A NEW DELAY-DEPENDENT STABILITY CRITERION FOR NEUTRAL DELAY SYSTEMS WITH NORM-BOUNDED UNCERTAINTY

Q.-L. Han

Faculty of Informatics and Communication
Central Queensland University
Rockhampton, Qld 4702, AUSTRALIA
Tel: + 61 7 4930 9270
Fax: + 61 7 4930 9729
E-mail: q.han@cqu.edu.au

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Abstract

The robust stability problem for uncertain linear delay-differential systems of neutral type is investigated. The norm-bounded uncertainty appears in all system matrices. A new delay-dependent stability criterion is derived. The criterion is formulated in the form of a linear matrix inequality (LMI). Numerical examples show that the criterion is much less conservative than those in the literature.

1 Introduction

The problems of stability and stabilization of delay-differential systems of neutral type have received considerable attention in the last two decades, see for example, [12, 18, 22]. The practical examples of neutral delay-differential systems include the distributed networks containing lossless transmission lines [2], and population ecology [15]. Many stability conditions are based on a matrix measure and a matrix norm [13, 17] or a simple Lyapunov functional [21, 23]. Although it is easy to check these conditions, the conditions required the matrix measure to be negative or the parameters to be tuned. The results are usually more conservative. Recently, a linear matrix inequality (LMI) technique [1] has been employed to investigate the stability of neutral systems and less conservative criteria have been obtained, see for example, [4, 9, 10, 11, 16, 20, 19].

In this paper, the robust stability of neutral systems under norm-bounded uncertainties in all system matrices is studied by using decomposition technique of discrete-delay term matrix. A delay-dependent criterion that is formulated in terms of an LMI is obtained. Numerical examples show that the results obtained in this paper are less conservative than some existing results in the literature.

Notation: For a symmetric matrix W, "$W > 0$" denotes that W is positive definite matrix. Let I be an identity matrix of appropriate dimension. $\mathcal{C}([-h,0],\mathbb{R}^n)$ stands for the set of continuous $\mathbb{R}^n$ valued functions on $[-h,0]$ and let $x_t \in \mathcal{C}([-h,0],\mathbb{R}^n)$ be a segment of system trajectory defined as $x_t(\theta) = x(t+\theta)$, $-h \le \theta \le 0$ and denotes $\|\phi\|_c = \sup_{-h\le\theta\le 0}\|\phi(\theta)\|$ as the norm for $\phi \in \mathcal{C}([-h,0],\mathbb{R}^n)$. Use $\|\ \|$ to stand for either the Euclidean vector norm or the induced matrix 2-norm and denote $\sigma_{\max}(W)$ as the maximum singular value of the matrix W.

2. Problem statement

Consider the uncertain linear neutral delay-differential system

$$\frac{d}{dt}[x(t) - C(t)x(t-h)] = A(t)x(t) + B(t)x(t-h) \quad (1)$$

$$x(\theta) = \phi(\theta),\ \forall\,\theta \in [-h,\ 0] \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state, $h > 0$ is the constant time-delay, $\phi(\cdot)$ is a continuous vector valued initial function, $A(t) \in \mathbb{R}^{n\times n}$, $B(t) \in \mathbb{R}^{n\times n}$ and $C(t) \in \mathbb{R}^{n\times n}$ are uncertain matrices, not known completely, except that they are within a compact set Ω which we will refer to as the uncertainty set

$$\big(A(t),\ B(t),\ C(t)\big) \in \Omega \subset \mathbb{R}^{n\times 3n} \text{ for all } t\in[0,\infty)$$

Throughout this paper, we assume that

A1. The system $x(t)-C(t)x(t-h)=0$ is asymptotically stable.

In this paper, we will attempt to formulate a practically computable criterion to check the stability of system described by (1)-(2).

3. Main results

In order to improve the delay bound, similar to the retarded system [6], decompose the matrix $B(t)$ as $B(t)=B_1+B_2(t)$, where B_1 is a constant matrix. System (1) can be rewritten as in the following form

$$\frac{d}{dt}[x(t)-C(t)x(t-h)+B_1\int_{t-h}^{t} x(\xi)d\xi] = [A(t)+B_1]x(t)+B_2(t)x(t-h) \qquad (3)$$

Define the operator $\mathcal{D}:\ \mathcal{C}([-h,0],\mathbb{R}^n)\to\mathbb{R}^n$ as

$$\mathcal{D}x_t = x(t)-C(t)x(t-h)+B_1\int_{t-h}^{t} x(\xi)d\xi$$

Remark 1. The nominal systems of systems (1) and (3) are

$$\dot{x}(t)-C\dot{x}(t-h)=Ax(t)+Bx(t-h) \qquad (4)$$

and

$$\frac{d}{dt}[x(t)-Cx(t-h)+B_1\int_{t-h}^{t} x(\xi)d\xi] = \big(A+B_1\big)x(t)+B_2x(t-h) \qquad (5)$$

respectively. The corresponding operator $\mathcal{D}_{\text{nominal}}:\ \mathcal{C}([-h,0],\mathbb{R}^n)\to\mathbb{R}^n$ for system (5) is

$$\mathcal{D}_{\text{nominal}}x_t = x(t)-Cx(t-h)+B_1\int_{t-h}^{t} x(\xi)d\xi$$

It is easy to prove that assuming that the operator $\mathcal{D}_{\text{nominal}}$ is stable, the asymptotic stability of system (4) is equivalent to that of system (5). Therefore, no additional dynamics in the sense defined in [7, 8] appear in system (5) compared with system (4).

In fact, it is obvious that the asymptotic stability of system (5) implies that of system (4). In the following we will show that the asymptotic stability of system (4) also implies that of system (5). The characteristic functions of systems (4) and (5) are

$$f(s)=\det(sI-sCe^{-hs}-A-Be^{-hs})$$

and

$$g(s)=\det\left[s(I-Ce^{-hs}+B_1\frac{1-e^{-hs}}{s})-(A+B_1)-B_2e^{-hs}\right]$$

By the asymptotic stability of system (4), we have

$$f(s)\neq 0 \text{ for } \forall\,\mathrm{Re}\,s\geq 0$$

Simple computation to obtain

$$\begin{aligned} g(s) &= \det\left[s(I-Ce^{-hs}+B_1\frac{1-e^{-hs}}{s})-(A+B_1)-B_2e^{-hs}\right] \\ &= \det\left[sI-sCe^{-hs}+B_1(1-e^{-hs})-(A+B_1)-B_2e^{-hs}\right] \\ &= \det(sI-sCe^{-hs}-A-Be^{-hs}) \\ &= f(s) \end{aligned}$$

Then

$$g(s)\neq 0 \text{ for } \forall\,\mathrm{Re}\,s\geq 0$$

Noting that the operator $\mathcal{D}_{\text{nominal}}$ is stable, system (5) is asymptotically stable according to [12].

For the stability of the considered system, we can conclude that

Proposition 1. Under A1 and given a scalar $\bar{h}>0$, system described by (1) and (2) is asymptotically stable for any constant time-delay h satisfying $0\leq h\leq \bar{h}$ if the operator $\mathcal{D}$ is stable and there exist $n\times n$ matrices $P>0$, $R>0$ and $W>0$ such that the LMI (6), as shown at the bottom of the last page of the paper, holds, where

$$\Xi_{11}(t)=-[A(t)+B_1]^T P-P[A(t)+B_1]-\bar{h}R-W$$

$$\Xi_{12}(t)=[A(t)+B_1]^T PC(t)-PB_2(t)$$

$$\Xi_{22}(t)=W+B_2^T(t)PC(t)+C^T(t)PB_2(t)$$

Proof. See the full version of the paper [11].

Remark 2. The operator $\mathcal{D}$ is stable if $\mathcal{D}x_t = x(t) - C(t)x(t-h) + B_1 \int_{t-h}^{t} x(\xi)d\xi = 0$ is asymptotically stable. A sufficient condition is that the inequality $\|C(t)\| + \bar{h}\|B_1\| \le 1-\delta < 1$ holds for a sufficiently small $\delta > 0$. If one uses the sufficient condition to replace the fact that the operator $\mathcal{D}$ is stable, the assumption **A1** is also implied by the condition. In this case, the assumption **A1** is not necessarily needed.

Remark 3. If $B_1 = 0$, through some simple variable changes one can easily obtain that system (1)-(2) is asymptotically stable if $\|C(t)\| \le 1-\delta < 1$ for a sufficiently small $\delta > 0$ and there exist $n \times n$ matrices $\tilde{P} > 0$, $\tilde{W} > 0$ satisfying the following LMI

$$\begin{bmatrix} -A^T(t)\tilde{P} - \tilde{P}A(t) - \tilde{W} & A^T(t)\tilde{P}C(t) - \tilde{P}B(t) \\ C^T(t)\tilde{P}A(t) - B^T(t)\tilde{P} & \tilde{W} \end{bmatrix} > 0$$

which does not include any information on the time-delay. Therefore, the corresponding criterion is independent of delay.

Remark 4. To compare our result with that in [16], we now consider the stability of nominal system (4). For this case, let $B_1 = B$, $B_2 = 0$. In light of Proposition 1, system (4) is asymptotically stable for any constant time-delay h satisfying $0 \le h \le \bar{h}$ if the operator $\mathcal{D}_{\text{nominal}}$ is stable and there exist $n \times n$ matrices $P > 0$, $R > 0$ and $W > 0$ such that

$$\begin{bmatrix} (1,1) & (A+B)^T PC & -\bar{h}(A+B)^T PB \\ C^T P(A+B) & W & 0 \\ -\bar{h}B^T P(A+B) & 0 & \bar{h}R \end{bmatrix} > 0 \quad (7)$$

with

$$(1,1) \triangleq -(A+B)^T P - P(A+B) - \bar{h}R - W$$

By Theorem 1 in [16], system (4) is asymptotically stable if there exist $n \times n$ matrices $P > 0$ and $R > 0$ such that

$$\|C\| + \bar{h}\|B\| < 1$$

and

$$(A+B)^T P + P(A+B) + (h+1)R$$
$$+h(A+B)^T PBR^{-1}B^T P(A+B)$$
$$+(A+B)^T PCR^{-1}C^T P(A+B) < 0 \quad (8)$$

According to Remark 1 in [16], h in (8) can be replaced by $\bar{h}$, then rewrite (8) in the following LMI

$$\begin{bmatrix} (1,1) & (A+B)^T PC & -\bar{h}(A+B)^T PB \\ C^T P(A+B) & R & 0 \\ -\bar{h}B^T P(A+B) & 0 & \bar{h}R \end{bmatrix} > 0 \quad (9)$$

with

$$(1,1) \triangleq -(A+B)^T P - P(A+B) - (\bar{h}+1)R$$

If $W = R$, then (7) reduces to (9). Noting that R and W in (7) are two free variables, (7) is less restrictive than (9).

Now we consider the norm bounded uncertainty described by

$$A(t) = A + \Delta A(t), \quad B(t) = B + \Delta B(t), \quad C(t) = C + \Delta C(t) \quad (10)$$

where

$$[\Delta A(t) \quad \Delta B(t) \quad \Delta C(t)] = LF(t)[E_a \quad E_b \quad E_c] \quad (11)$$

where $F(t) \in \mathbb{R}^{p \times q}$ is an unknown real and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$\sigma_{\max}(F(t)) \le 1 \quad (12)$$

and L, E_a and E_b are known real constant matrices which characterize how the uncertainty enters the nominal matrices A and B.

Let $B = B_1 + B_2$, then $B_2(t) = B_2 + \Delta B(t)$. Now we state the following result.

Proposition 2. Under A1 and given a scalar $\bar{h} > 0$, the system described by (1) and (2), with uncertainty described by (10) to (12) is asymptotically stable for any constant time-delay h satisfying $0 \le h \le \bar{h}$ if the operator $\mathcal{D}$ is stable and there exist $n \times n$ matrices $X > 0$, $Y > 0$, $Z > 0$ and a scalar $\mu > 0$ such that the LMI (13), as shown at the bottom of the last page of the paper, holds, where

$$\Psi_{11} = -(A+B_1)^T X - X(A+B_1) - \bar{h}Y - Z - \mu E_a^T E_a$$

$$\Psi_{12} = (A+B_1)^T XC - XB_2 - \mu E_a^T E_b$$

$$\Psi_{22} = Z + B_2^T XC + C^T XB_2 - \mu E_b^T E_b - E_c^T E_c$$

Proof. See the full version of the paper [11].

Remark 5. The efficiency of Proposition 2 depends on the decomposition of matrix *B*. The matrix B_1 was chosen such that the operator $\mathcal{D}$ is stable and $A(t)+B_1$ is "more stable" than matrix $A(t)$. The decomposition idea was first introduced by Goubet-Batholomeus *et al.* [6] for the retarded case. Now we consider how to decompose the matrix *B*. For the case that the matrix *B* is decomposed as $B = B_1 + B_2$, where $B_1 = -\beta I$, $B_2 = B - B_1$ and $\beta > 0$. First we choose the initial value β_0 by solving the following optimization problem

minimize β

subject to

$$\beta > 0$$

$$\|C(t)\| + \bar{h}\beta \le 1-\delta < 1 \text{ (for sufficiently small } \delta > 0\text{)}$$

$$\begin{bmatrix} (A-\beta I)^T P + P(A-\beta I) + E_a^T E_a & PL \\ L^T P & -I \end{bmatrix} < 0$$

Use this decomposion to solve the LMI (13) by employing Matlab LMI Toolbox [5]. If it is feasible, increase β (such as $\beta_n = \beta_{n-1} + 0.0001$) and then resolve the LMI (13) and $\|C(t)\| + \bar{h}\beta \le 1-\delta < 1$ until for some $\beta > 0$ the LMI (13) is not feasible or $\|C(t)\| + \bar{h}\beta \le 1-\delta < 1$ is not satisfied. For each $\beta > 0$, there is a corresponding $\bar{h}$. From these $\bar{h}$'s, find the largest one $\bar{h}_{\max}$. The corresponding $B_1 = -\beta I$ is the 'optimal' choice. For non-diagonal decomposition, it's more complicated but the idea is the same.

Concerning how to solve the LMI (13), a Matlab m-function is written which automatically generates the LMI (13) and then solves this LMI using LMI Solver FEASP in LMI toolbox [5]. The inputs to the function are system matrices and the time-delay. The function returns whether the LMI is feasible. If feasible, it also gives matrices $X > 0$, $Y > 0$, $Z > 0$ and scalar $\mu > 0$ as outputs.

4. Examples

To illustrate the stability criterion, the following examples are presented.

Example 1. Consider the following uncertain linear retarded system [3]

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]x(t-h) \qquad (14)$$

where

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \; B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$$

and $\Delta A(t)$ and $\Delta B(t)$ are unknown matrices satisfying $\|\Delta A(t)\| \le 0.2$ and $\|\Delta B(t)\| \le 0.2$, $\forall t$. The above system is of the form of (10) to (13) with $C = \Delta C(t) = 0$, $L = 0.2I$ and $E_a = E_b = I$, $E_c = 0$.

By the criterion in [3], system (14) is robustly stable for any h satisfying $0 \le h \le 0.4437$. Using the stability criterion in this paper, decomposing matrix *B* as

$$B_1 = \begin{bmatrix} -0.21 & 0 \\ -0.96 & -0.09 \end{bmatrix}, \; B_2 = \begin{bmatrix} -0.79 & 0 \\ -0.04 & -0.91 \end{bmatrix}$$

the maximum value of $\bar{h}_{\max}$ for the system to have guaranteed robust stability is $\bar{h}_{\max} = 2.1186$ which gives $\bar{h}_{\max}$ more than four times the result in [3]. Therefore, for this example, the robust stability criterion in this paper gives a less conservative result than that in [3]. Other results surveyed by de Souza and Li [3] are much more conservative.

Example 2. Consider the following uncertain linear neutral delay-differential system

$$\frac{d}{dt}\{x(t) - [C + \Delta C(t)]x(t-h)\}$$
$$= [A + \Delta A(t)]x(t) + [B + \Delta B(t)]x(t-h) \qquad (15)$$

where matrices *A* and *B* are the same as Example 1 and

$$C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \; 0 \le c < 1$$

and $\Delta A(t)$, $\Delta B(t)$ and $\Delta C(t)$ are unknown matrices satisfying $\|\Delta A(t)\| \le \alpha$, $\|\Delta B(t)\| \le \alpha$ and $\|\Delta C(t)\| \le \alpha$ $\forall t$. The considered system is of the form of (10) to (13) with $L = \alpha I$ and $E_a = E_b = E_c = I$.

Decompose the matrix *B* as

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & -0.38 \end{bmatrix}, \; B_2 = \begin{bmatrix} -1 & 0 \\ -1 & -0.62 \end{bmatrix}$$

For $\alpha = 0.2$, the maximum value $\bar{h}_{\max}$ is listed in the following table for various parameter *c*. As *c* increases, $\bar{h}_{\max}$ decreases.

c	0.00	0.10	0.20
$\bar{h}_{\max}$	1.3248	1.2070	1.0414
c	0.30	0.40	0.50
$\bar{h}_{\max}$	0.8404	0.6139	0.3646

For $c = 0.10$, we now consider the effect of uncertainty bound α on the maximum time-delay for stability $\bar{h}_{\max}$. The following table illustrates the numerical results for different α. We can see that $\bar{h}_{\max}$ decreases as α increases.

α	0.00	0.10	0.20	0.30
$\bar{h}_{\max}$	2.2526	1.7680	1.2070	0.4782

Example 3. Consider the following uncertain linear neutral delay-differential system [17]

$$\frac{d}{dt}\{x(t)-[C+\Delta C(t)]x(t-h)\}$$
$$=[A+\Delta A(t)]x(t)+[B+\Delta B(t)]x(t-h) \qquad (16)$$

where

$$A=\begin{bmatrix}-2 & 1\\ -1 & -1\end{bmatrix},\ B=\begin{bmatrix}-1 & -1\\ 1 & -2\end{bmatrix},\ C=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},$$

$$\Delta A=\begin{bmatrix}a_1(t) & 0\\ 0 & a_2(t)\end{bmatrix},\ \Delta B=\begin{bmatrix}b_1(t) & 0\\ 0 & b_2(t)\end{bmatrix},\ \Delta C=\begin{bmatrix}c_1 & 0\\ 0 & c_2\end{bmatrix}$$

where $|a_i(t)| \le 0.2$, $|b_i(t)| \le 0.2$, $|c_i| \le 0.2$ for $i = 1, 2$, $\forall t$. The above system is of the form of (10) to (13) with $L = 0.2I$ and $E_a = E_b = E_c = I$.

By the criterion in [17], system (16) is robustly stable for any h satisfying $0 \le h \le 0.2257$. Decomposing the matrix B as

$$B_1=\begin{bmatrix}-0.75 & -0.25\\ 0.20 & -0.55\end{bmatrix},\ B_2=\begin{bmatrix}-0.25 & -0.75\\ 0.80 & -1.45\end{bmatrix}$$

and using the criterion in this paper, the maximum value of $\bar{h}_{\max}$ for the system to have guaranteed robust stability is $\bar{h}_{\max} = 0.6447$.

It is clear that for this example, the stability criterion in this paper gives a less conservative result than that in [17].

Example 4. Consider the following uncertain linear neutral delay-differential system [16]

$$\dot{x}(t)-C\dot{x}(t-h)=Ax(t)+Bx(t-h) \qquad (17)$$

where

$$A=\begin{bmatrix}-0.9 & 0.2\\ 0.1 & -0.9\end{bmatrix},\ B=\begin{bmatrix}-1.1 & -0.2\\ -0.1 & -1.1\end{bmatrix},\ C=\begin{bmatrix}-0.2 & 0\\ 0.2 & -0.1\end{bmatrix}$$

Decomposing the matrix B as

$$B_1=\begin{bmatrix}-0.41 & 0.04\\ -0.07 & -0.31\end{bmatrix},\ B_2=\begin{bmatrix}-0.69 & -0.24\\ -0.03 & -0.79\end{bmatrix}$$

and using the criterion in this paper, the maximum value of $\bar{h}_{\max}$ for the system to be asymptotically stable is $\bar{h}_{\max} = 1.6991$. By the criteria in [16] and [4], system (17) is asymptotically stable for any h satisfying $0 \le h \le 0.3$, and $0 \le h \le 0.74$, respectively. It is clear that for this example, the stability criterion in this paper gives a much less conservative result than these in [16, 4].

5. Conclusion

The stability problem for uncertain neutral delay-differential systems has been addressed. A stability criterion has been derived. Numerical examples have shown significant improvements over some existing results.

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$$\Xi(t,\bar{h}) = \begin{bmatrix} \Xi_{11}(t) & \Xi_{12}(t) & -\bar{h}[A(t)+B_1]^T PB_1 \\ \Xi_{12}^T(t) & \Xi_{22}(t) & -\bar{h}B_2^T(t)PB_1 \\ -\bar{h}B_1^T(t)P[A(t)+B_1] & -\bar{h}B_1^T PB_2(t) & \bar{h}R \end{bmatrix} > 0 \tag{6}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & -\bar{h}(A+B_1)^T XB_1 & -XL & (A+B_1)^T XL \\ \Psi_{12}^T & \Psi_{22} & -\bar{h}B_2^T XB_1 & C^T XL & B_2^T XL \\ -\bar{h}B_1^T X(A+B_1) & -\bar{h}B_1^T XB_2 & \bar{h}Y & -B_1^T XL & 0 \\ -L^T XP & L^T XC & -L^T XB_1 & \mu I & L^T XL \\ L^T X(A+B_1) & L^T XB_2 & 0 & L^T XL & I \end{bmatrix} > 0 \tag{13}$$