ON THE REDUCTION OF AN ARBITRARY 2-D POLYNOMIAL MATRIX TO GSS FORM

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Abstract

In this paper, it is shown that any arbitrary 2-D polynomial system matrix can be reduced by zero coprime system equivalence to a generalized state space form. The exact form of both the generalized state space (GSS) system matrix and the transformation linking it to the original system matrix are established.

1 Introduction

State space models play an important role in the theory of 1-D finite-dimensional linear systems. In recent years attempts have been made towards extending the state space representation to more general systems, e.g. time-delay systems or systems described by partial differential equations. Another extension from 1-D to 2-D is the 2-D discrete state space model which has a number of variants as given by Roesser (1975), Attasi (1973) or Fornasini-Marchesini (1976). One of the limitations of these models is that they can only be used to describe 2-D proper transfer functions. In other words, they are suitable only for the representation of northeast quarter plane 2-D systems. Several authors have suggested a generalized state space description for 2-D systems. Zak (1984) suggested a generalized model based on Roesser's model while Kaczorek (1988) proposed a model based on that of Fornasini-Marchesini. Since the natural description of a system is not necessarily in a state space form, it is desirable to transform such a description into a state space one. The reduction of an arbitrary 2-D polynomial system matrix to 2-D generalized state space form was first studied by Pugh et al.(1998). Their algorithm involves the application of a two stage reduction procedure which includes the removal of factors from certain matrices to ensure that the transformations linking the original system matrix with the final GSS form are polynomial. The method does not give a priori the form of neither the resulting 2-D GSS system matrix nor the transformation linking it to the original 2-D polynomial system matrix.

In the present work, we show that every 2-D polynomial system matrix is equivalent to a 2-D generalized state space form. The exact form of both the GSS system matrix and the transformation linking it with the original system matrix will be given.

The transformation linking the original system matrix to its corresponding GSS form is shown to be *zero coprime equivalence*. This type of equivalence has been studied by Levy (1981) , Johnson (1993) and Pugh et al. (1996) and has been shown to provide the connection between all least order polynomial realizations of a given 2-D transfer function matrix (Pugh et al. 1998).

2 System Matrices in GSS Form

Consider the following 2-D singular state space system as given by Kaczorek (1988):

$$\begin{aligned} Ex(i+1,j+1) =& A_1x(i+1,j) + A_2x(i,j+1) \\ &+ A_0x(i,j) + B_1u(i+1,j) \\ &+ B_2u(i,j+1) + B_0u(i,j) \qquad (1) \\ y(i,j) =& Cx(i,j) + Du(i,j) \qquad (2) \end{aligned}$$

where $x(i,j)$ is the state vector, $u(i,j)$ is the input vector, $y(i,j)$ is the output vector, E, A_0, A_1, A_2, B_0, B_1, B_2, C and D are constant real matrices of appropriate dimensions and E may be singular.

Then, taking the $2-D$ *z-transform* of (1) and (2) and assuming zero boundary conditions yields

$$\left[\begin{array}{c|c} szE - sA_1 - zA_2 - A_0 & sB_1 + zB_2 + B_0 \\ \hline -C & D \end{array}\right] \left[\begin{array}{c} \overline{x}(s,z) \\ -\overline{u}(s,z) \end{array}\right] = \left[\begin{array}{c} 0 \\ -\overline{y}(s,z) \end{array}\right] \qquad (4)$$

The polynomial matrix over $\mathbb{R}[s,z]$,

$$P(s,z) = \left[\begin{array}{c|c} szE - sA_1 - zA_2 - A_0 & sB_1 + zB_2 + B_0 \\ \hline -C & D \end{array}\right] \qquad (5)$$

in (4), is a sytem matrix in GSS form. The general form of a 2-D polynomial system matrix is given by

$$P(s,z) = \left[\begin{array}{cc} T(s,z) & U(s,z) \\ -V(s,z) & W(s,z) \end{array}\right] \qquad (6)$$

where $T(s,z), U(s,z), V(s,z)$ and $W(s,z)$ are respectively $r\times r$, $r\times n$, $m\times r$ and $m\times n$ polynomial matrices with $T(s,z)$ invertible, in which case the system matrix in (6) is said to be regular. The transfer function matrix of the system matrix in (6) is given by

$$G(s,z) = V(s,z)T^{-1}(s,z)U(s,z) + W(s,z) \qquad (7)$$

Definition 1 *Two polynomial matrices $P_1(s,z)$ and $S_1(s,z)$ of appropriate dimensions, are said to be zero left coprime if the compound matrix $\begin{bmatrix} P_1(s,z) & S_1(s,z) \end{bmatrix}$ has full rank for all complex values of the indeterminate pair (s,z). Similarly, $P_2(s,z)$ and $S_2(s,z)$, of appropriate dimensions, are said to be zero right coprime if the compound matrix $\begin{bmatrix} P_2^T(s,z) & S_2^T(s,z) \end{bmatrix}^T$ has full rank for all complex values of the indeterminate pair (s,z).*

Definition 2 *Two polynomial system matrices $P_1(s,z)$ and $P_2(s,z) \in \mathbb{P}(m,n)$, where $\mathbb{P}(m,n)$ denotes the class of $(r+m)\times(r+n)$ polynomial matrices where m,n are fixed positive integers and r is variable and ranges over all integers greater than $max(-m,-n)$, are said to be zero coprime system equivalent (z.c.s.e.) if they are related by the following*

$$\underbrace{\begin{bmatrix} M(s,z) & 0 \\ X(s,z) & I_m \end{bmatrix}}_{S_1(s,z)} \underbrace{\begin{bmatrix} T_1(s,z) & U_1(s,z) \\ -V_1(s,z) & W_1(s,z) \end{bmatrix}}_{P_2(s,z)}$$

$$= \underbrace{\begin{bmatrix} T_2(s,z) & U_2(s,z) \\ -V_2(s,z) & W_2(s,z) \end{bmatrix}}_{P_1(s,z)} \underbrace{\begin{bmatrix} N(s,z) & Y(s,z) \\ 0 & I_n \end{bmatrix}}_{S_2(s,z)} \qquad (9)$$

where $P_1(s,z), S_1(s,z)$ are zero left coprime and $P_2(s,z), S_2(s,z)$ are zero right coprime and $M(s,z)$, $N(s,z)$, $X(s,z)$ and $Y(s,z)$ are polynomial matrices of appropriate dimensions.

The transformation of *z.c.s.e.* is an extension of Fuhrmann's *strict system equivalence* from the 1-D to the 2-D setting and has been shown by Jonhson (1993) to preserve important properties of the system matrix $P(s,z)$

Lemma 1 *(Johnson 1993) The transformation of z.c.s.e. preserves the transfer function and the invariant polynomials of the matrices:*

(i) $T_i(s,z), i=1,2.$

(ii) $P_i(s,z), i=1,2.$

(iii) $\begin{bmatrix} T_i(s,z) & U_i(s,z) \end{bmatrix}, i=1,2.$

(iv) $\begin{bmatrix} T_i(s,z) \\ -V_i(s,z) \end{bmatrix}, i=1,2.$

3 Reduction to GSS Form

Let $P(s,z)$ be a 2-D $(r+m)\times(r+n)$ polynomial system matrix given by (6). First write $P(s,z)$ as

$$P(s,z) = \sum_{i=0}^{p}\sum_{j=0}^{q} P_{i,j}s^i z^j \qquad (10)$$

where $P_{i,j}$, $i=0,1,...,p$ and $j=0,1,...,q$ are $(r+m)\times(r+n)$ real constant matrices. Now construct the matrices

$$E = \begin{bmatrix} \multicolumn{4}{c}{0_{(r+n)(pq-1),(r+n)pq}} \\ E_q & E_{q-1} & \cdots & E_1 \end{bmatrix} \qquad (11)$$

where

$$E_j = \begin{bmatrix} P_{p,j} & P_{p-1,j} & \cdots & P_{1,j} \end{bmatrix}, j=1,2,\ldots,q. \qquad (12)$$

$$A_0 = Diag(-I_{(r+n)(pq-1)}, -P_{0,0}), \qquad (13)$$

$$A_1 = \begin{bmatrix} \multicolumn{5}{c}{0_{(r+n)p(q-1),(r+n)pq}} \\ 0_{(r+n)(p-1),(r+n)(pq-p+1)} & \multicolumn{4}{c}{I_{(r+n)(p-1)}} \\ 0_{(r+m),(r+n)p(q-1)} & -P_{p,0} & -P_{p-1,0} & \cdots & -P_{1,0} \end{bmatrix}, \qquad (15)$$

and

$$A_2 = \begin{bmatrix} 0_{(r+n)p(q-1),(r+n)p} & I_{(r+n)p(q-1)} \\ \multicolumn{2}{c}{0_{(r+n)(p-1),(r+n)pq}} \\ \multicolumn{2}{c}{A_{2,q} \quad A_{2,q-1} \quad \cdots \quad A_{2,1}} \end{bmatrix} \qquad (16)$$

where

$$A_{2,j} = \begin{bmatrix} 0_{(r+m),(r+n)(p-1)} & -P_{0,j} \end{bmatrix}, j=1,2,\ldots,q. \qquad (17)$$

Theorem 1 *Let the matrices E, A_0, A_1 and A_2 be as constructed in (11), (13), (15) and (16) respectively, then the $[(r+n)pq+2m]\times[(r+n)pq+m+n]$ polynomial system matrix in GSS form (5):*

$$Q(s,z) = \left[\begin{array}{cc|c} szE - sA_1 - zA_2 - A_0 & -Z_m & 0 \\ -Z_n & 0 & I_n \\ \hline 0 & I_m & 0 \end{array}\right] \qquad (18)$$

where $Z_n = \begin{bmatrix} 0_{n,[(r+n)pq+r+m-n]} & I_n \end{bmatrix}$ and $Z_m^T = \begin{bmatrix} 0_{m,[(r+n)pq-m]} & I_m \end{bmatrix}$ is related to the original system matrix $P(s,z)$ by the following :

$$S_1(s,z)P(s,z) = Q(s,z)S_2(s,z) \qquad (19)$$

where

$$S_1(s,z) = \left[\begin{array}{c|c} 0_{(r+n)(pq-1),r} & 0_{(r+n)(pq-1),m} \\ I_r & 0_{r,m} \\ 0_{(m+n),r} & 0_{(m+n),m} \\ \hline 0_{m,r} & I_m \end{array}\right],$$

$$S_2(s,z) = \left[\begin{array}{c|c} \multicolumn{2}{c}{Y_1} \\ \multicolumn{2}{c}{Y_2} \\ \multicolumn{2}{c}{\vdots} \\ \multicolumn{2}{c}{Y_q} \\ -V & W \\ \hline 0_{n,r} & I_n \end{array}\right], \tag{21}$$

$$Y_j = \left[\begin{array}{cccc} s^{p-1}z^{q-j} & s^{p-2}z^{q-j} & \dots & z^{q-j} \end{array}\right]^T \otimes I_{r+n}, \tag{22}$$

where $j = 1, 2, \dots, q$ and $\otimes$ denotes the matrix Kronecker product.

Proof. From the construction of $Q(s,z)$ and with appropriate partitioning of the matrices $S_1(s,z)$, $P(s,z)$, $Q(s,z)$ and $S_2(s,z)$, it can be easily verified that

$$S_1(s,z)P(s,z) = Q(s,z)S_2(s,z) \tag{23}$$

$$= \left[\begin{array}{cc} 0_{(r+n)(pq-1),r} & 0_{(r+n)(pq-1),n} \\ T & U \\ 0_{m+n,r} & 0_{m+n,n} \\ -V & W \end{array}\right] \tag{24}$$

■

Lemma 2 *The matrices in* (19)*, $Q(s,z)$ and $S_1(s,z)$ are zero left coprime and $P(s,z)$ and $S_2(s,z)$ are zero right coprime.*

Proof. This follows from the fact that the minor obtained by deleting the columns $(r+n)(pq-1)+1, \dots, (r+n)(pq-1)+r+n$ of the matrix

$$\left[\begin{array}{cc} Q(s,z) & S_1(s,z) \end{array}\right] \tag{25}$$

is equal to ± 1 and the minor obtained by deleting the rows $1, \dots, r+npq$ and the rows $r+n+1, \dots, r+m+2n$ of the matrix

$$\left[\begin{array}{c} P(s,z) \\ S_2(s,z) \end{array}\right] \tag{26}$$

is equal to 1. ■

Theorem 2 *If $P(s,z)$ is an arbitrary $(r+m)\times(r+n)$ polynomial system matrix over $\mathbb{R}[s,z]$ given by* (6) *and $Q(s,z)$ is the corresponding $[(r+n)pq+2m]\times[(r+n)pq+m+n]$ 2-D system matrix in GSS form* (18)*, then $P(s,z)$ and $Q(s,z)$ are z.c.s.e.*

Proof. The result follows immediately from Theorem 1 and Lemma 2. ■

Example 1 *Consider the 2×2 system matrix $P(s,z)$ over $\mathbb{R}[s,z]$ given by*

$$P(s,z) \equiv \left[\begin{array}{c|c} t(s,z) & u(s,z) \\ \hline -v(s,z) & w(s,z) \end{array}\right] \tag{27}$$

where

$$t(s,z) = (z^2+1)s^2 - (2z^2 - z - 3)s + z^2 - 4z + 1, \tag{28}$$
$$u(s,z) = (z^2 - z)s^2 - (z^2 - 2)s + z^2 - z, \tag{29}$$
$$v(s,z) = -(z+2)s^2 + (z^2 - z)s + 4z + 1, \tag{30}$$
$$w(s,z) = (2z^2 - z)s^2 + 5zs + z^2 - z + 3 \tag{31}$$

Here $r = m = n = 1$ and $p = q = 2$.
The transfer function of the system matrix $P(s,z)$ is given by :

$$\begin{aligned} G^{[P]}(s,z) = & \frac{1}{(z^2+1)s^2 - (2z^2-z-3)s + z^2 - 4z + 1} \\ & \times[-(z^3+z^2-2z)s^4 + (z^4 - z^3 + 3z^2 - 2z - 4)s^3 \\ & - (z^4 - 4z^3 + 2z)s^2 + (z^4 - 6z^3 + 13z + 2)s \\ & + 4z^3 - 2z^2 - 2z + 3] \end{aligned} \tag{32}$$

and the invariant polynomials of $P(s,z)$ are computed as :

$$\begin{aligned} \varepsilon_1^{[P]} &= 1 \\ \varepsilon_2^{[P]} &= (2z^4 - 2z^3 + z^2 + z)s^4 - \left(-3z^4 + 8z^3 + 8z^2 - 4\right)s^3 \\ &+ \left(2z^4 - 16z^3 + 13z^2 + 12z + 3\right)s^2 \\ &+ \left(-z^4 + 2z^3 - 24z^2 + 13z + 11\right)s \\ &+ z^4 - z^3 + 5z^2 - 14z + 3 \end{aligned} \tag{33}$$

Writing $P(s,z)$ in the form (10)*, the coefficient matrices $P_{i,j}$ are given by*

$$P_{0,0} = \left[\begin{array}{cc} 1 & 0 \\ -1 & 3 \end{array}\right], P_{0,1} = \left[\begin{array}{cc} -4 & -1 \\ -4 & -1 \end{array}\right], P_{0,2} = \left[\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array}\right],$$
$$P_{1,0} = \left[\begin{array}{cc} 3 & 2 \\ 0 & 0 \end{array}\right], P_{1,1} = \left[\begin{array}{cc} 1 & 0 \\ 1 & 5 \end{array}\right], P_{1,2} = \left[\begin{array}{cc} -2 & -1 \\ -1 & 0 \end{array}\right],$$
$$P_{2,0} = \left[\begin{array}{cc} 1 & 0 \\ 2 & 0 \end{array}\right], P_{2,1} = \left[\begin{array}{cc} 0 & -1 \\ 1 & -1 \end{array}\right], P_{2,2} = \left[\begin{array}{cc} 1 & 1 \\ 0 & 2 \end{array}\right] \tag{36}$$

Then, constructing the 10×10 polynomial system matrix in GSS form $Q(s,z)$ corresponding to (18) *gives*

$$Q(s,z) \equiv \left[\begin{array}{ccccc|c} I_2 & 0 & -zI_2 & 0 & 0 & 0 \\ 0 & I_2 & 0 & -zI_2 & 0 & 0 \\ 0 & 0 & I_2 & -sI_2 & 0 & 0 \\ Q_1 & Q_2 & Q_3 & Q_4 & -E_2 & 0 \\ 0 & 0 & 0 & -E_2^T & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \tag{37}$$

where

$$Q_1 = \left[\begin{array}{cc} sz & sz \\ 0 & 2sz \end{array}\right], Q_2 = \left[\begin{array}{cc} -2sz+z & -sz+z \\ -sz & z \end{array}\right],$$

$$Q_3 = \left[\begin{array}{cc} s & -sz \\ s(z+2) & -sz \end{array}\right],$$

$$Q_4 = \begin{bmatrix} s(z+3)-4z+1 & 2s-z \\ sz-4z-1 & 5sz-z+3 \end{bmatrix},$$

and E_2 is the second column of I_2.

The matrices E, A_0, A_1 and A_2 corresponding to $(11, 13, 15$ and $16)$ are given by

$$E = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & -2 & -1 & 0 & -1 & 1 & 0 \\ 0 & 2 & -1 & 0 & 1 & -1 & 1 & 5 \end{bmatrix}, \tag{41}$$

$$A_0 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -3 \end{bmatrix}, \tag{42}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & -3 & -2 \\ 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \end{bmatrix}, \tag{43}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 4 & 1 \end{bmatrix} \tag{44}$$

By virtue of Theorem 1 and Theorem 2, the polynomial matrix $P(s,z)$ in (27) and the corresponding system matrix in GSS form $Q(s,z)$ in (37) are related by the z.c.s.e. transformation $S_1(s,z)P(s,z) = Q(s,z)S_2(s,z)$, where

$$S_1(s,z) = \left[\begin{array}{c|c} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline 0 & 1 \end{array}\right], \tag{45}$$

and

$$S_2(s,z) \equiv \left[\begin{array}{c|c} n(s,z) & y(s,z) \\ \hline 0 & 1 \end{array}\right] \tag{46}$$

where

$$n(s,z) = \begin{bmatrix} sz \\ 0 \\ z \\ 0 \\ s \\ 0 \\ 1 \\ 0 \\ (z+2)s^2 - (z^2+z)s - 4z - 1 \end{bmatrix}$$

and

$$y(s,z) = \begin{bmatrix} 0 \\ sz \\ 0 \\ z \\ 0 \\ s \\ 0 \\ 1 \\ (2z^2 - z)s^2 + 5sz + z^2 - z + 3 \end{bmatrix}$$

In fact it can be easily verified that

$$S_1(s,z)P(s,z) = Q(s,z)S_2(s,z) \equiv \begin{bmatrix} 0_{6,1} & 0_{6,1} \\ t(s,z) & u(s,z) \\ 0_{2,1} & 0_{2,1} \\ -v(s,z) & w(s,z) \end{bmatrix} \tag{49}$$

where $t(s,z)$, $u(s,z)$, $v(s,z)$ and $w(s,z)$ are given by (28), (29), (30) *and* (31) *respectively.*

The matrices $Q(s,z), S_1(s,z)$ are zero left coprime and the matrices $P(s,z), S_2(s,z)$ are zero right coprime since the matrices

$$\begin{bmatrix} Q(s,z) & S_1(s,z) \end{bmatrix}$$

and

$$\begin{bmatrix} P(s,z) \\ S_2(s,z) \end{bmatrix}$$

have respectively a 10×10 and a 2×2 minor which is equal to 1.
The transfer function of the system matrix $Q(s,z)$ is given by :

$$\begin{aligned} G^{[Q]}(s,z) =& \frac{1}{(z^2+1)s^2 - (2z^2 - z - 3)s + z^2 - 4z + 1} \\ & \times[-(z^3 + z^2 - 2z)s^4 + (z^4 - z^3 + 3z^2 - 2z - 4)s^3 \\ & - (z^4 - 4z^3 + 2z)s^2 + (z^4 - 6z^3 + 13z + 2)s \\ & + 4z^3 - 2z^2 - 2z + 3] \\ =& G^{[P]}(s,z) \end{aligned} \tag{52}$$

and the invariant polynomials of $Q(s,z)$ *are :*

$$\begin{aligned}\varepsilon_{1}^{[Q]} &= \varepsilon_{2}^{[Q]} = \varepsilon_{3}^{[Q]} = \varepsilon_{4}^{[Q]} = \varepsilon_{5}^{[Q]} = \varepsilon_{6}^{[Q]} = \varepsilon_{7}^{[Q]} = \varepsilon_{8}^{[Q]} = 1 \\ &= \varepsilon_{1}^{[P]}, \\ \varepsilon_{10}^{[Q]} &= (2z^4 - 2z^3 + z^2 + z)s^4 + (-3z^4 + 8z^3 + 8z^2 - 4)s^3 \\ &+ (2z^4 - 16z^3 + 13z^2 + 12z + 3)s^2 \\ &+ (-z^4 + 2z^3 - 24z^2 + 13z + 11)s \\ &+ z^4 - z^3 + 5z^2 - 14z + 3 \\ &= \varepsilon_{2}^{[P]}. \end{aligned} \tag{53}$$

which is in accord with Lemma 1.

4 Conclusions

In this paper, a new reduction procedure to GSS form of an arbitrary 2-D system matrix has been presented. The exact nature of the equivalence transformation linking the original system matrix with its corresponding GSS form has been set out and shown to be that of *zero coprime system equivalence*. Despite the fact that the resulting 2-D system matrix may be larger in size than the one obtained by the algorithm used by Pugh et al. (1998), the method presented in this paper has the advantage of providing a priori both the final 2-D system matrix in GSS form and the transformation relating it to the original polynomial system matrix. To reduce the size of the resulting system matrix while preserving its GSS form, a constant *z.c.s.e.* transformation may be used.

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