APPLICATION OF TWO-DEGREE-OF-FREEDOM CONTROL TO ELECTRODYNAMIC SHAKER USING ADAPTIVE FILTER BASED ON H_∞ FILTER

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Abstract

In this paper, we propose a method of applying a two-degree-of-freedom (2DOF) control with adaptive filter since such control of an electrodynamic shaker that is not permitted to employ the iteration control method. At first, a mathematical model and an uncertainty weighting are introduced, and a feedback controller is designed using μ-synthesis. 2DOF controller is constituted. Next, an adaptive filter is added for the purpose of improving the control performance. Uncertainty of the system is considered, and an adaptive filter based on the H_∞ filtering problem is employed. Lastly, we show the performance of the controller by experiment using an actual equipment with non-linear characteristic.

1 Introduction

The controller of the multi-axis shaking system is required to provide not only stable control, but also good replication of the given reference waveform as the response waveform of the movement of the shaking table. However, when the mass of a test piece cannot be ignored in comparison with that of the table, the interaction between the shaking table and the test piece becomes significant. As a result of this interaction, the transfer function of the shaking system takes on an antiresonant characteristic at the resonant frequency of the test piece and control becomes more difficult. To avoid this difficulty, existing controllers usually employ an open-loop method using iterative compensation by repeating excitations to obtain a better response waveform for the reference. However, the need for a new method in which iteration control is unnecessary is increasing in such cases as testing in civil engineering [4].

On the other hand, a 2DOF controller is often used for its advantage of convenience in that the design of system stability and the control performance can be treated independently. There has been some research in which elements of adaptive control were employed in the feedforward block to improve the control performance when accurate modeling is difficult, and those controllers yielded good results [6, 9]. We have applied 2DOF controller to the multi-axis electrodynamic shaking system using μ-synthesis in feedback control and an adaptive filter, and good performance is achieved [8]. In the above technique, the transfer characteristic of the ”secondary path” that means the path from the adaptive filter output to the control point is expediently assumed to be known. In above control system, the transfer characteristic of the secondary pass from the filter output to the error signal is expediently assumed to known characteristic. But uncertainty by the influence of the test piece exists in the actual secondary path, and it is desired that an adaptive filter with the uncertainty considered is employed.

In this paper, an adaptive filter based on H_∞ filter is used [7]. This method has been applied to active noise control [7] and adaptive equalizers over wireless channels [5], and a good performance is achieved compared with general Filtered-X LMS method. The secondary path is considered as known in these examples. However the uncertainty existing in the secondary path can be assumed as a disturbance. Because this method is based on H_∞ filtering problem which finds an optimal solution under the assumption of worst case for the unknown disturbances. In the proposed controller constitution, an uncertainty inevitably exists in the secondary path in which the controlled system is contained. Therefore, this adaptive filter is suitable method for this control constitution.

This paper is organized as follows. In section 2 a mathematical model and uncertainty weighting function for the system are introduced. A feedback controller is designed using μ-synthesis and 2DOF controller is constituted in section 3. Then, an adaptive filter based on H_∞ filter is applied in section 4. Section 5 provides some experiment results. We finally conclude with section 6.

2 Model of electrodynamic shaker

It is difficult to express the precise characteristic of an actual plant with a mathematical model, and some uncertainty remains in the model. In this section, the outline of the electrodynamic shaker is explained, and we introduce a nominal model and an uncertainty weighting function for the system. In this time, we evaluate a control performance for a plant which has nonlinear characteristic by using a compact experimental shaker.

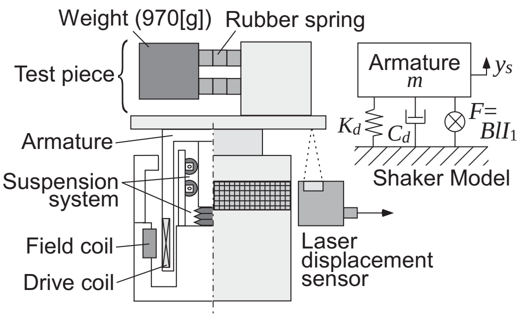

Figure 1: Overview of Electro-Dynamic shaking system

2.1 Modeling of electrodynamic shaker

The electrodynamic shaker shown in the Figure 1 is considered as the target. To make the shaker have a nonlinear characteristic, a friction is generated by increasing the clamping pressure of the suspension system. As a low frequency band that the influence of the friction is appeared saliently is targeted of the experiment, the armature displacement of the shaker is chosen as the observed quantity, and it is measured by the laser displacement sensor. The simplified model of the electrodynamic shaker can be shown as a vibration system of 1 degree of freedom, as in Figure 1. The transfer function from the input voltage u_s of the amplitude to the displacement y_s of the armature is shown, as follows:

$$\begin{aligned} G_s' &= \frac{BlG_a}{mL_d s^3 + \beta_1 s^2 + \beta_2 s + K_d R_d} \\ \beta_1 &= mR_d + C_d L_d \\ \beta_2 &= K_d L_d + C_d R_d + (Bl)^2 \end{aligned} \tag{1}$$

where m is armature mass, K_d is stiffness coefficient of suspension, C_d is damping coefficient of suspension, L_d is inductance of drive coil, R_d is resistance of drive coil, l is length of drive coil, B is magnetic flux density, G_a is amplifier gain. G_s which is a quantity the characteristic of antialiasing filter and AC coupling of the amplitude is added to this transfer characteristic G_s' is defined as the nominal model.

Also, we assume that a resonant specimen is applied, and the 970g specimen is supported by a rubber spring, as shown in Figure 1, and is put on the armature. The first-order resonant frequency of the specimen is approximately 13Hz. As an influence of the resonance, a peak notch appeared in the transfer characteristic of the shaker in the neighborhood of the resonant frequency.

2.2 Modeling uncertainty

For investigating the influence of the above friction, the transfer function of the plant is measured by using an input signal of white noise input that rms value differs respectively. That result is shown in Figure 2. The nonlinear characteristic for the level of the input voltage is existed in a static state. Considering a modeling of this influence, as parameters of the suspension

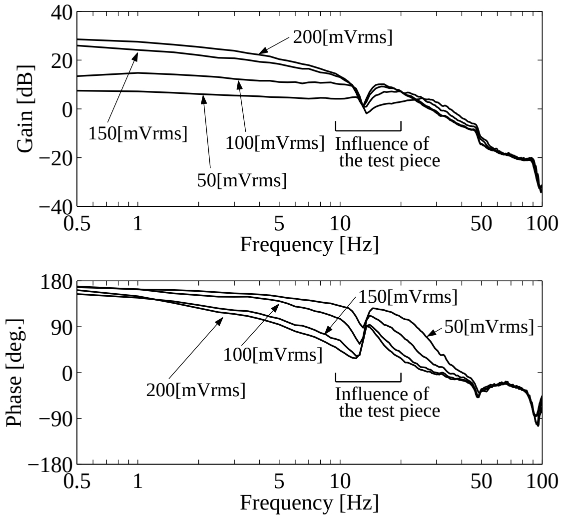

Figure 2: Transfer function of the system

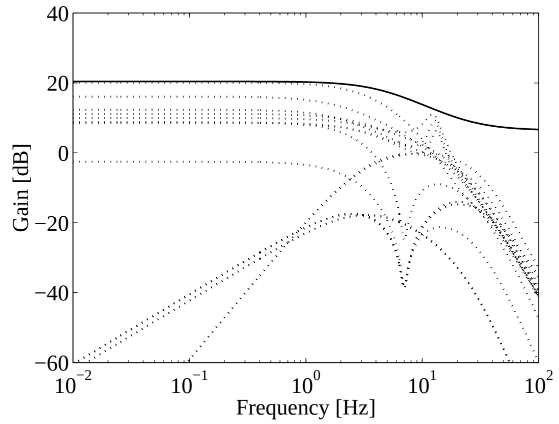

Figure 3: Uncertainty and weighting function W_a

system are changed mainly, the characteristic that corresponds to an input voltage made can be identified. The characteristic when the transfer function is measured by the white noise set at 150mVrms is defined as the nominal model, and the perturbation range of the parameter is found by setting the other characteristic as the uncertainty.

Also, a stable excitation must be executed regardless of the characteristic of the specimen mounted on the armature, but a considerable influence of the resonance appears in the transfer characteristic of shaker. We describe an uncertainty to maintain stability against this influence.

We consider additional uncertainty for parameter perturbations of the shaker and the influence of the specimen. The additional uncertainties are shown by dashed lines in Figure 3. Here, the magnitude of the weighting function W_a is chosen to cover all the perturbations as follows:

$$W_a = \frac{2.1(s+150)}{s+30}. \tag{2}$$

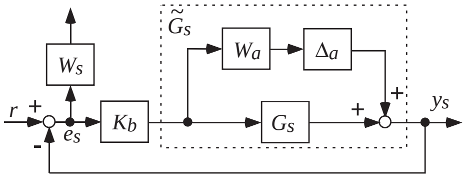

Figure 4: Feedback structure

Also, we assume that uncertainties which cannot be considered exist, and hence the magnitude of the weighting function is assumed to increase in the high-frequency band.

3 Design of 2DOF controller

We design the controller of an electrodynamic shaker. The design of the controller is carried out using MATLAB.

3.1 Control objectives

We choose the following control objectives for the shaker.

a) Stabilize the system even when an uncertainty exists.
b) Maintain the performance wherein the reference signal is replicated
well.

We must design a controller to maintain robust stability against the uncertainty model. Moreover, we would like to design a controller that maintains control performance with this uncertainty. Thus, the controller for this system is designed by μ-synthesis[2]. We fabricate a 2DOF controller to improve the transient response.

3.2 μ-synthesis

Let us consider the feedback structure shown in Figure 4, where r is a control reference, e_s is a control error, K_b is the feedback controller. A weighting function W_s for the replication performance of the reference signal is considered. For a precise replication of a seismic wave, it is considered that the gain of the low-frequency band can be enlarged as long as the uncertainty is satisfied; then we can get

$$W_s = \frac{10 \cdot \alpha \cdot 0.4 \cdot 2\pi}{s + 0.4 \cdot 2\pi} \cdot \frac{0.8 \cdot 2\pi}{s + 0.8 \cdot 2\pi} \cdot \left(\frac{4 \cdot 2\pi}{s + 4 \cdot 2\pi} \right)^2 \cdot \frac{s^2}{s^2 + 0.89s + 0.39}. \tag{3}$$

α is the adjustment parameter which we choose to be 1.2. Also, the characteristic of high pass filter is included to make it possible to be applied to the AC coupling of the amplifier.

As stated above, the design objective for stability and control performance is formalized as the requirements for a closed-loop transfer function with weighting functions. Therefore, the generalized plant P, which is shown in Figure 5, is constructed to make the control objectives fit the μ-synthesis framework.

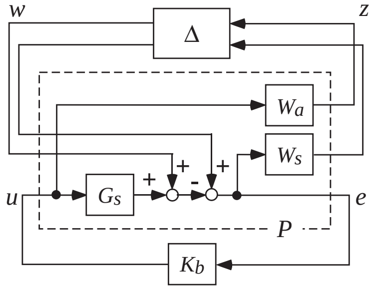

Figure 5: μ-Synthesis

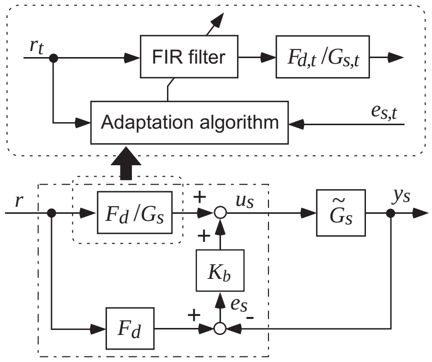

Figure 6: Block diagram of 2DOF control

Here, the block structure Δ of the uncertainty is defined as

$$\Delta := \left\{ \mathrm{diag}\left(\Delta_a, \Delta_{perf}\right), \Delta_a \in \mathcal{C}^{1\times 1}, \Delta_{perf} \in \mathcal{C}^{1\times 1} \right\}, \tag{4}$$

where $|\Delta_a| \leq 1$, $|\Delta_{perf}| \leq 1$, and Δ_{perf} is a fictitious uncertainty block for considering robust performance. Next, consider this generalized plant P partitioned as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}. \tag{5}$$

We define LFT on P by K_b as follows:

$$F_l(P, K_b) := P_{11} + P_{12}K_b(I - P_{22}K_b)^{-1}P_{21}. \tag{6}$$

The robust performance condition is equivalent to the following structured singular value μ.

$$\sup_{\omega \in \mathcal{R}} \mu_\Delta \left(F_l(P, K_b)(j\omega) \right) < 1 \tag{7}$$

Since the controller satisfies this condition, we employ D-K iteration procedure. We obtained a controller which satisfies Equation (7) after two iterations, and the degree of this controller was reduced from 16 states to 10 states.

3.3 Construction of 2DOF controller

The 2DOF controller shown in the part enclosed within the chained line in Figure 6 is constructed to improve the transient response to the reference value. If feedback control operates as designed, the transfer function from the reference r to

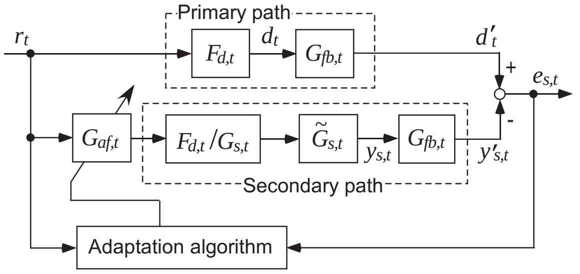

Figure 7: Block diagram of arranging the feedback term

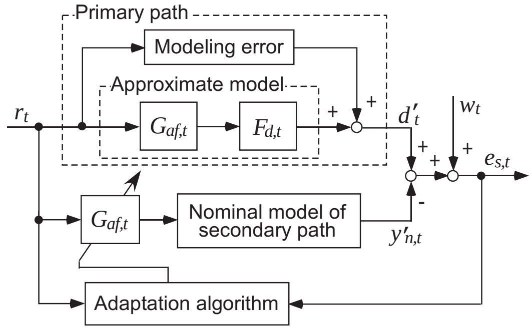

Figure 8: Block diagram of replacing the primary path by its approximate model

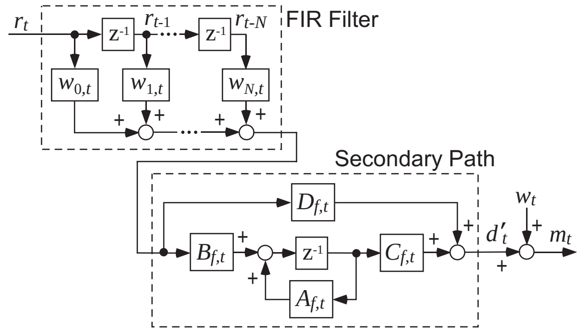

Figure 9: The approximate model of the primary path

the displacement y_s becomes equal to the reference model F_d. Therefore, a desirable response can be achieved by selecting an appropriate characteristic as a reference model F_d.

For a precise replication of a reference signal, a small phase delay characteristic at low frequency is desirable. We employ a low-pass Butterworth filter whose cut-off frequency is 20Hz. Also, F_d is set to the 5th order so that F_d/G_s will be proper.

4 Adaptive filter based on H_∞ filter

Adding an adaptive filter to the feedforward term enables to compensate plant perturbation by online updating. Here, the feedback controller is designed to maintain robust stability and performance against the uncertainty as section 3. And, the adaptive filter is designed with the same care for the uncertainty, we can constitute a more robust controller. So the description hereafter is targeted discrete system, the controller designed at contiguous system is discretized via the Tustin transform.

4.1 Application to electrodynamic shaker

When an adaptive filter is used instead of F_d/G_s, it takes time until the filter converges, and the transient response is likely to worsen. Hence, an adaptive filter is inserted in series with the existent feedforward block, as shown in the part enclosed within the dotted line in Figure 6. The filter is structured to compensate the difference between the nominal model and the actual plant. When the controller is constituted like this, the transfer function of the secondary pass from the filter output to the error signal needs to be considered. But, the uncertainty is existed in the secondary path of the actual plant. Then, the estimation-based adaptive filtering (EBAF) method based on the H_∞ estimation problem is employed so that the method can be applicable to this situation.

We prepare that the system of Figure 6 is to be interpreted as an estimation problem. To clarify the description, the feedback term is arranged as Figure 7 with using $G_{fb,t} = 1/(1 + \tilde{G}_{s,t}K_{b,t})$. Here, $G_{af,t}$ denotes the characteristic of the adaptive filter. The objective in the adaptive filter is to generate a filter output such that the output of the secondary path $y'_{s,t}$ is close to the output d'_t of the primary path. When the adaptive filter properly adjusted, the characteristic from r_t to d'_t must be equivalent to the characteristic from r_t to $y'_{s,t}$ because $y'_{s,t}$ becomes to be equal to d'_t. And, it is considered that the primary path can be constituted by an approximate model to which the characteristic from r_t to $y'_{s,t}$ is copied. Therefore, the system of Figure 7 is arranged as the system of Figure 8. On the other hand, the secondary path is generally known, however the uncertainty is existed in the secondary path in this case. Then, the uncertainty of the secondary path is newly considered as a disturbance, and it is assumed that the secondary path is equivalent to the nominal model. Also, because the nominal model is copied as for that of the primary path, the difference of characteristic with the actual plant is defined as the modeling error.

As stated above, the control objective of the adaptive filter is formalized as the problem that estimates the desired signal d'_t under the situation with the disturbance. Here, the measured quantity used in the estimation process is defined. The measured quantity m_t is defined as the sum of the reference quantity d'_t and the disturbance w_t. However, m_t is calculated by adding the output $y'_{n,t}$ of the nominal model and the error, because w_t cannot be directly measured.

$$m_t \equiv d'_t + w_t = e_{s,t} + y'_{n,t} \tag{8}$$

4.2 Configuration of H_∞ filtering problem

The approximate model of the primary path is shown in Figure 9, where $A_{f,t}, B_{f,t}, C_{f,t}, D_{f,t}$ is the state-space parameters of the secondary path, $W_t = [w_{0,t}, w_{1,t}, \ldots, w_{N,t}]^T$ is the filter weight coefficient, θ_t is state variable of the secondary path. Also, we assume $x_t^T = [W_t^T \theta_t^T]^T$ for the overall system, the state-space representation of the system Γ of Figure 9 is

$$\begin{bmatrix} W_{t+1} \\ \theta_{t+1} \end{bmatrix} = \begin{bmatrix} I_{(N+1)\times(N+1)} & 0 \\ B_{f,t}h_t^T & A_{f,t} \end{bmatrix} \begin{bmatrix} W_t \\ \theta_t \end{bmatrix} \equiv F_t x_t \quad (9)$$

where $h_t = [r_t, r_{t-1}, ..., r_{t-N}]^T$. The measured output defined in Equation (8) is

$$m_t = \begin{bmatrix} D_{f,t}h_t^T & C_{f,t} \end{bmatrix} \begin{bmatrix} W_t \\ \theta_t \end{bmatrix} + w_t \equiv H_t x_t + w_t. \quad (10)$$

And, the estimated quantity is defined as follow

$$z_t = \begin{bmatrix} L_{1,t} & L_{2,t} \end{bmatrix} \begin{bmatrix} W_t \\ \theta_t \end{bmatrix} \equiv L_t x_t. \quad (11)$$

Here, we consider $m_t \in \mathcal{R}^{1\times 1}$, $z_t \in \mathcal{R}^{1\times 1}$, $\theta_t \in \mathcal{R}^{N_s \times 1}$ and $W_t \in \mathcal{R}^{(N+1)\times 1}$. Also, we assume that F_t is not nonsingular at all time.

In the above system Γ, for a given $\gamma > 0$, The H_∞ filtering problem is formalized as seeking $\hat{z}_t = \mathcal{F}(m_{t_0}, ..., m_{t_1})$ satisfies as follow

$$\sup_{w_t, x_{t_0}} \frac{\sum_{t=t_0}^{t_1-1} \|z_t - \hat{z}_t\|^2}{x_{t_0}^T \Pi_0^{-1} x_{t_0} + \sum_{t=t_0}^{t_1-1} \|w_t\|^2} < \gamma^2. \quad (12)$$

where the system Γ is defined in the finite horizon $[t_0, t_1]$, $\sum_{t=t_0}^{t_1} \|w_t\|^2 < \infty$, and $\Pi_0 > 0$ is weighting matrix for x_{t_0}.

4.3 Computing algorithm of H_∞ filtering problem

To solve this H_∞ filtering problem, a following minimax problem is given, and an existence condition of a solution is introduced. Equation (12) is arranged as the following cost function:

$$\tilde{J}(\hat{z}, m, x_{t_0}; t_0, \tau) := \sum_{t=t_0}^{\tau-1} \|z_t - \hat{z}_t\|^2 - \gamma^2 \left(x_{t_0}^T \Pi_0^{-1} x_{t_0} + \sum_{t=t_0}^{\tau-1} \|m_t - H_t x_t\|^2 \right) \quad (13)$$

where τ is fixed value in $[t_0, t_1]$ and $\{m_t : t_0 \le t \le \tau\}$ is fixed value. The minimax problem is considered for this cost function as follow

$$\min_{\hat{z}} \max_{x_{t_0}} \tilde{J}(\hat{z}, m, x_{t_0}; t_0, \tau). \quad (14)$$

In regard to this minimax problem, the following theorem is given [1, 3].

Theorem 1 *The following condition (A) is considered.*

(A) It is assumed that the discrete-time Ricatti equation

$$\begin{aligned} M_{t+1} &= F_t \Sigma_t F_t^T \\ \Sigma_t^{-1} &= M_t^{-1} + H_t^T H_t - \gamma^{-2} L_t^T L_t \quad (15) \\ M_{t_0} &= \Pi_0 \end{aligned}$$

has the solution Σ_t of the positive matrix in the finite horizon $[t_0, \tau]$.

In this time, if the condition (A) consists, then a solution of minimax problem Equation (14) exists, and one of the computing algorithm of H_∞ filter of the system Γ is given by

$$\hat{z}_t = L_t \hat{x}_t + L_t M_t H_t^T (I + H_t M_t H_t^T)^{-1} (m_t - H_t \hat{x}_t) \quad (16)$$

where $t_0 \le t \le t_1$, and $\hat{x}_t$ is given by

$$\begin{aligned} \hat{x}_{t+1} &= F_t \hat{x}_t + F_t M_t H_t^T (I + H_t M_t H_t^T)^{-1} (m_t - H_t \hat{x}_t) \\ \hat{x}_{t_0} &= 0 \quad (17) \end{aligned}$$

Also, M_t is calculated by time update algorithm Equation (15).

5 Experiment

To implement the controller with the processing board, the controller is discretized via the Tustin transform at the sampling frequency of 512Hz. The control performance is evaluated by the result of the transient response at that excitation.

5.1 Setting of H_∞ filter

The secondary path of Figure 9 is set at the reference model $F_{d,t}$. And, L_t is chosen as $L_t = H_t$ for the estimation problem that estimation quantity z_t is to be equivalent to d'_t. The filter length is set at 16 taps to reduce the calculation amount. γ is chosen as $\gamma = 1$. The initial quantity Π_0 of Equation (15) is chosen so that the error may be reduced within the range that can satisfy the stability and performance. Also, we compare with Filtered-X LMS method that has been conventionally used. The filter length is set at 16 taps because of suiting to EBAF method, and a convergence factor is similarly chosen within the requirement.

5.2 Experiment results of 2DOF controller

The results of using the reference waveform when the peak value is 1mm are shown in Figure 10. The rms error (= rms value of the error waveform / rms value of the reference waveform 100%), the maximum error and its appearance time are written in the control results respectively.

The result of 2DOF controller without the adaptive filter is shown that the response waveform differs from the reference waveform vastly. In this result, even when such an uncertainty exists, the control itself is stable. But, because the nominal model differs from the actual plant, good performance is not yielded.

5.3 Experiment results by applying EBAF method

First, the experiment results of using Filtered-X LMS method in the same condition are shown in Figure 11. Comparing with the results of Figure 10, the rms error increase slightly.

Next, the experiment results of using EBAF method are shown in Figure 12. Comparing with the results of Figure 10,11,

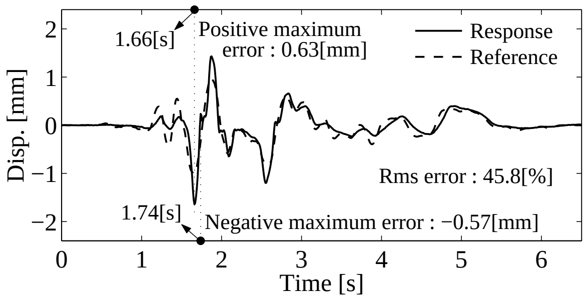

Figure 10: Results of 2DOF controller

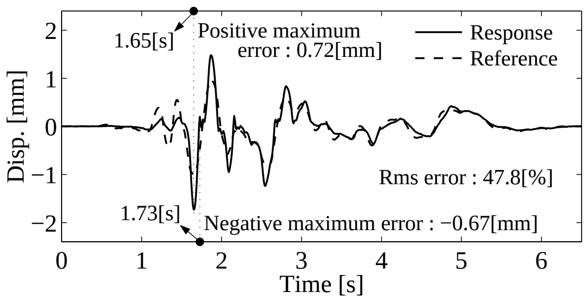

Figure 11: Results of control with Filtered-X LMS

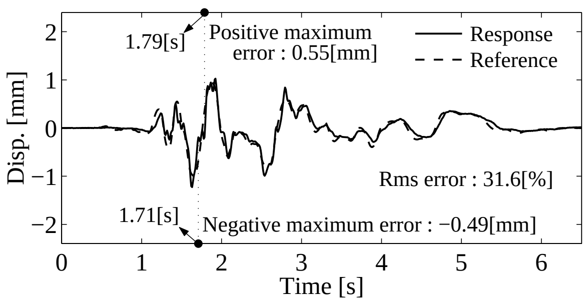

Figure 12: Results of control with EBAF

when EBAF method is applied, the track ability to the reference waveform maintains good performance, and the rms error is decreased to approximately 13% compared with 2DOF controller. The maximum error is improved compared with the other control results, too. Especially, for a time period 1.5∼2s that the reference signal varies widely, the response signal in other control results overshoots the reference, whereas the response of EBAF method is suppressed specifically.

6 Conclusion

In this study, we assumed that a conventional open-loop method using iterative compensation by repeating excitations could not be employed for the electrodynamic shaker control, and we constructed a controller useful for such a situation. First, a feedback controller was designed to achieve the robust performance for the uncertainty by μ-synthesis, and a 2DOF controller was constructed. And, for compensating the difference from the nominal model, an adaptive filter based on H_∞ filtering problem was employed, because the uncertainty was existed in the actual plant. When that effect was evaluated by the experiment, it was found that adding EBAF method could yield a good result of control. Also, comparing with Filtered-X LMS method, it was confirmed that the proposed controller was especially effective for the condition that uncertainty was existed.

Finally, the controller which has more robustness could be constituted for the uncertainty by adding an adaptive filter based on H_∞ filter to 2DOF controller with using μ-synthesis, and it was shown that the proposed controller is useful in the assumed situation.

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