IMPLEMENTATION OF A FUZZY LYAPUNOV-BASED CONTROL STRATEGY FOR A MACRO-MICRO MANIPULATOR

A. Mannani, H.A. Talebi Y.N. Asbagh

M7823272@cic.aut.ac.ir, Alit@cic.aut.ac.ir, Younaas@engineer.com
Department of Electrical Engineering, Amirkabir University of Technology, Tehran, 15914, Iran
Fax: (+98-21) 6406469

Keywords: Lyapunov function, nonminimum-phase, tracking, fuzzy control, macro-micro manipulators.

Abstract

The experimental results for the implementation of a fuzzy tracking controller on a macro-micro manipulator (M3) are shown and compared to those of a joint-PD controller. The test bed is a nonlinear nonminimum-phase MIMO system with a complex dynamic model and pronounced joint frictions. The controller belongs to a class of fuzzy controllers based on Lyapunov-function reasoning. The control strategy assumes no *a priori* knowledge about the system dynamics, however it uses the structural properties of the system model. Although the control structure can be of the form of many standard fuzzy control techniques, it may also be interpreted as a nonlinear MIMO PID. It is shown that the performance of the proposed controller is better than that of the joint-PD and can be further improved using fuzzy control techniques.

1 Introduction

Fuzzy control, with its various methodologies has been the focus of attention for the past several years. Most of the works however, suffer from the lack of systematic design and suitable framework for stability and performance analysis.

Takagi-Sugeno-model-based fuzzy controllers, which formulate stability and performance indices in terms of LMIs, are one of the solutions [15]. Practical system modelings are prone to error and uncertainties. So in that case, model-free control methods are the best solutions. This however, will complicate the stability analysis of the closed-loop system.

In [20] and [2], fuzzy relations and phase portrait of error were used respectively to derive the fuzzy rules for controlling a flexible link. In [18], by dividing the trajectory into piecewise sliding surfaces, two fuzzy controllers were designed for hitting motion and sliding in each region. In [11], a fuzzy hyperbolic state space model together with its stability and optimal performance are discussed. The authors also introduce a fuzzy Lyapunov synthesis for a SISO system. The idea is to choose a Lyapunov function candidate and derive the fuzzy rules to make its derivative negative where the only *a priori* knowledge is the output relative degree.

For a macro-micro manipulator (M3) the macro links show considerable vibrations during the motion which results in end-point position errors of the micro part. A PDE with time-varying boundary conditions governs these vibrations. Due to non-collocated nature of sensors and actuators of macro manipulator, the system shows nonminimum-phase behavior. The structural damping, frictions and the couplings between the macro and micro parts also contribute to the modeling error and make the control of M3, a difficult problem.

In [8], the command filtering prevents the excitation of the macro flexible modes. The effect of the geometric characteristics of the micro part on the system zeros were considered in [4]. In [16], the P-PED method transfers the vibration energy of the macro part to the micro manipulator and a frequency matching algorithm was used in [17]. In [7], the inertial forces of the high-bandwidth micro manipulator damp the vibration of the macro part. In [14], a two-stage controller determines the micro trajectory to damp the macro vibrations and subsequently brings the micro to rest. In [19], a VSC and a predictive controller do the tracking control for the micro and macro parts respectively. In [3], a stable feedforward neural network identifies the M3 dynamics online and two controllers for macro and micro parts are used in an inverse dynamics scheme for output tracking and vibration suppression. Just few works (such as [12]) consider the nonminimum-phase characteristics of the system.

To extend the idea of fuzzy Lyapunov synthesis to a MIMO problem, this paper presents a methodology that uses only the output relative degree and the structural properties of the system model where the latter are direct results of the physical laws. No other *a priori* system knowledge is assumed. The control decision rule provides a framework for stability and performance analysis and essentially differs from [11]. Although, the control structure can be of the form of many standard fuzzy control techniques, it may also be interpreted as a nonlinear MIMO PID.

In this paper, first the problem formulation is brought in section 2 and then in sections 3 and 4, the proposed controller and experimental results are presented respectively.

2 Manipulator Model

The M3 dynamics with locked macro joints are given by [1,5]:

$$M(\theta, q)\begin{pmatrix}\ddot{\theta}\\ \ddot{q}\end{pmatrix} + \begin{pmatrix} h_1(X) + f_1(\dot{\theta}, \theta) + g_1(X) \\ h_2(X) + f_2\dot{q} + g_2(X) + Kq \end{pmatrix} = \begin{pmatrix} u \\ 0 \end{pmatrix} \quad (1)$$

where $X^T = [\theta^T, q^T, \dot{\theta}^T, \dot{q}^T]$. $\theta \in \Re^n$ and $q \in \Re^m$ are the vectors of micro joint variables and flexible modes respectively. h_1, h_2 are the coriolis/centripetal and g_1, g_2 are gravity forces. f_1 is the joint friction, f_2 is the structural damping term, and M and K are the positive-definite mass and stiffness matrices, respectively. Define $N = M^{-1} = \begin{pmatrix} N_1 & N_2 \\ N_2^T & N_3 \end{pmatrix}$. Equation (1) can be written as:

$$\begin{pmatrix} \ddot{\theta} \\ \ddot{q} \end{pmatrix} = \begin{pmatrix} N_1 & N_2 \\ N_2^T & N_3 \end{pmatrix} \left[-\begin{pmatrix} h_1 + f_1 + g_1 \\ h_2 + f_2\dot{q} + g_2 + Kq \end{pmatrix} + \begin{pmatrix} u \\ 0 \end{pmatrix} \right] \quad (2)$$

The end-effector of the micro can be expressed as:

$$y = \theta + \phi q \quad (3)$$

where ϕ is a constant matrix. y has to track a reference trajectory while the closed-loop and internal dynamics are stable. Successive differentiation of the output yields:

$$\ddot{y} = A + Bu \quad \text{(4-a)}$$

$$B = (N_1 + \phi N_2^T),\ A = -B(h_1 + f_1 + g_1)$$
$$-(N_2 + \phi N_3)(h_2 + Kq + f_2\dot{q} + g_2) \quad \text{(4-b)}$$

It is well known that since M is positive definite, N and hence N_1 are positive definite [12]. Also, by continuity, B is positive definite in a neighborhood of $\phi = 0$.

3 The Proposed Control Strategy

Most practical systems have complex nonlinear and uncertain models. Their dynamics however, are based on physical laws, resulting in some structural mathematical properties. Although the exact model is difficult to derive, the properties are known and held in the presence of uncertainties.

The proposed control scheme is a fuzzy model-free controller based on first selecting a Lyapunov function candidate and then making its derivative negative according to the structural properties of the system dynamics. The idea of fuzzy Lyapunov synthesis has also been suggested in [11], where no direct insight to the system structural dynamic properties was given and hence the stability analysis was less tractable there.

3.1 The Stability Analysis

The idea is explained through the tracking control of M3. To begin, let us choose a Lyapunov function candidate as

$$V = \frac{1}{2} R^T R \quad (5)$$

where $R = (\alpha e + \beta \dot{e} + \gamma \int_0^t e d\tau)$, $e = y - y_r$ and y_r is the reference trajectory and α, β and γ are the time-varying weight matrices. The time derivative of V can be given by

$$\dot{V} = R^T((\gamma + \dot{\alpha})e + (\alpha + \dot{\beta})\dot{e} + \dot{\gamma}\int_0^t e d\tau - \beta \ddot{y}_r) + R^T \beta (Bu + A) \quad (6)$$

We first consider the control term in Equation (6)

$$C_1 = R^T \beta B u \quad (7)$$

If we can make βB a positive-definite matrix, then by the following choice of the control input, the negative-definiteness of C_1 is ensured:

$$u = -\zeta R \quad , \quad \zeta > 0 \quad (8)$$

where ζ is a positive-definite matrix. It can be shown that B is indeed positive-definite in the neighborhood of $\phi = 0$. Hence by choosing any $\beta > 0$, the positive-definiteness of βB term is guaranteed. The idea is to make C_1 large enough such that it dominates the effects of C_2 and C_3 where

$$C_2 = R^T((\gamma + \dot{\alpha})e + (\alpha + \dot{\beta})\dot{e} + \dot{\gamma}\int_0^t e d\tau - \beta \ddot{y}_r)$$

$$C_3 = R^T \beta A. \quad (9)$$

Moreover, it can be observed that the C_2 term does not explicitly depend on the system dynamics and it is only a function of design parameters α, β, γ, measurable signals $e, \dot{e}$ and the reference trajectory. Consequently, a fuzzy rule-base like that in [11] can be developed to enhance the stability. As far as C_3 term is concerned, it can be divided to

$$C_3 = C_{3m} + C_{3um}$$

where C_{3m} and C_{3um} are the matched and unmatched uncertainties respectively given by

$$C_{3m} = -R^T \beta B(h_1 + f_1 + g_1) \quad (10)$$

$$C_{3um} = -R^T \beta (N_2 + \phi N_3)(h_2 + Kq + f_2\dot{q} + g_2) \quad (11)$$

The C_{3m} term can be overcome by a large ζ in Equation (8). The C_{3um} depends on the deflection variables q and it can be shown that it is bounded by [9,10]:

$$\mu = \left\| R^T \beta \right\| \times \kappa_1(\varepsilon_1, \varepsilon_2) \times \kappa_2 \quad (12)$$

with $\varepsilon_1, \varepsilon_2$ as lower bounds for $\det(M_3)$ and $\det(M)$ and

$$\left\| N_2 + \phi N_3 \right\| \le \kappa_1(\varepsilon_1, \varepsilon_2), \quad \left\| \frac{\partial M_3}{\partial \theta} + Kq + f_2\dot{q} + g_2 \right\| \le \kappa_2$$

Thus to make ensure $\dot{V}$ negative, C_1 should be greater than μ. Since both B and C_3 depend on the system internal dynamics, it should be stable to make the scheme effective.

3.2 Internal Dynamics and Performance Measures

The major issue in the analytical design of the model-free controller is that since no information about the system model is available, the stability of the zero dynamics of the system cannot be determined mathematically. However, the relationship between the flexible modes of the macro manipulators and the internal dynamics can be used as a measure of internal dynamics stability.

The Lyapunov function candidate can be chosen based on tracking objectives. For a system whose internal dynamics are not stable and/or other performance indices should also be considered three approaches may be suggested:

1. Define a Lyapunov function candidate to include indices for internal dynamics stability and other performance

measures, too. This is often very complicated.

2. Tune the parameters of the Lyapunov function candidate based on performance measures and/or internal stability.

3. Add other control terms to the main controller in C_1 such that each term satisfies one (or more) of the other performance measures and/or internal stability.

Note that in selecting the last two approaches the main control term C_1 should be kept effective and hence $\dot{V} < 0$ still holds.

For the control of M3 two other objectives were also considered. The first one is to smooth and bound the control signal for actuators. This may be done using the following TS rules with some smooth membership functions:

If u_i is small, Then control = u_i (13-a)

If u_i is large, Then control = MAX $i = 1,...,n$ (13-b)

where u_i is the i'th component of the control vector and MAX is less than the actuators' maximum output torque.

The second goal is to make the internal dynamics of the system bounded. To this end[1], three approaches can be taken:

I. Based on the third approach, the control signal can be further limited:

If abs (deflection) is high, Then (LM)$_i$ is 0

If abs (deflection) is small, Then (LM)$_i$ is 1

(control signal)$_i$ = (LM)$_i$ * u_i $i = 1,...,n$ (14)

Adding this limiting control term is equivalent to selecting ζ in Equation (8) as a diagonal matrix rather than a scalar. This would not affect the stability as long as $\zeta > 0$. The cost we pay for less vibration is however a slower response.

II. Rearranging the terms in Equation (4), we have

$$\phi(\ddot{q} + Kq + f_2\dot{q}) = Bu - (N_1 + \phi N_2^T)(h_1 + f_1 + g_1)$$
$$-(N_2 + \phi N_3)h_2 - N_2(Kq + f_2\dot{q}) - \ddot{\theta} \quad (15)$$

Qualitatively speaking, adding a damping term to Equation (8) will help the damping of the internal modes. This is similar to the term used in [12,13] based on a sensitivity analysis. The new term is added as

$$u = -\zeta[R + (Z_1 * \dot{q} + Z_2 * q) * Z_3 * abs(R)] \quad (16)$$

where Z_1 and Z_2 are determined based on the modal damping and the bounds on positive-definite matrices f_2 and K. $Z_3 * abs(R)$ make the control term C_1 still effective. $Z_3 \in [0,1]$ may be determined using some fuzzy rules based on the macro deflection. Due to damping time constant of the modes, this term is effective for slow trajectories.

III. Based on the second approach given above, the parameters of each PID controller can be tuned according to some performance measures. To explain the idea, consider a sample step response in Figure 1. Four regions are classified in Table 1. To reduce the overshoots, derivative coefficient should be decreased. At the rising portion of the curve, the slope of the reference trajectory is high and the proportional and derivative terms can be selected accordingly. The tuning rules are summarized in Tables 1 and 2. A small integral term can also be added to ensure zero static errors. This tuning is equivalent to choosing the time varying parameters of the Lyapunov function such that $\beta B > 0$ and $C_1 < 0$.

[1]A similar idea in [6] is used to suppress the vibrations and track the reference trajectory with one control input with sliding mode controllers.

4 Experimental Results

The actual planar M3 test bed consists of one flexible macro and two rigid similar micro links (Figure 2) whose parameters are listed in Table 3. The system model can be derived based on the Euler-Lagrange method [1,5]. The macro link is made of two parallel ($5cm$ apart) stainless-steel beams, which are fixed at both ends, hence both the gravitational and torsional effects are negligible. The micro links are made of aluminum with circular cross sections. The actuators are three Maxon DC servomotors, each equipped with 500 count./rev. incremental encoders. All joints undergo considerable friction. To maintain the locked-joint condition, two brakes plus a joint-PD controller kept the initial position of the macro links unchanged. The first vibration mode of a pseudo-clamped beam was used [1] and fedback through the strain gauges -installed near the base of flexible beams- after signal conditioning. The software was written in Watcom C and implemented in MATLAB xPC Target environment. The Data acquisition was done using Advantec DAQ cards with a $1ms$ sampling period and the ODE 4 (Runge-Kutta) solver.

4.1 The Definitions of the Output

In [9], it is shown that in macro locked-joint condition, the B matrix in Equation (4) is independent of the deflection modes. Through numerical simulation the eigenvalues of the B matrix were plotted against the variations of θ where they always remained positive [10].

Two definitions for the two-dimensional output vector were used. In the first one, let the output vector be defined as

$$y = \theta + \begin{pmatrix} \phi_1 \\ 0 \end{pmatrix} q \quad (17)$$

where $\phi_1 q$ is the macro tip pseudo-angle. It can be seen that the second element is minimum-phase while the first one may not. The second definition is adopted from the method suggested in [12] and is based on the cartesian task space coordinates. To derive this definition, note that the endpoint of the manipulator and its desired trajectory in cartesian space may be given by $Y = h(q,\theta)$ and $Y_d = h(q_d, \theta_d)$ respectively. The idea is to define the output y such that

$$h(q,\theta) = h(q_d, \theta_d) \quad (18)$$

holds against the variations in q and θ. Up to the first degree of approximation, this would yield [12]:

$$y = \theta + WJ_{\theta}^{-1}(q_d, \theta_d)J_q(q_d, \theta_d)q \quad (19)$$

where J_θ and J_q are related jacobians and J_θ is required to be nonsingular in the task space. W is a weighting matrix and can be used to define minimum-phase outputs. Assuming a cartesian coordinates with its x axis along with the non-deflected beam and the beam deflections measured along its y axis, it yields the following definition for the output:

$$y = \theta + \frac{p_1(x)}{L\sin(\theta_{3d} - \theta_{2d})}\begin{pmatrix} \sin(\theta_{3d}) \\ -\sin(\theta_{2d}) \end{pmatrix} q \quad (20)$$

where $p_1(x)$ is the first mode shape, L is the length of the micro links and θ_{2d} , θ_{3d} are the desired joint angles.

4.2 Reference Trajectories

Three reference trajectories were considered for tracking. The first one is a 0.5 *Hz* sinusoidal trajectory:

$$y_d = (\pi/6)\sin(\pi t)\begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (21)$$

The second one is a quintic trajectory:

$$y_d = (\pi/2)s(t)\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$s(t) = \begin{cases} 320t^5 - 416t^4 + 164t^3 - 16t^2 + t & t < .5\,\text{sec}. \\ 1 & t \geq .5\,\text{sec}. \end{cases} \quad (22)$$

and the last one is a ramp trajectory with different slopes for output vector elements to avoid singularities

$$y_d = \begin{pmatrix} 2.5t \\ 3t \end{pmatrix} \text{ for } t < .5 \text{ and } y_d = \begin{pmatrix} 1.25 \\ 1.5 \end{pmatrix} \text{ for } t \geq .5. \quad (23)$$

4.3 Comparing the Responses[2] and Conclusion

Figure 3 depicts the results of applying the joint-PD controller to the first output in Equation (17) for tracking of the quintic trajectory. The PD gains were selected such that smaller and larger gains would result in worse responses. As seen the controller can not damp the vibrations. Figure 4 repeats the test when the PD controller uses the feedback of the elastic mode, too which results in vibration damping as well.

In figures 4 to 7, the output in Equation (17), tracks the quintic and the sinusoidal trajectories, first using only the basic controller in Equation (8) (with feedback from both rigid and elastic modes) and the next time by adding fuzzy control terms in section 3.2 to improve the responses. Although the parameters of the controllers have not been optimally tuned, some improvements like more vibration damping and/or better time responses are achieved.

Figures 8 to 11 show the responses of the second output in Equation (20) for tracking quintic and ramp trajectories. In practice, to avoid the singularities and instability, the coefficients of q in Equation (20) were limited to a small range against the variations in θ_{2d} and θ_{3d} . In this time, the fuzzy terms are more effective in damping the vibrations.

As a conclusion, the stability reasoning in section 3.1 provides a suitable framework to apply fuzzy control techniques to this plant. It is also inherently robust to the plant model uncertainties (like unmodeled dynamics) and behaves well against the complex model of the system. Thus it can be regarded as a good starting point for analysis and design of the model-free controllers.

References

[1] F. Bellezza, L. Lanari, G. Ulivi, "Exact Modeling of the Flexible Slewing Link", IEEE ICRA, pp. 734-739, (1990).

[2] C. Chen, Y. Yin, "Fuzzy Logic Control of a Moving Flexible Manipulator", IEEE CCA, pp. 315-320, (1999).

[3] P. Cheng, R.V. Patel, "Vibration Control of a Flexible Macro-Micro Manipulator System Using Neural Networks", IFAC, (2002).

[4] W.W. Chiang, R. Kraft, R.H. Jr. Cannon, "Design and Experimental Demonstration of Rapid, Precise End-Point Control of a Wrist Carried by a Very Flexible Manipulator", *Int. J. Rob. Res.,* **Volume** 10, No. 1, pp. 30-40, (1991).

[5] A.R. Frazer, R.W. Daniel, "Perturbation Techniques for Flexible Manipulators", Kluwer Acad. Publish., (1991).

[6] N.M. Kwok, C.K. Lee, "Control of a Flexible Manipulator Using a Sliding Mode Controller with a Fuzzy-Like Weighting Factor", IEEE ISIE, pp. 52-57, (2001).

[7] J.Y. Lew, D.J. Trudnowski, "Vibration Control of a Micro-Macro Manipulator System", *IEEE Cont. Sys. Mag.,* pp. 26-31, (1996).

[8] D.P. Magee, W.J. Book, "Filtering Micro-Manipulator Wrist Commands to Prevent Flexible Base Motion", American Control Conference, (1995).

[9] A. Mannani, "Fuzzy control of a macro-micro manipulator and its implementation", M.Sc. thesis, Amirkabir University of Technology, (2002).

[10] A. Mannani, H.A. Talebi, "A fuzzy Lyapunov-based control strategy for a macro-micro manipulator", IEEE CCA, pp. 368-373, (2003).

[11] M. Margaliot, G. Langholz, "New Approaches in Fuzzy Modeling and Control", World Scientific Pub. Co., (2000).

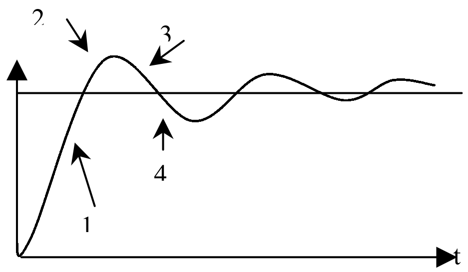


Figure 1: Four regions of a sample step response.

$\dot{e} \backslash e$	+	-
+	1/0	BB/CC
-	1/1	1/0

Table 1: Rules for P/D Coeff.

$\dot{y}_r$	High	Small
BB	Low	High
CC	Big	Medium

Table 2: Rules for BB and CC.

[2] Figures show actuators' torques (Nm), macro deflection (m), outputs (rad) versus time (s) respectively.

Macro link Length (m) , width (mm) , thickness (mm)	0.59, 70, 1.25
Macro link hub radius (mm)	30
Macro link tip equivalent mass (Kg) , inertia (Kgm^2)	2, $\leq$ .07
Macro link natural frequency (Hz)	1.5
Macro link hub inertia (Kgm^2)	0.63
Micro link length (m) , radius (mm) , thickness (mm)	0.17, 25, 2
Micro link joint mass (Kg) , inertia (Kgm^2)	0.326, 0.003

Table 3: Parameters of the test bed.

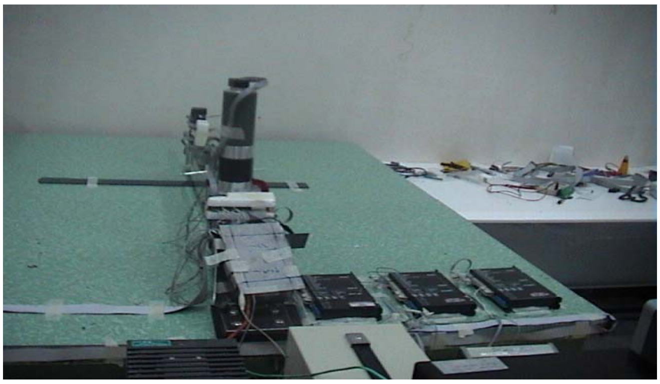


Figure 2: Actual Macro-Micro Manipulator test bed.

[12] M. Moallem, R.V. Patel, "A Vibration Control Strategy for a Boom-Mounted Manipulator System for High-Speed Positioning", IEEE IROS, pp. 299-304, (1999).
[13] M. Moallem, K. Khorasani, R.V. Patel, "An Inverse Dynamics Sliding Control Technique for Flexible Multi-Link Manipulators", American Cont. Conf., pp. 1407-1411, (1997).
[14] I. Sharf, "Active Damping of a Large Flexible Manipulator with a Short-Reach Robot", *ASME J. Dyn. Sys. Meas. Cont.*, pp. 705-713, (1996).
[15] K. Tanaka, H.O. Wang, "Fuzzy Control Systems Design and Analysis", John Wiley and Sons Inc., (2001).
[16] M.A. Torres, S. Dubowsky, A.C. Posini, "Vibration Control of Deployment Structures' Long-Reach Space Manipulators: The P-PED Method", IEEE ICRA., pp. 2498-2504, (1996).
[17] J.V. Vliet, I. Sharf, "A Frequency Matching Algorithm for Active Damping of Macro-Micro Manipulator Vibrations", IEEE IROS, pp. 782-787, (1998).
[8] W.J. Wang, H.R. Lin, "Fuzzy Control Design for the Trajectory Tracking on Uncertain Nonlinear Systems", *IEEE Trans. on Fuzzy Sys.* , **Volume** 7, No. 1, pp. 53-62, (1999).
[19] W. Yim, S. N. Singh, "Nonlinear Inverse and Predictive End Point Trajectory Control of Flexible Macro-Micro Manipulators", *ASME J. Dyn. Sys. Meas. Cont.*, **Volume** 119, pp. 412-420, (1997).
[20] B. Yoo, S. Jeong, W. Ham, " Hybrid Control of Flexible Manipulator Based on Fuzzy Relations", IEEE ICRA, pp. 817-823, (1996).

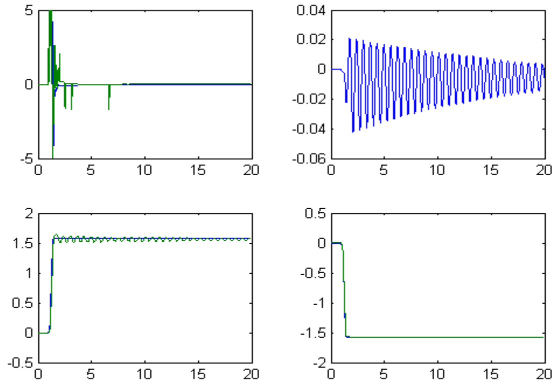
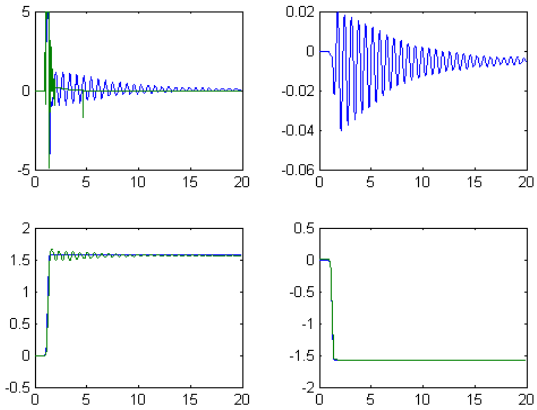
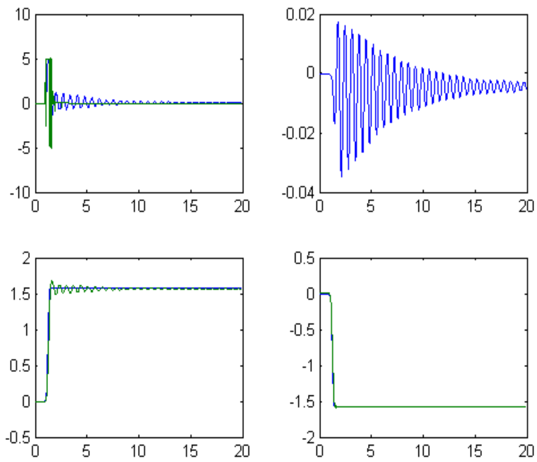
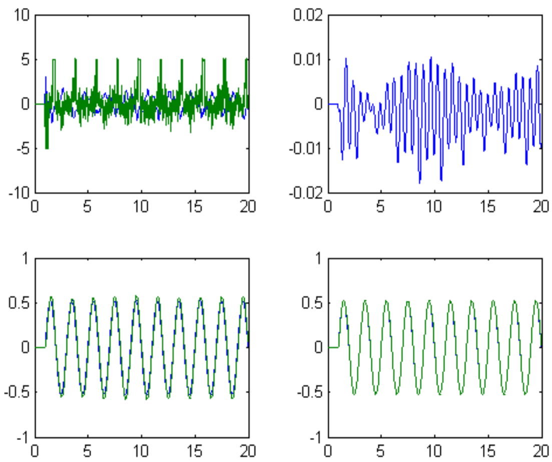


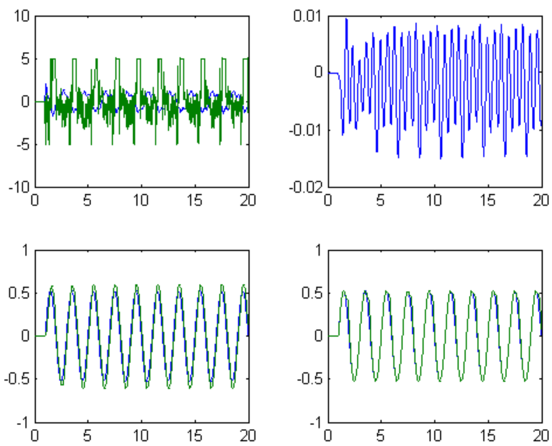
Figure 3: The response of the first output to the quintic trajectory with joint-PD controller.



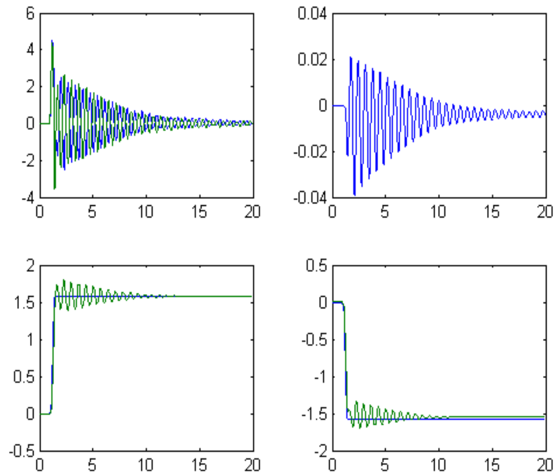


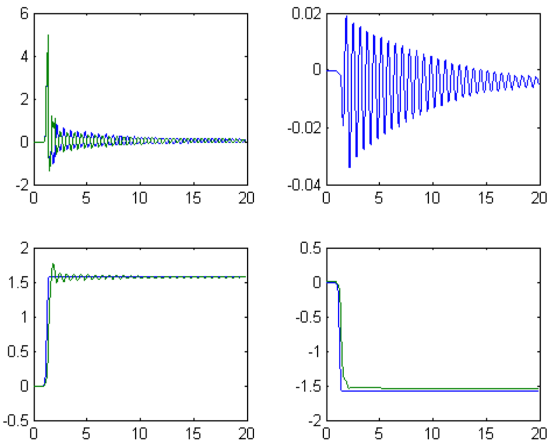
Figures 4,5:The responses of the first output to the quintic trajectory with the controller in Equation (8) without/with using approach III.



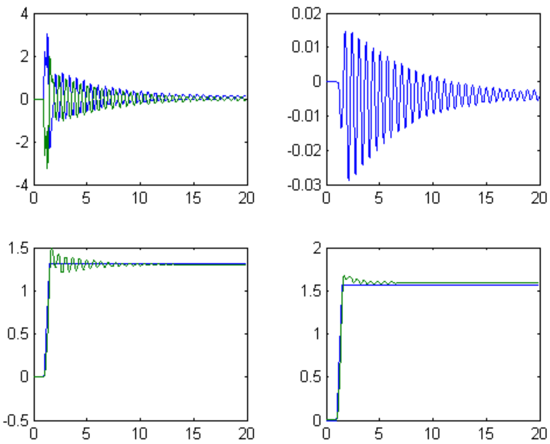


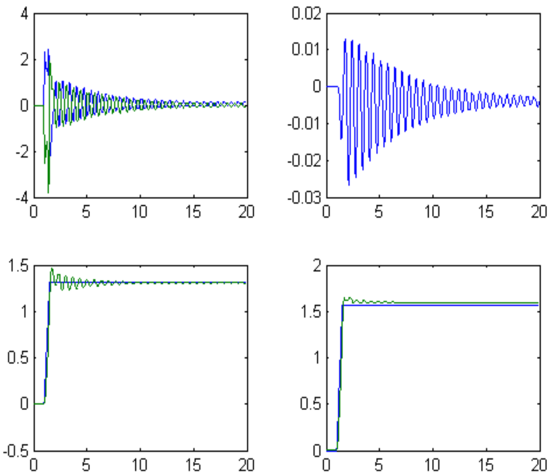
Figures 6,7: The responses of the first output to the sinusoidal trajectory with the controller in Equation (8) without/with using approach I.





Figures 8,9: The responses of the second output to the quintic trajectory with the controller in Equation (8) without/with using approaches I, II and III (all together).





Figures 10,11: The responses of the second output to the ramp trajectory without/with using approaches I and II (together).