\ell\|_\infty} \quad \text{or} \quad \|\ell\|_\infty < \frac{1}{\|Q\,M\|_\infty} \qquad (9)$$

Thus the robust stability strongly depends on the model M and how the *Y-parameter* Q is selected.

Consider the practical form of the optimal regulator (using M in (1)) of the *GTDOF* system based on the available model M of the process

$$R_{\text{opt}} = \frac{P_w G_w M_+^{-1}}{1 - P_w G_w M_- z^{-d}} = \frac{\left(P_w G_w M_+^{-1}\right)}{1 - \left(P_w G_w M_+^{-1}\right)\left(M_+ M_- z^{-d}\right)} = \frac{Q}{1 - QM} \qquad (10)$$

where

$$Q = P_w G_w M_+^{-1} \qquad (11)$$

is the nominal *Y-parameter* depending on the model of the plant, which gives back (2) as $Q_o = Q\big|_{M=S} = P_w G_w S_+^{-1}$. The dependence on the inverse stable part is direct and visible, however, G_w generally depends on the inverse unstable part. We can now state that R_{opt} is also an *ARS* controller (but do not forget that only for the model M and not for the true process S).

Analyze the basic robust stability condition (9) obtained for *ARS* regulators in case of the *generic scheme*, where the optimal regulator is given by (10) and $Q = P_w G_w M_+^{-1}$ from (11). We get

$$|QM\ell| = \left|P_w G_w M_+^{-1} M \ell\right| = \left|P_w G_w M_- z^{-d}\,\ell\right| = |P_w \ell| \qquad (12)$$

where $|G_w M_-| = 1$, (because M_- is monic by definition and G_w is monic by construction), furthermore $\left|z^{-d}\right| = 1$ (which is well known) were used, thus finally

$$\sup_\omega |\ell| \le 1/|P_w| \quad \text{or} \quad \|\ell\|_\infty \le 1/\|P_w\|_\infty \qquad (13)$$

Because the right hand side of this inequality depends only on P_w, which is the reference model for the regulatory property of the *GTDOF* system, this means that this is a special controller structure, where the performance of the closed-loop is directly influenced by the robustness limit (via the selected P_w).

2 Model errors of the time-delay mismatch

Let us compute the relative model error ℓ for an *IS* plant, where the model uncertainty comes only from a time-delay mismatch. The delay-free term is assumed to be known exactly, so $\overline{M}_- = 1$ and $M_+ = S_+$. In this case

$$\ell = \ell_d = \frac{\Delta}{M} = \frac{S - M}{M} = \frac{S_+ z^{-d} - S_+ z^{-d_m}}{S_+ z^{-d_m}} = z^{-(d - d_m)} - 1 \qquad (14)$$

Assume an equivalent continuous time plant with time-delay τ and a model with time-delay τ_m. The analogous equivalence means

$$\ell = \ell_\tau = e^{-\Delta\tau s} - 1 \qquad (15)$$

where $\Delta\tau = \tau - \tau_m$. The robust stability condition (13) for the continuous time case is now

$$\sup_\omega |\ell_\tau| = \sup_\omega \left|e^{-j\Delta\tau\omega} - 1\right| \le 1/|P_w(\omega)| \qquad (16)$$

Assume a first order reference model

$$P_w = \frac{1}{1 + sT_w} \tag{17}$$

and using the first order Taylor expansion of the exponential term one can get a good approximation of (16) for small deviations

$$\left| 1 - \frac{\tau_m}{\tau} \right| \le \frac{T_w}{\tau} \tag{18}$$

The interpretation of (18) is very simple: for small T_w, which means high closed-loop performance, the model time delay τ_m must be close to the true delay τ. So it is obtained that the admissible time-delay mismatch is limited by the inverse of the performance. It could be furthermore very interesting how this limit influences the robustness of the loop.

Detailed investigation of this limiting behavior needs further numerical computations. Simple calculations give that the sensitivity function of the *GTDOF* system having time-delay mismatch for the discrete-time case is (assuming $G_w = 1$)

$$E = \frac{1 - P_w z^{-d_m}}{1 + \ell P_w z^{-d_m}} = \frac{1 - P_w z^{-d_m}}{1 + \ell_d P_w z^{-d_m}} \tag{19}$$

and the continuous time equivalent follows as

$$E = \frac{1 - P_w e^{-s\tau_m}}{1 + \ell P_w e^{-s\tau_m}} = \frac{1 - P_w e^{-s\tau_m}}{1 + \ell_\tau P_w e^{-s\tau_m}} \tag{20}$$

For P_w given by (17) the sensitivity function (20) becomes

$$E = \frac{1 + sT_w - e^{-s\tau_m}}{1 + sT_w + P_w\left(e^{-s\tau} - e^{-s\tau_m}\right)} \tag{21}$$

The well-known Nyquist stability margin (the simplest robustness measure) is defined by

$$\begin{aligned} \rho_m &= \rho_{min}(R) = \min_{\omega}\left|\rho(\omega, R)\right| = \min_{\omega}\left|1 + RS\right| = \\ &= \min_{\omega}\left|1 + Y(j\omega)\right| = \frac{1}{\|E\|_\infty} \end{aligned} \tag{22}$$

which is the distance between the point $(-1 + 0j)$ and the closest point of the open-loop transfer function $Y(j\omega)$. The reciprocal value of the norm is $\|E\|_\infty$. Unfortunately there is no simple analytical solution to obtain how the robustness the closed-loop depends on the time-delay mismatch and on the performance. It is, however, possible to compute the graphical plot of a complex functional relationship $\rho_m = \rho_{min}(\tau_m/\tau, T_w/\tau)$ with the help of MATLAB.

As a result Fig. 2 shows the function $\rho_{min}(T_w/\tau)$ for $\tau_m = 0.5\tau, \tau, 2\tau$. For the ideal $\tau_m = \tau$ (no mismatch) case ρ_{min} depends only on our design goal (T_w) and on the plant time-delay (τ), more exactly on their relative value T_w/τ. The best robustness measure is $\rho_{min}(0) = 0.5$ for cases when the reference model P_w requires a very fast transient response from the time-delay process and the measure is $\rho_{min}(\infty) = 1$, if τ is negligible comparing to the time lag of P_w. It can be well seen that either under- or over-estimation of the time-delay causes considerable decrease of the robustness. Virtually ρ_{min} is more sensitive for over-estimation. (The left ends of the plots correspond to the robust stability limit.) While the no mismatch case provides an all stabilizing property for any performance requirement, in case of a non zero time-delay mismatch one can always expect the violation of the robustness stability limit for higher performance design.

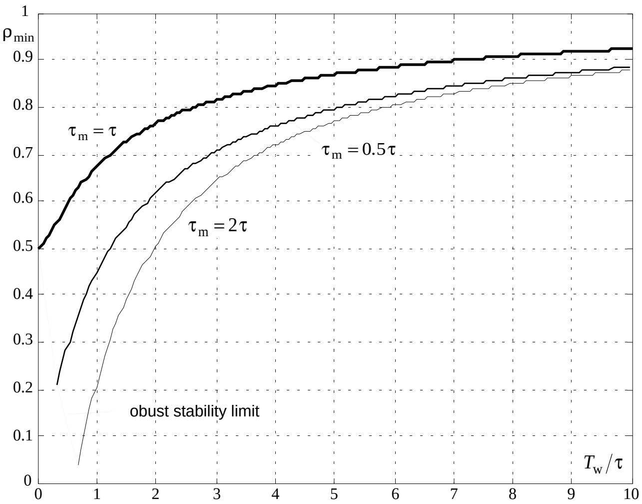

Figure 2: The function $\rho_{min}(T_w/\tau)$ for $\tau_m = 0.5\tau, \tau, 2\tau$

It may be more reasonable to plot the $\rho_{min}(\tau_m/\tau)$ function parametrized by T_w/τ as Fig. 3 shows. One can see how extremely sensitive the robustness is for high performance requirement, when T_w/τ is small and how this sensitivity decreases when T_w/τ is large for low performance design. It is also interesting to observe, that for small mismatch the over-estimation of the delay gives higher ρ_{min}, however, for large mismatch ρ_{min} is somewhat more sensitive, as it is shown in Fig. 2.

In a relatively wide range of T_w/τ, the over-estimation of the time-delay by τ^*/τ improves (i.e. increases) the ρ_{min} to ρ^*_{min} according to the maxima of the curves observable in Fig. 3. The over-estimation is less than 25% and the improvement is marginal, less than 5% as Fig. 4 shows.

If we assume that the time-delay mismatch is less than 20% in a practical case, the robustness degradation is always less than 10% for $T_w/\tau \geq 0.5$, which can be well seen in Fig. 3. So if we want to speed up the open-loop process to a time constant, considerable less than the delay, then it can be done only using a quite accurate knowledge of the time-delay. Contrary, if someone can expect a considerable variation in the time delay then only a less demanding (slower) design is much more reliable and robust.

(The jags of both figures origin from the relative accuracy of the numerical computations. Do not forget that the Nyquist plot of a time-delay process has infinite number of winds around the origin and sometimes even the radius of the external wind is quite small. So it is not easy to find such frequency scaling which provides to determine both ρ_{min} (i.e. $\|E\|_\infty$) and the robust stability limit at the same time within a proper accuracy.)

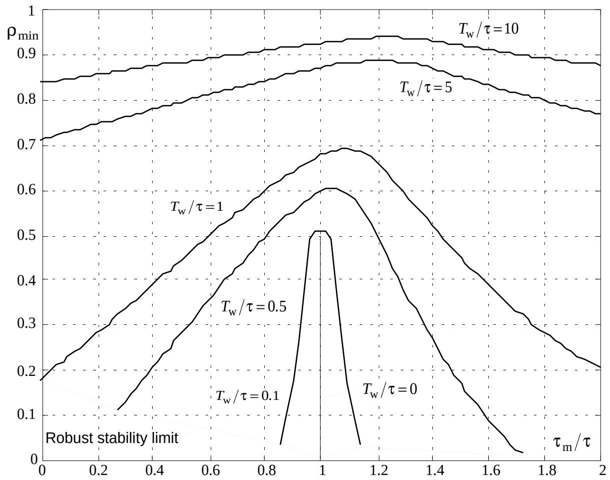

Figure 3: The function $\rho_{min}(\tau_m/\tau)$ parametrized by T_w/τ

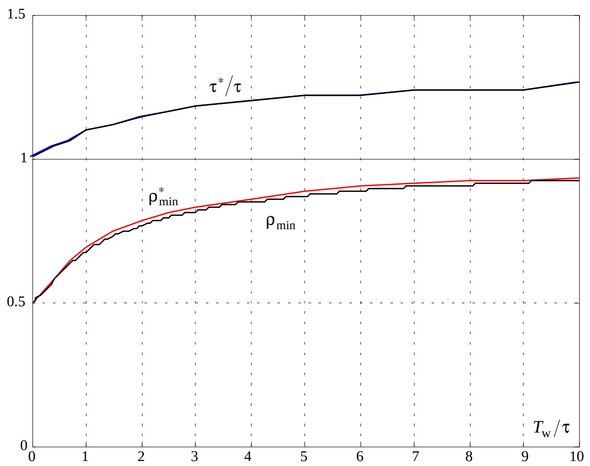

Figure 4: Influence of the time-delay over-estimation

The above results strengthen the conservative practical design experience that the time-delay is practically equivalent to an *IU* zero, i.e. invariant.

Further investigations [7] showed that the influence of time constant mismatch strongly depends on the plant parameters (τ/T), too, so it is not easy to observe similar relationships as obtained for time-delay mismatch. Another interesting observation is that the maximum of ρ_{min} also strongly depends on the performance requirement (T_w/τ), because an $\mathcal{H}_\infty$ optimal regulator would need the optimal computation of G_w, see [5]. As a conclusion for time constant mismatch it is not possible to obtain so general observations we got for the time-delay mismatch. This is why practically all general design procedure, requiring robust stability margin optimization, uses specific method.

The only similar observation was if our performance requirement is not so demanding (larger T_w/τ), then the robustness sensitivity is smaller.

3 Conclusions

Most of the widely applied identification and adaptive control methods assume an apriori known time-delay. It is not easy (although possible) to incorporate the iterative or adaptive estimation of the delay into the recursive methods. Therefore one can always assume a time-delay uncertainty or mismatch at all practical applications. It was discussed here how this mismatch influences the robustness degradation and the reachable closed-loop performance.

The investigations show that bandwidth higher than the bandwidth of the delay term ($T_w < \tau$) can be reached only for a considerable lower robustness and at the same time a much more accurate knowledge of the time-delay is necessary. This corresponds to the practical design experience that the corner frequency of a delay term corresponds to an unstable zero, i.e., similarly invariant. So the acceptable performance domain means $T_w \geq \tau$.

A certain slight overestimation of the time-delay improves the robustness, however, a higher overestimation causes considerable robustness degradation again. This observation can be used for model predictive algorithms, too.

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