An Adaptive PID-type Iterative Learning Controller for Unknown Nonlinear Systems with Initial State Errors

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Abstract

In this paper, a new adaptive PID-type iterative learning controller (ILC) is proposed for a class of repeatable nonlinear systems with unknown nonlinearities. The initial state errors are allowed to be nonzero and varying for each iteration. The main structure of the adaptive PID-type ILC is constructed based on a time-varying boundary layer which is designed to overcome the problem of initial state errors and further eliminate the possible undesirable chattering behavior. Although the optimal gains of PID-type ILC for a best approximation are generally unknown, adaptive algorithms with projection mechanisms are derived between successive iterations to ensure the stability and convergence of the learning system. It is shown that all the adjustable parameters and the internal signals remain bounded for all iterations, and the norm of tracking error vector at each time instant will asymptotically converge to a tunable residual set.

1 Introduction

As is widely known, iterative learning control (ILC) has become one of the most effective control strategies for nonlinear dynamic system in dealing with repeated tracking control or periodic disturbance rejection. The ILC system improves the control performances by some simple self-tuning processes without using accurate system models and can be applied to some practical applications such as robotics, servo motors, etc. In most studies for traditional D-type or P-type ILC algorithms [1]–[8], the control input is directly updated by a learning mechanism using the information of error and input in the previous iteration. It is necessary to assume that the nonlinearities of nonlinear plants satisfying global Lipschitz continuous condition. Recently, the ILC algorithm which tunes control parameters between successive iterations have been widely studied [9]–[13]. In general, this type of ILC is referred as an adaptive iterative learning controller (AILC). One of the most interesting features of AILC schemes is that the requirement of Lipschitz continuous condition could be relaxed.

The robustness of the ILC system against initial state error is also an important issue in practical environment. Although the related works [2]–[4] showed that tracking error will be bounded in the presence of bounded initial state error and the size of tracking error after learning will be small if the magnitude of initial state error is small. There was no information on how to adjust the size of the bounds. In [6], the tracking error can be estimated in terms of the initial state error and the parameters of PD-type ILC algorithm. Later, a PID-type ILC algorithm [7] and an operator algorithm were further studied in [8], respectively. However, the PID-type ILC in [7] only deals with a class of nonlinear systems with fixed initial state errors. For iterative learning control of nonlinear systems with variable initial state errors, it is still a challenge for traditional D-type, P-type, PD-type and PID-type ILCs. Unfortunately, the problem of initial state errors seems more difficult in the design of AILC since they are assumed to be zero in all the related AILC works [9]–[14].

In this paper, an adaptive PID-type ILC for a class of nonlinear systems with unknown plant nonlinearities, and without the requirement of Lipschitz continuous condition is proposed. It is assumed that the system will probably exist the initial state errors, which may be varying and large. A concept of time-varying boundary layer, i.e., the width of the boundary layer is decreased along the time axis, is used to design the PID-type ILC. Motivated by [15], the PID-type controller is used as an approximator for an optimal controller. Since the optimal PID gains for the best approximation is generally unavailable, the parameters of the PID-type ILC are tuned between successive iterations to ensure the stability and convergence. Under this learning controller, the nonlinearities, especially the nonlinear input (control) gain, of the controlling plant can be unknown. This is an important feature since most of the related works dealing with the similar nonlinear control design problem need certain information of the input gain, e.g., the neural network or fuzzy system based adaptive control in [16]–[22] and adaptive iterative learning control in [10], [14]. We show that the norm of tracking error vector will asymptotically converge to a tunable residual set whose size depends on the width of boundary layer. Furthermore, all adjustable parameters as well as the internal signals will remain bounded.

This paper is organized as follows. In section 2, we propose the PID-type controller and discuss the approximation error between the PID-type controller and the optimal controller. The plant description, control objective and design steps of the proposed adaptive PID-type ILC are presented in section 3. Analysis of stability and learning performance will be studied extensively in section 4. A repetitive tracking control of a Chua's chaotic circuit is demonstrated in section 5. Finally a conclusion is made in section 6.

2 Approximation of the optimal controller using PID controller

In this paper, we apply a PID-type controller to design an adaptive iterative learning controller for repeatable nonlinear systems. In general, the form of a PID-type controller with inputs $e(t)$, $\int_0^t e(\tau)d\tau$, $\dot{e}(t)$ and output $u_{PID}(t)$ is given by

$$u_{PID}(t) \;=\; K_P e(t) + K_I \int_0^t e(\tau)d\tau + K_D\dot{e}(t) \tag{1}$$

where $e(t)$ is the output error which will be defined later, K_P is the proportional gain, K_I is the integral gain, and K_D is the derivative gain. Let $W = [K_P, K_I, K_D]^\top \in \mathbb{R}^3$ and $Z = [e(t), \int_0^t e(\tau)d\tau, \dot{e}(t)]^\top \in \mathbb{R}^3$ be the gain vector and the input vector of the PID-type controller. Then (1) can be further described in a matrix form as follows :

$$u_{PID}(t) \;=\; u_{PID}(Z, W) \;=\; W^\top Z \tag{2}$$

It is emphasized that the PID-type controller (1) or (2) in this paper is used to approximate the optimal controller $u_\star$. Define the optimal weight as follows :

$$W^* \equiv \arg\min_{|W|<M_W} \left[\sup_{|Z|<M_Z} |u_{PID}(Z,W) - u_\star|\right] \tag{3}$$

where M_W is the bound of W and M_Z is the bound of Z. The minimum functional approximation error between the optimal PID-type controller $u_{PID}(Z, W^*)$ and $u^\star$ is defined as

$$\epsilon_M \equiv u_{PID}(Z, W^*) - u_\star \tag{4}$$

Note also that the minimum approximation error ϵ_M is assumed to satisfy $|\epsilon_M| \le \theta^*$, in which θ^* is a prescribed small positive constant. Generally, gains of the PID-type controller are often tuned via suitable adaptation laws in adaptive control since it is not easy to get the optimal gains for the PID-type controller.

Theorem 1 : Define the estimation errors of the PID gains as $\widetilde{W} \equiv W - W^*$ and $Z \equiv [e(t), \int_0^t e(\tau)d\tau, \dot{e}(t)]^\top \in \mathbb{R}^3$. The functional approximation error ϵ_f will satisfy

$$\epsilon_f \equiv u_{PID}(Z,W) - u_\star = \widetilde{W}^\top Z + r \tag{5}$$

and the residual term r can be bounded by

$$|r| < \theta^* \tag{6}$$

Proof : The functional approximation error ϵ_f satisfies

$$\begin{aligned} \epsilon_f &= u_{PID}(Z,W) - u_\star \\ &= u_{PID}(Z,W) - u_{PID}(Z,W^*) + u_{PID}(Z,W^*) - u_\star \\ &= \left(W^\top Z - W^{*\top} Z\right) + \epsilon_M \\ &= \widetilde{W}^\top Z + \epsilon_M \end{aligned} \tag{7}$$

If we let $r{=}\epsilon_M$, then we have $|r| = |\epsilon_M| \le \theta^*$. Q.E.D.

3 Adaptive PID-type Iterative Learning Controller

In this section, we consider a class of nonlinear systems which can perform a given task repeatedly over a finite time interval $[0, T]$ as follows :

$$\begin{aligned} \dot{x}_1^j(t) &= x_2^j(t) \\ \dot{x}_2^j(t) &= x_3^j(t) \\ &\vdots \\ \dot{x}_n^j(t) &= -f(X^j(t)) + b(X^j(t))u^j(t) \\ y^j(t) &= x_1^j(t) \end{aligned} \tag{8}$$

where $X^j(t) = [x_1^j(t), \cdots, x_n^j(t)]^\top \in \mathbb{R}^{n\times 1} \times [0,T]$ is the state vector, $u^j(t)$ is the control input, $y^j(t)$ is the system output, $f(X^j(t))$ and $b(X^j(t))$ are unknown real continuous nonlinear functions of state. Here, j denotes the index of iteration and $t \in [0,T]$. The control objective is to force the state vector $X^j(t) = [x_1^j(t), x_2^j(t), \cdots, x_n^j(t)]^\top = [x_1^j(t), \dot{x}_1^j(t), \cdots, x_1^{(n-1),j}(t)]^\top$ to follow some specified desired trajectory $X_d(t) = [x_d(t), \dot{x}_d(t), \cdots, x_d^{(n-1)}(t)]^\top$ for all $t \in [0,T]$ as close as possible even there exists initial state errors. In order to achieve the above control objective, some assumptions on the nonlinear system and desired trajectory are given as follows :

(A1) There exists a positive but unknown lower bound b_L, such that $0 < b_L < b(X^j(t))$ for all $X^j(t) \in \mathbb{R}^n \times [0,T]$.

(A2) Let the state errors $e_1^j(t), \cdots, e_n^j(t)$ be defined as $e_1^j(t) = x_1^j(t) - x_d(t), e_2^j(t) = \dot{x}_1^j(t) - \dot{x}_d(t), \cdots, e_n^j(t) = x_1^{(n-1),j}(t) - x_d^{(n-1)}(t)$, and the output error $e^j(t)$ be defined as $e^j(t) = e_1^j(t)$. The initial state errors at each iteration are not necessarily fixed, but assumed to satisfy $|e_i^j(0)| \le \varepsilon_i$ for some known positive constants $\varepsilon_i, i = 1, \cdots n$.

(A3) The desired state trajectory $X_d(t) = [x_d(t), \dot{x}_d(t), \cdots, x_d^{(n-1)}(t)]^\top$ is measurable and bounded.

In order to illustrate the idea of the proposed learning control scheme, we use the following four steps to explain the design approach.

• **Step 1**. Based on the assumptions on the nonlinear plant (8), we define a switching function as follows :

$$s^j(t) = c_1 e_1^j(t) + c_2 e_2^j(t) + \cdots + c_{n-1} e_{n-1}^j(t) + e_n^j(t) \tag{9}$$

where $c_1, \cdots, c_{n-1}$ are the coefficients of a Hurwitz polynomial $\Delta(D) = D^{n-1} + c_{n-1}D^{n-2} + \cdots + c_1$. It is noted that there exists a known constant ε^* such that the initial value of $s^j(t)$ will satisfy $|s^j(0)| \le c_1\varepsilon_1 + c_2\varepsilon_2 + \cdots + \varepsilon_n \equiv \varepsilon^*$ by assumption (A2). In order to overcome the uncertainty from initial state errors, a new function $s_\phi^j(t)$ is introduced as follows :

$$s_\phi^j(t) = s^j(t) - \phi(t)\mathbf{sat}\left(\frac{s^j(t)}{\phi(t)}\right) \tag{10}$$

where **sat** is the saturation function defined as

$$\mathbf{sat}\left(\frac{s^j(t)}{\phi(t)}\right) = \begin{cases} 1 & \text{if } s^j(t) > \phi(t) \\ \frac{s^j(t)}{\phi(t)} & \text{if } |s^j(t)| \le \phi(t) \\ -1 & \text{if } s^j(t) < -\phi(t) \end{cases}$$

and $\phi(t)$ is the width of the boundary layer which is time-varying depending on time t, but not related to the iteration number j. $\phi(t)$ is designed to satisfy the differential equation

$$\dot{\phi}(t) + k\phi(t) = 0 \tag{11}$$

with $k > 0$ and initial condition $\phi(0) = \varepsilon^*$. Of course, $0 < \varepsilon^* e^{-kT} \leq \phi(t) \leq \varepsilon^*, \forall t \in [0,T]$. There are three important properties of the function $s^j_\phi(t)$ useful for the technical analysis of convergence and stability. Firstly, according to the definition of (10), it can be easily shown that $s^j_\phi(0) = 0$ and $s^j_\phi(t)\mathbf{sat}\left(\frac{s^j(t)}{\phi(t)}\right) = |s^j_\phi(t)|$. Secondly, the derivative of $(s^j_\phi(t))^2$ with respective to time t can be computed as

$$
\begin{aligned}
\frac{d}{dt}\left(s^j_\phi(t)\right)^2 &= 2s^j_\phi(t)\dot{s}^j_\phi(t) \\
&= \begin{cases} 2s^j_\phi(t)\left(\dot{s}^j(t) - \dot{\phi}(t)\right) & \text{if } s^j(t) > \phi(t) \\ 0 & \text{if } |s^j(t)| \leq \phi(t) \\ 2s^j_\phi(t)\left(\dot{s}^j(t) + \dot{\phi}(t)\right) & \text{if } s^j(t) < -\phi(t) \end{cases} \\
&= 2s^j_\phi(t)\Big(\dot{s}^j(t) - \mathbf{sgn}\left(s^j_\phi(t)\right)\dot{\phi}(t)\Big) \qquad (12)
\end{aligned}
$$

where **sgn** denotes the traditional sign function. Thirdly, $s^j(t)$ and $s^j_\phi(t)$ will satisfy the following equation

$$
\begin{aligned}
& 2s^j_\phi(t)\left\{-ks^j(t) - \mathbf{sgn}\left(s^j_\phi(t)\right)\dot{\phi}(t)\right\} \\
&= 2s^j_\phi(t)\Bigg\{-ks^j_\phi(t) - k\phi(t)\mathbf{sat}\left(\frac{s^j(t)}{\phi(t)}\right) \\
&\quad - \mathbf{sgn}\left(s^j_\phi(t)\right)\dot{\phi}(t)\Bigg\} \\
&= -2k\left(s^j_\phi(t)\right)^2 - 2|s^j_\phi(t)|\left(\dot{\phi}(t) + k\phi(t)\right) \\
&= -2k\left(s^j_\phi(t)\right)^2 \qquad (13)
\end{aligned}
$$

according to the design of (10) and (11).

• **Step 2**. To give the motivation of our proposed control strategy, we differentiate $s^j(t)$ along the system trajectory (8) with respective to time t as follows :

$$
\dot{s}^j(t) = \sum_{i=1}^{n-1} c_i e^j_{i+1}(t) - x^{(n)}_d(t) - f(X^j(t)) + b(X^j(t))u^j(t) \qquad (14)
$$

If the nonlinear functions $f(X^j(t))$ and $b(X^j(t))$ are completely known, we can define the certainty equivalent controller as

$$
u^j_\star(t) = \frac{f(X^j(t)) + x^{(n)}_d(t) - \sum_{i=1}^{n-1} c_i e^j_{i+1}(t) - ks^j(t)}{b(X^j(t))} \qquad (15)
$$

with the positive constant k the same as that in (11). Then substituting (14) and (15) into (12), and use the fact of (13) will lead to $\frac{d}{dt}(s^j_\phi(t))^2 = -2k(s^j_\phi(t))^2$. This implies $s^j_\phi(t) = 0$ for all $t \in [0,T]$ and $j \geq 1$ since $s^j_\phi(0) = 0$. However, $f(X^j(t))$ and $b(X^j(t))$ are in general unknown or only partially known. Hence (12) can only be rewritten as

$$
\begin{aligned}
& \frac{d}{dt}\left(s^j_\phi(t)\right)^2 \\
&= 2s^j_\phi(t)\Big(\dot{s}^j(t) - \mathbf{sgn}\left(s^j_\phi(t)\right)\dot{\phi}(t)\Big) \\
&= -2k\left(s^j_\phi(t)\right)^2 + 2s^j_\phi(t)b(X^j(t))\left(u^j(t) - u^j_\star(t)\right) \qquad (16)
\end{aligned}
$$

To achieve the control objective, the proposed adaptive PID-type ILC will be divided into two parts, which is given by

$$
u^j(t) = u^j_{L_1}(t) + u^j_{L_2}(t) \qquad (17)
$$

and

$$
\begin{aligned}
u^j_{L_1}(t) &= u_{PID}(Z^j(t), W^j(t)) - \mathbf{sat}\left(\frac{s^j(t)}{\phi(t)}\right)\theta^j(t) \qquad (18) \\
u^j_{L_2}(t) &= -\gamma_w s^j_\phi(t) Z^{j\top}(t) Z^j(t) - \gamma_\theta s^j_\phi(t) \qquad (19)
\end{aligned}
$$

where $Z^j(t) = [e^j(t), \int_0^t e^j(\tau)d\tau, \dot{e}^j(t)]^\top \in \mathbb{R}^3 \times [0,T]$ and $W^j(t) = [K^j_P(t), K^j_I(t), K^j_D(t)]^\top \in \mathbb{R}^3 \times [0,T]$ are the input vector and the gain vector of the PID controller, respectively, and $\gamma_w > 0$ and $\gamma_\theta > 0$. Here, the PID controller in (18) is used to compensate for the certainty equivalent controller. Now if we do not consider the effect of $u^j_{L_2}(t)$ in this moment and substitute (18) into (16), we have

$$
\begin{aligned}
& \frac{1}{b(X^j(t))}\frac{d}{dt}\left(s^j_\phi(t)\right)^2 \\
&= -\frac{2k}{b(X^j(t))}\left(s^j_\phi(t)\right)^2 + 2s^j_\phi(t)\Bigg[\widetilde{W}^{jT}(t)Z^j(t) \\
&\quad - \mathbf{sat}\left(\frac{s^j(t)}{\phi(t)}\right)\theta^j(t) + r^j(t) + u^j_{L_2}(t)\Bigg] \\
&\leq -\frac{2k}{b(X^j(t))}\left(s^j_\phi(t)\right)^2 + 2s^j_\phi(t)\widetilde{W}^{j\top}(t)Z^j(t) \\
&\quad - 2|s^j_\phi(t)|\widetilde{\theta}^j(t) + 2s^j_\phi(t)u^j_{L_2}(t) \qquad (20)
\end{aligned}
$$

by using the results (5) and (6) given in Theorem 1.

• **Step 3** : Since the optimal parameters K^*_P, K^*_I, K^*_D and θ^* for an optimal approximation are generally unknown, the control parameters at time t of jth iteration will be tuned via some suitable adaptive laws between successive iteration. The adaptation algorithms for control parameters at (next) $j+1$th iteration $W^{j+1}(t)$ and $\theta^{j+1}(t)$ are given as follows :

$$
\begin{aligned}
W^{j+1}_p(t) &= W^j(t) - \gamma_w s^j_\phi(t) Z^j(t) \qquad (21) \\
\theta^{j+1}_p(t) &= \theta^j(t) + \gamma_\theta |s^j_\phi(t)| \qquad (22)
\end{aligned}
$$

and

$$
\begin{aligned}
W^{j+1}(t) &= proj\left(W^{j+1}_p(t)\right) \\
&= \left[proj(K^{j+1}_{p,P}(t)), proj(K^{j+1}_{p,I}(t)), proj(K^{j+1}_{p,D}(t))\right]^\top \qquad (23) \\
\theta^{j+1}(t) &= proj\left(\theta^{j+1}_p(t)\right) \qquad (24)
\end{aligned}
$$

where $proj$ denotes the projection mechanism :

$$
proj\left(z^{j+1}_p(t)\right) = \begin{cases} \bar{z} & \text{if } z^{j+1}_p(t) \geq \bar{z} \\ -\bar{z} & \text{if } z^{j+1}_p(t) \leq -\bar{z} \\ z^{j+1}_p(t) & \text{otherwise} \end{cases}
$$

with $\bar{z}$ being the upper bound of $|z^*|$ (z^* belongs to an element of $\{W^*, \theta^*\}$). According to the projection algorithm, it is noted that the parameter errors will be bounded for all iterations and for all $t \in [0,T]$.

4 Analysis of Stability and Convergence

If we define the projected parameter errors as $\widetilde{W}(t) = W(t) - W^*$, $\widetilde{\theta}^j(t) = \theta^j(t) - \theta^*$ and unprojected parameter errors as $\widetilde{W}_p(t) = W_p(t) - W^*$, $\widetilde{\theta}^j_p(t) = \theta^j_p(t) - \theta^*$, respectively, then we have $\widetilde{W}^{j\top}_p(t)\widetilde{W}^j_p(t) \geq \widetilde{W}^{j\top}(t)\widetilde{W}^j(t)$ and $(\widetilde{\theta}^j_p(t))^2 \geq (\widetilde{\theta}^j(t))^2$.

Furthermore, it is easy to show by subtracting the optimal control gains on both side of (21), (22) that

$$\begin{aligned}\widetilde{W}_p^{j+1}(t) &= \widetilde{W}^j(t) - \gamma_w s_\phi^j(t) Z^j(t)\\ \widetilde{\theta}_p^{j+1}(t) &= \widetilde{\theta}^j(t) + \gamma_\theta |s_\phi^j(t)|\end{aligned}$$

Now we are ready to state the main results in the following theorem.

Theorem 2 : Consider the nonlinear system (8) satisfying the assumptions (A1)–(A3). If we define $E^j(t) = [e_1^j(t), e_2^j(t), \cdots e_{n-1}^j(t)]^\top$, then the proposed adaptive PID-type ILC guarantees :

(t1) $\lim_{j\to\infty} s_\phi^j(t) = s_\phi^\infty(t) = 0$, $\forall t \in [0,T]$.

(t2) $\lim_{j\to\infty} |s^j(t)| = |s^\infty(t)| \le \phi(t) = e^{-kt}\varepsilon^*$, $\forall t \in [0,T]$.

(t3) All adjustable control parameters and internal signals are bounded $\forall t \in [0,T]$ and $\forall j \ge 1$.

(t4) Let λ be the positive constant such that $\Delta(D-\lambda)$ is still a Hurwitz polynomial, then

$$\begin{aligned}\lim_{j\to\infty} \|E^j(t)\| &= \|E^\infty(t)\|\\ &\le m_1 e^{-\lambda t}\|E^\infty(0)\| + m_1\varepsilon^* \frac{e^{-kt}-e^{-\lambda t}}{\lambda - k}\end{aligned}$$

for some positive constant m_1 and $t \in [0,T]$, and

$$\lim_{j\to\infty} |e_n^j(t)| = |e_n^\infty(t)| \le \sum_{i=1}^{n-1} c_i |e_i^\infty(t)| + e^{-kt}\varepsilon^*, \forall t \in [0,T]$$

Proof :
(t1) Define the cost functions of performance as :

$$\begin{aligned}V^j(t) &= \int_0^t \left[\frac{1}{\gamma_w}\widetilde{W}^{j\top}(\tau)\widetilde{W}^j(\tau) + \frac{1}{\gamma_\theta}\left(\widetilde{\theta}^j(\tau)\right)^2\right] d\tau\\ V_p^j(t) &= \int_0^t \left[\frac{1}{\gamma_w}\widetilde{W}_p^{j\top}(\tau)\widetilde{W}_p^j(\tau) + \frac{1}{\gamma_\theta}\left(\widetilde{\theta}_p^j(\tau)\right)^2\right] d\tau\end{aligned}$$

then we can derive

$$\begin{aligned}&V^{j+1}(t) - V^j(t)\\ &\le V_p^{j+1}(t) - V^j(t)\\ &= \int_0^t \left[\frac{1}{\gamma_w}\widetilde{W}_p^{j+1\top}(\tau)\widetilde{W}_p^{j+1}(\tau) + \frac{1}{\gamma_\theta}\left(\widetilde{\theta}_p^{j+1}(\tau)\right)^2\right.\\ &\qquad \left. - \frac{1}{\gamma_w}\widetilde{W}^{j\top}(\tau)\widetilde{W}^j(\tau) - \frac{1}{\gamma_\theta}\left(\widetilde{\theta}^j(\tau)\right)^2\right] d\tau\\ &= \int_0^t \left[-2s_\phi^j(\tau)\widetilde{W}^{j\top}(\tau)Z^j(\tau) + \gamma_w (s_\phi^j(\tau))^2 Z^{j\top}(\tau)Z^j(\tau)\right.\\ &\qquad \left. + 2|s_\phi^j(\tau)|\widetilde{\theta}^j(\tau) + \gamma_\theta(s_\phi^j(\tau))^2\right] d\tau \qquad (25)\end{aligned}$$

If we integrate (20) over time interval $[0,t]$, $t \in (0,T]$ as follows :

$$\begin{aligned}&\int_0^t \frac{1}{b(X^j(\tau))}\frac{d}{d\tau}(s_\phi^j(\tau))^2 d\tau = \int_{(s_\phi^j(0))^2}^{(s_\phi^j(t))^2} \frac{1}{b(X^j(\tau))} d(s_\phi^j(\tau))^2\\ &\le \int_0^t \left[-\frac{2k}{b(X^j(\tau))}(s_\phi^j(\tau))^2 + 2s_\phi^j(\tau)\widetilde{W}^{j\top}(\tau)Z^j(\tau)\right.\\ &\qquad \left. - 2|s_\phi^j(\tau)|\widetilde{\theta}^j(\tau) + 2s_\phi^j(\tau)u_{L_2}^j(\tau)\right] d\tau\end{aligned}$$

then after some manipulations, we can find

$$\begin{aligned}&\int_0^t \left[-2s_\phi^j(\tau)\widetilde{W}^{j\top}(\tau)Z^j(\tau) + 2|s_\phi^j(\tau)|\widetilde{\theta}^{jT}(\tau)\right] d\tau\\ &\le \int_0^t \left[-\frac{2k}{b(X^j(\tau))}(s_\phi^j(\tau))^2 + 2s_\phi^j(\tau)u_{L_2}^j(\tau)\right] d\tau\\ &\qquad - \int_{(s_\phi^j(0))^2}^{(s_\phi^j(t))^2} \frac{1}{b(X^j(\tau))} d(s_\phi^j(\tau))^2 \qquad (26)\end{aligned}$$

Since $s_\phi^j(0) = 0$ due to step 1, the design of $u_{L_2}^j(t)$ in step 2 is now clear if we substitute (26) and (19) into (25), and show that

$$\begin{aligned}&V^{j+1}(t) - V^j(t)\\ &\le \int_0^t \left[-\frac{2k}{b(X^j(\tau))}(s_\phi^j(\tau))^2 + 2s_\phi^j(\tau)u_{L_2}^j(\tau)\right.\\ &\qquad \left. + \gamma_w(s_\phi^j(\tau))^2 Z^{j\top}(\tau)Z^j(\tau) + \gamma_\theta(s_\phi^j(\tau))^2\right] d\tau\\ &\qquad - \int_0^{(s_\phi^j(t))^2} \frac{1}{b(X^j(\tau))} d(s_\phi^j(\tau))^2\\ &\le -\int_0^{(s_\phi^j(t))^2} \frac{1}{b(X^j(\tau))} d(s_\phi^j(\tau))^2 \qquad (27)\end{aligned}$$

Thus, we have

$$\int_0^{(s_\phi^j(t))^2} \frac{1}{b(X^j(\tau))} d(s_\phi^j(\tau))^2 \le V^j(t) - V^{j+1}(t) \le V^1(t)$$

for iteration $j \ge 1$. This implies $\int_0^{(s_\phi^j(t))^2} \frac{1}{b(X^j(\tau))} d(s_\phi^j(\tau))^2$ is bounded $\forall t \in [0,T]$ and $j \ge 1$ since $V^1(t)$ is bounded $\forall t \in [0,T]$ due to projection algorithms (23)-(24). On the other hand, $V^j(t)$ will converge to some positive function since $V^j(t)$ is positive definite and monotonically decreasing by the fact of (27). Hence, $V^{j+1}(t) - V^j(t)$ converges to zero and

$$\begin{aligned}&\lim_{j\to\infty} \int_0^{(s_\phi^j(t))^2} \frac{1}{b(X^j(\tau))} d(s_\phi^j(\tau))^2\\ &= \int_0^{(s_\phi^\infty(t))^2} \frac{1}{b(X^\infty(\tau))} d(s_\phi^\infty(\tau))^2 = 0, \forall\, t \in [0,T]\end{aligned}$$

since $b(X^\infty(t)) > 0$. Therefore, we have

$$\lim_{j\to\infty} s_\phi^j(t) = s_\phi^\infty(t) = 0, \forall\, t \in [0,T]$$

This proves (t1) of theorem 2.

(t2) The boundedness of $s^j(t)$ at each iteration over $[0,T]$ can be concluded from equation (10) because $\phi(t)$ is always bounded and the bound of $s^\infty(t)$ will satisfy

$$\lim_{j\to\infty} |s^j(t)| = |s^\infty(t)| \le \phi(t) = e^{-kt}\varepsilon^*, \forall\, t \in [0,T]$$

This proves (t2) of theorem 2.

(t3) Boundedness of $s^j(t)$ implies boundedness of $e_1^j(t), e_2^j(t), \cdots, e_n^j(t)$. Together with the fact that all the adjustable parameters are bounded due to projection algorithms, (t3) of theorem 2 is guaranteed.

(t4) To find the learning performance of each state tracking error at the final iteration, we consider the following state space equation :

$$\begin{bmatrix} \dot{e}_1^\infty(t) \\ \dot{e}_2^\infty(t) \\ \vdots \\ \dot{e}_n^\infty(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -c_1 & -c_2 & \cdots & -c_n \end{bmatrix} \begin{bmatrix} e_1^\infty(t) \\ e_2^\infty(t) \\ \vdots \\ e_n^\infty(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} s^\infty(t)$$

or simply

$$\dot{E}^\infty(t) = A_c E^\infty(t) + B_c s^\infty(t) \tag{28}$$

by using assumption (A2) and the definition of switching function $s^j(t)$ in (9). Solution of (28) in time domain is given by

$$E^\infty(t) = e^{A_c t} E^\infty(0) + \int_0^t e^{A_c(t-\tau)} B_c s^\infty(\tau) d\tau \tag{29}$$

where the state transition matrix $e^{A_c t}$ satisfies $\|e^{A_c t}\| \leq m_1 e^{-\lambda t}$ for some suitable positive constant m_1. Taking norms on (29), it yields

$$\begin{aligned} & \|E^\infty(t)\| \\ \leq \;& m_1 e^{-\lambda t}\|E^\infty(0)\| + m_1 \int_0^t e^{-\lambda(t-\tau)}\|B_c\||s^\infty(\tau)|d\tau \\ \leq \;& m_1 e^{-\lambda t}\|E^\infty(0)\| + m_1 \int_0^t e^{-\lambda(t-\tau)} e^{-k\tau}\varepsilon^* d\tau \\ \leq \;& m_1 e^{-\lambda t}\|E^\infty(0)\| + m_1 \varepsilon^* \frac{e^{-kt} - e^{-\lambda t}}{\lambda - k} \end{aligned}$$

Finally, tracking performance of $e_n^\infty(t)$ which is shown in (t4) can be easily found by using (9). This concludes (t4) of theorem 2.

5 Simulation Example

In this section, we apply the proposed adaptive PID-type ILC to a Chua's chaotic circuit with variable initial states to repetitive track a desired trajectory. The typical Chua's chaotic circuit is a simple nonlinear oscillator circuit, which includes very rich bifurcation and chaotic phenomena. The transformed dynamic equation of a Chua's circuit [22] is given by

$$\begin{aligned} \dot{x}_1^j(t) &= x_2^j(t) \\ \dot{x}_2^j(t) &= x_3^j(t) \\ \dot{x}_3^j(t) &= \frac{14}{1805} x_1^j(t) - \frac{168}{9025} x_2^j(t) + \frac{1}{38} x_3^j(t) \\ & \quad - \frac{2}{45}\left(\frac{28}{361} x_1^j(t) + \frac{7}{95} x_2^j(t) + x_3^j(t)\right)^3 + u^j(t) \\ y^j(t) &= x_1^j(t) \end{aligned}$$

where $x_1^j(t)$, $x_2^j(t)$ and $x_3^j(t)$ denote the system states, $y^j(t)$ is the system output. The control objective is to control the state $X^j(t) = [x_1^j(t), x_2^j(t), x_3^j(t)]^\top$ to track the desired trajectory $X_d(t) = [x_d(t), \dot{x}_d(t), \ddot{x}_d(t)]^T = [\sin(t), \cos(t), -\sin(t)]^\top$ for $t \in [0, 10]$ as close as possible even initial state error exists. The design steps are given in the following :

(D1) The switching function is simply equal to $s^j(t) = c_1 e_1^j(t) + c_2 e_2^j(t) + e_3^j(t)$ where $e_1^j(t) = x_1^j(t) - \sin t$, $e_2^j(t) = x_2^j(t) - \cos t$ and $e_3^j(t) = x_3^j(t) + \sin t$. The modified switching function with time-varying boundary layer is designed as $s_\phi^j(t) = s^j(t) - \phi(t)\mathbf{sat}\left(\frac{s^j(t)}{\phi(t)}\right)$ with $\dot{\phi}(t) + k\phi(t) = 0$ and $k > 0$.

(D2) Design the controller $u^j(t)$ as in (17) with the two iterative learning control components as in (18) and (19), respectively. In this case, the initial gains of the PID-type ILC are given as :

$$W^1(t) \;=\; [K_P^1(t), K_I^1(t), K_D^1(t)]^\top = [0.1, 0.5, 0.1]^\top$$

for all $t \in [0, 10]$. The compensated force for approximation error of the PID-type controller in (18) is designed with initial control parameter $\theta^1(t) = 0.1$.

(D3) Finally, the parameters $W^j(t)$ and $\theta^j(t)$ are updated for next iteration by using the projection type adaptation algorithms (21)–(24). In general, the upper bounds on the optimal parameters are not easy to estimate for an arbitrary optimal controller. For real implementation of iterative controlling the plant, suitable values of the upper bounds are usually selected as large as possible. In most of our simulations, these upper bounds are all set to be 10.

To begin with this example, the parameters of k, γ_w, γ_θ, c_1 and c_2 are chosen as $k = 4$, $\gamma_w = \gamma_\theta = 100$ and $c_1 = c_2 = 4$. For a practical situation the Chua's chaotic circuit may have variable initial states at the beginning of each iteration. In other words, $X^j(0) = [x_1^j(0), x_2^j(0), x_3^j(0)]^\top = [rand, rand, rand]^\top$ for all $j \geq 1$, in which $rand$ is a generator of random number between the interval $[-1, 1]$. Here the variable initial states are chosen as

$$\begin{aligned} X^1(0) &= [0.9003, -0.5377, 0.1603]^\top \\ X^2(0) &= [-0.0280, 0.7826, 0.3931]^\top \\ X^3(0) &= [-0.0871, -0.9630, 0.4821]^\top \\ X^4(0) &= [-0.0829, 0.1731, 0.4379]^\top \\ X^5(0) &= [0.6327, 0.3573, -0.4856]^\top \end{aligned}$$

Therefore, this implies there exist initial state errors for $e_1^j(0) = x_1^j(0) - \sin(0) = x_1^j(0)$, $e_2^j(0) = x_2^j(0) - \cos(0) = x_2^j(0) - 1$ and $e_3^j(0) = x_3^j(0) + \sin(0) = x_3^j(0)$. Since $s^j(0) = c_1 e_1^j(0) + c_2 e_2^j(0) + e_3^j(0)$, the initial value of $\phi(t)$ is selected as $\phi(0) = \varepsilon^* = \max_j \left(|c_1 e_1^j(0)| + |c_2 e_2^j(0)| + |e_3^j(0)|\right) = 9.8963$. From Figure 1, simulation results are discussed as follows :

(R1) Figure 1(a) shows that the supremum value of $|s_\phi^j(t)|$, i.e., $\sup_{t\in[0,10]} |s_\phi^j(t)|$ with respective to iteration j. It is noted that the technical result given in (t1) of theorem 2 is verified by observing the asymptotical convergence of $|s_\phi^j(t)|$ in Figure 1(a).

(R2) We demonstrate the trajectory of $s^j(t)$ for iteration $j = 5$ in Figure 1(b). This fact satisfies the technical result (t2) of theorem 2. It is also shown that the tracking error trajectory can be bounded by a tunable pre-specified time-varying boundary layer $\phi(t)$ and the proposed adaptive PID-type ILC successfully overcome the variable initial state errors. Furthermore, the comparisons of the system states $x_1^5(t)$, $x_2^5(t)$ and $x_3^5(t)$ as well as desired states $x_d(t)$, $\dot{x}_d(t)$ and $\ddot{x}_d(t)$ are shown in Figure 1(c)-(e). The trajectories of PID gains $K_P^5(t)$, $K_I^5(t)$ and $K_D^5(t)$ of the propose adaptive PID-type ILC are also shown in Figure 1(f)-(h).

(R3) Finally, the bounded control input in the technical result (t3) of theorem 2 is proven in Figure 1(i). It is obvious that the smooth control force $u^5(t)$ at 5th iteration is obtained by using the time-varying boundary layer design.

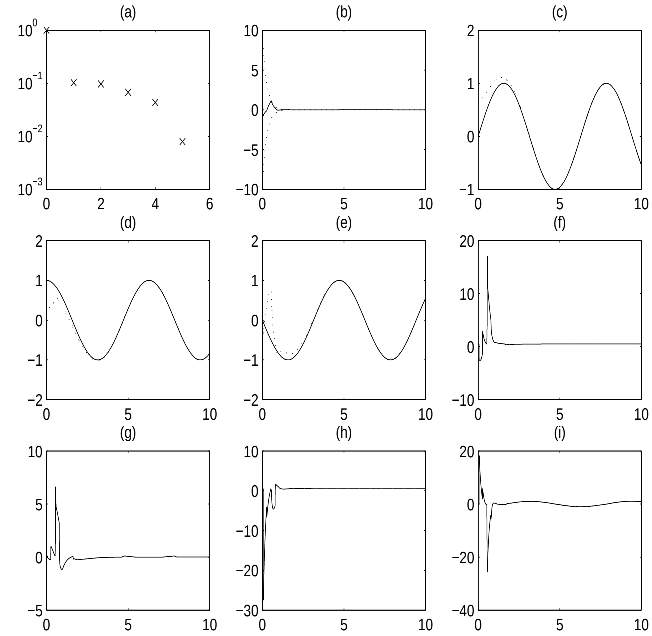

Figure 1 :
(a) $\sup_{t\in[0,10]} |s_\phi^j(t)|$ versus iteration j.
(b) $s^5(t)$ (solid line) and $\phi(t), -\phi(t)$ (dashed line) versus t.
(c) $x_d(t)$ (solid line) and $x_1^5(t)$ (dashed line) versus time t.
(d) $\dot{x}_d(t)$ (solid line) and $x_2^5(t)$ (dashed line) versus time t.
(e) $\ddot{x}_d(t)$ (solid line) and $x_3^5(t)$ (dashed line) versus time t.
(f) $K_P^5(t)$ versus time t.
(g) $K_I^5(t)$ versus time t.
(h) $K_D^5(t)$ versus time t.
(i) $u^5(t)$ versus time t.

6 Conclusion

In this paper, we proposed a method for designing an adaptive PID-type ILC for a class of nonlinear systems with varying initial state errors and unknown bounds on plant nonlinearities. The main PID-ILC in this AILC is used to follow the unknown certainty equivalent controller. To deal with the problems of initial state errors at each iteration and the possible undesirable chattering behavior, a technique of time-varying layer is adopted. Based on the Lyapunov like analysis, the proportional, integral and derivative gains of the PID-type ILC are adjusted between successive iterations to achieve a better learning performance. We show that the tracking error can asymptotically converge to a tunable residual set as iteration goes to infinity, and all adjustable parameters and the internal signals remain bounded. From the simulation result of iterative learning control for a Chua's circuit, the feasibility of the proposed adaptive PID-type ILC is clearly demonstrated.

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