CHATTERING REDUCTION VIA FUZZY LOGIC: APLLICATION TO STEPPER MOTOR

Bernardo Rincón Márquez, Alexander G. Loukianov and Edgar N. Sanchez

CINVESTAV IPN Unidad Guadalajara,
Apartado Postal 31-438, Guadalajara, Jal. 44550, México.
e mails:brincon [louk, sanchez]@gdl.cinvestav.mx

Keywords: sliding mode, nonlinear system, disturbance rejection, fuzzy control.

Abstract

In this paper, one method of robust control based on sliding mode and fuzzy logic techniques is presented. It combines hierarchical control with high gain approach for multivariable and nonlinear systems; in order to eliminate chattering in presence of disturbances. Simulation results are presented to illustrate the applicability of the approach

1 Introduction

A simple and good technique of robust control is the sliding mode one (Utkin, 1992; Utkin, et al., 1999), since it is composed of two clear steps: selection of the sliding surface, such that the sliding mode equation on this surface is robust in presence of disturbances, and design of a discontinuous control which stabilizes the projection motion of the closed loop system on the sliding surface subspace.

It is known that the sliding mode motion is invariant with respect to disturbance, which satisfies the matching condition (Drajenovic, 1969). There are two cases for controlling systems with unmatching condition: measured disturbances, and, unmeasured ones. For the second case, the problem can be solved using high gain control, but it can produce the "chattering" (Utkin, et al., 1999) or oscillations on the sliding surface due to the imperfections in the control devices.

In this paper we propose a new control scheme using combination of the sliding mode control, block control (Luk'yanov, 1998) and fuzzy logic control (Driankov, et al., 1996) techniques to eliminate the chattering in the closed-loop system with both the matched and unmatched unknown perturbations. Note the sliding mode fuzzy logic controller was investigated for the system with matched perturbations in (Palm, et al., 1997; Alexík and Vittek, 1994; Scibile and Kouvaritakis, 2001; Wong, et al., 2001; Ha, et al., 2001; Kaynak, et al., 2001)

2 Control Method

Consider a single input single output system (SISO) nonlinear system subject to a disturbances

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{b}(\mathbf{x})u + \mathbf{g}(\mathbf{x})w \tag{1}$$

$$y = h(\mathbf{x}) \tag{2}$$

where $\mathbf{x} = (x_1,..., x_n)^T$ is the state vector, u is the control input, y is the output, w represents an external disturbance which is unknown but bounded, $\mathbf{f}(\mathbf{x})$ and $\mathbf{b}(\mathbf{x})$ are sufficiently smooth and bounded functions, $\mathbf{g}(\mathbf{x})$ is an unknown but bounded function, and $\mathbf{f}(0) = 0$.

We assume that there exists a nonlinear transformation that reduces the system (1) to the so-called Block Controllable Form with disturbances (Luk'yanov, 1998):

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + b_1(x_1)x_2 + g_1(x_1)w \\ \dot{x}_i &= f_i(\bar{\mathbf{x}}_i) + b_i(\bar{\mathbf{x}}_i)x_{i-1} + g_i(\bar{\mathbf{x}}_i)w, \quad i = 2,..., n-1 \qquad (3) \\ \dot{x}_n &= f_n(\mathbf{x}) + b_n(\mathbf{x})u + g_n(\mathbf{x})w \\ y &= x_1 \qquad (4) \end{aligned}$$

with $\bar{\mathbf{x}}_i = (x_1,..., x_i)^T$, $b_i \neq 0$, $f_i(\bar{\mathbf{x}}_i)$ and $b_i(\bar{\mathbf{x}}_i)$ are sufficiently smooth and bounded functions.

If the disturbance w satisfies the matching condition (Drajenovic, 1969), that is, there is a scalar function $\lambda(\mathbf{x})$ such that

$$\mathbf{g}(\mathbf{x}) = \mathbf{b}(\mathbf{x})\lambda(\mathbf{x}) \tag{5}$$

then it is easy to show that $g_1(x_1) = 0$ and $g_i(\bar{\mathbf{x}}_i) = 0$, $i = 2,..., n-1$ in (3), and therefore, sliding mode motion is invariant with respect to external disturbance. The aim of this paper is to design a discontinuous control that provides robustness, with not chattering, to the closed-loop system for the unmatched disturbances case, i.e. w does not satisfy the matching condition (5). In this case there is $g_i(\bar{\mathbf{x}}_i) \neq 0$, $i = 2,..., n-1$. The control procedure consists of the following:

Suppose that the output y requires to follow the reference signal r. Using the block control technique (Luk'yanov, 1998) we introduce the following recursive transformation:

Work supported by Conacyt, Mexico, under grant 36960A

$$z_1 = x_1 - r \equiv \Phi_1(x_1, r) \quad (6a)$$
$$z_2 = f_1(x_1) + b_1(x_1)x_2 - \dot{r} + k_1(x_1 - r) \equiv \Phi_2(\bar{\mathbf{x}}_2, \mathbf{r}_2) \quad (6b)$$
$$z_3 = \bar{b}_2(\bar{\mathbf{x}}_2, t)x_3 + f_2(\bar{\mathbf{x}}_2) + k_2\Phi_2(\bar{\mathbf{x}}_2, \mathbf{r}_2) \equiv \Phi_3(\bar{\mathbf{x}}_3, \mathbf{r}_3) \quad (6c)$$
$$z_{i+1} = \bar{b}_i(\bar{\mathbf{x}}_i)x_{i+1} + f_i(\bar{\mathbf{x}}_i) + k_i\Phi_i(\bar{\mathbf{x}}_i, \mathbf{r}_i) \equiv \Phi_{i+1}(\bar{\mathbf{x}}_{i+1}, \mathbf{r}_{i+1})$$
$$i = 3, ..., n \quad (6d)$$

where $\mathbf{z} = (z_1, ..., z_n)^T$ is a new variables vector, $k_i > 0$, $\mathbf{r}_2 = (r, r^{(1)})^T$, $\mathbf{r}_i = (r, r^{(1)}, ..., r^{(i-1)^T})$, $\bar{b}_i = \bar{b}_{i-1}b_i$. The transformation (6a)-(6d) reduces the system (3) to the following desired form:

$$\dot{z}_1 = -k_1 z_1 + z_2 + \bar{g}_1(z_1)w \quad (7a)$$
$$\dot{z}_i = -k_i z_i + z_{i+1} + \bar{g}_i(\bar{\mathbf{z}}_i)w, \; i = 2, ..., r-1 \quad (7b)$$
$$\dot{z}_n = \bar{f}_n(\mathbf{z}) + \bar{b}_n(\mathbf{z})u + \bar{g}_n(\mathbf{z})w \quad (7c)$$

where $\bar{\mathbf{z}}_i = (z_1, ..., z_i)^T$, $\bar{f}_n(\mathbf{z})$ is a continuous and bounded function, and $\bar{b}_n = \bar{b}_{n-1}b_n$.

In order to generate sliding mode in (7a)-(7c) a natural choice of the switching function is $s = z_n$ (6d). Then the desired dynamics of the closed-loop system for the case of unknown w, can be selected as

$$\dot{s} = -k_n sign(s) + \bar{g}_n(\mathbf{z})w, \quad k_n > 0 \quad (8)$$

From (7c) and (8) a discontinuous control strategy can be obtained of the following form:

$$u = -k_n \bar{b}_n^{-1}(\mathbf{z}) sign(s) + u_{eq}(\mathbf{z}) \quad (9)$$

where $u_{eq}(\mathbf{z})$ is the equivalent control calculated from $\dot{s} = 0$ in the absence of the disturbance as $u_{eq} = -\bar{b}_n^{-1}(\mathbf{z})\bar{f}_n(\mathbf{z})$.

In order to derive the stability condition, we use a positive definite function $V(s) = \frac{1}{2}s^2$. Then from

$$\dot{V} \le -[k_n - \bar{g}_n(\mathbf{z})w]|s|$$

we can obtain

$$k_n \ge |\bar{g}_n(\mathbf{z})w| \quad (10)$$

Under this condition the state converges to the surface $s = 0$ and the sliding mode motion occurs on this surface in a finite time. This motion is described by the following $(n-1)^{th}$ order system:

$$\dot{z}_1 = -k_1 z_1 + z_2 + \bar{g}_1(z_1)w \quad (11a)$$
$$\dot{z}_i = -k_i z_i + z_{i+1} + \bar{g}_i(\bar{\mathbf{z}}_i)w, \; i = 2, ..., n-2 \quad (11b)$$
$$\dot{z}_{n-1} = -k_{n-1}z_{n-1} + \bar{g}_{n-1}(\bar{\mathbf{z}}_{n-1})w \quad (11c)$$

The following assumption on the bounds of the unknown terms in (7a)-(7c), is stated:

A1) There exist positive constants q_{ij} and d_i such that

$$|\bar{g}_1(z_1)w| \le q_{11}|z_1| + d_1 \quad (12a)$$
$$|\bar{g}_2(\bar{\mathbf{z}}_2)w| \le q_{22}|z_2| + k_1 q_{21}|z_1| + d_2 \quad (12b)$$
$$|\bar{g}_3(\bar{\mathbf{z}}_3)w| \le q_{33}|z_3| + k_2 q_{32}|z_2| + k_1^2 q_{21}|z_1| + d_3 \quad (12c)$$
$$|\bar{g}_i(\bar{\mathbf{z}}_i)w| \le q_{i,i}|z_i| + \sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j}|z_j| + d_i,$$
$$i = 4, ..., n-1. \quad (12d)$$

To achieve the robustness property with respect to unknown but bounded uncertainty, the controller gains $k_1, \ldots, k_{n-1}$ have to be chosen hierarchically high. Thus, since $\bar{g}_1(z_1)w$ in (12a) does not depend on k_1, the value of this coefficient can be chosen so high that the term $k_1 z_1$ in (11a) will be dominate. By block linearization procedure, the term $\bar{g}_2(\bar{\mathbf{z}}_2)w$ in (12b) depends on k_1 but not on $k_2, \ldots, k_{r-1}$. Then for fixed k_1, the appropriate choice of k_2 value provides the domination of term $k_2 z_2$ in the second block of (11b), and so on.

In order to establish the required hierarchy of the control gains which ensures stability of the sliding mode motion (11a)-(11c), we choose a Lyapunov function candidate V for the system (11a)-(11c) as a sum of Lyapunov function candidates for the each block of (11a)-(11c), namely

$$V = \sum_{i=1}^{r-1} V_i, \quad V_i = \tfrac{1}{2}z_i^2, \quad i = 1, ..., n-1$$

and let us calculate the derivatives $\dot{V}_i$, $i = 1, ..., r-1$ step by step from the first block to the last block of (11a)-(11c).

At the first step, differentiating the Lyapunov function candidate $V_1 = \frac{1}{2}z_1^2$ along the trajectories of (11a) and using assumption A1, namely (12a), we get

$$\dot{V}_1 = -k_1 z_1^2 + z_1 z_2 + z_1 \bar{g}_1(z_1)w$$
$$= -|z_1|\left[(k_1 - q_{11})|z_1| - |z_2| - d_1\right]$$

which is negative in the region $|z_1| > \frac{1}{k_1 - q_{11}}|z_2| + \frac{d_1}{k_1 - q_{11}}$.

Therefore, the state ultimately enter the domain in subspace (z_1, z_2) defined by

$$|z_1| \le \alpha_{12}|z_2| + \beta_{12} \quad (13)$$

where the parameters α_{12} and β_{12} defined as

$$\alpha_{12} = (k_1 - q_{11})^{-1} \text{ and } \beta_{12} = \alpha_{12}d_1$$

are positive if the following condition holds:

$$k_1 > q_{11} \quad (14)$$

At the second step, following similar lines to those taken for the first block, the derivative $\dot{V}_2$ of the Lyapunov function candidate $V_2 = \frac{1}{2}z_2^2$ calculated along the trajectories of the second block of (11b), under conditions (12a), (128b) and (14), is given by

$$\begin{aligned}\dot{V}_2 &= -k_2 z_2^2 + z_2[z_3 + \overline{g}_2(z_1, z_2)w] \\ &\le -|z_2| \left[(k_2 - q_{22})|z_2| - |z_3| - k_1 q_{21}|z_1| - d_2 \right] \\ &\le -|z_2| \left[(k_2 - q_{22} - k_1 q_{21}\alpha_{12})|z_2| - |z_3| - k_1 q_{21}\beta_{12} - d_2 \right]\end{aligned}$$

which is negative if

$$(k_2 - q_{22} - k_1 q_{21}\alpha_{12})|z_2| - |z_3| - k_1 q_{21}\beta_{12} - d_2 > 0 .$$

Hence, the state ultimately enter the domain in the subspace (z_1, z_2, z_3) defined by

$$|z_2| \le \alpha_{23}|z_3| + \beta_{23}$$

and consequently

$$|z_1| \le \alpha_{13}|z_3| + \beta_{13}$$

where the scalar parameters α_{23}, β_{23} α_{13} and β_{13} defined as

$$\alpha_{23} = (k_2 - q_{22} - k_1 q_{21}\alpha_{12})^{-1},\ \beta_{23} = \alpha_{23}(k_1 q_{21}\beta_{12} + d_2),$$
$$\alpha_{13} = \alpha_{12}\alpha_{23} \quad \text{and} \quad \beta_{13} = \alpha_{12}\beta_{23} + \beta_{12}$$

are positive if the values of k_1 and k_2 satisfy the following inequalities

$$k_1 > q_{11} \quad \text{and} \quad k_2 > q_{22} + k_1 q_{21}\alpha_{12} \tag{15}$$

Proceeding in the same fashion for the i^{th} block of the system (11a)-(11c), then the convergence domain in the subspace $(z_1, z_2, ..., z_{i-2}, z_{i-1}, z_i)$, is

$$\begin{aligned}|z_1| &\le \alpha_{1,i}|z_i| + \beta_{1,i} \\ |z_2| &\le \alpha_{2,i}|z_i| + \beta_{2,i} \\ &\vdots \\ |z_{i-1}| &\le \alpha_{i-1,i}|z_i| + \beta_{i-1,i}\end{aligned} \tag{16}$$

where $\alpha_{j,i} = \alpha_{j,i-1}\alpha_{i-1,i}$,

$$\alpha_{i-1,i} = \left(k_{i-1} - q_{i-1,i-1} - \sum_{j=1}^{i-2} k_j^{(i-j)} q_{i-1,j}\alpha_{j,i-1}\right)^{-1},$$
$$\beta_{j,i} = \alpha_{j,i-1}\beta_{i-1,i} + \beta_{j,i-1}, \quad j = 1, ..., i-1 .$$

At the next step, taking again the derivative of the Lyapunov function $V_i = \frac{1}{2} z_i^2$ along the trajectories of the i^{th} block of (11a)-(11c), and using (12a)-(12d), we obtain

$$\begin{aligned}\dot{V}_i &= -k_i z_i^2 + z_i[z_{i+1} + \overline{g}_i(z_1, ..., z_i)w] \\ &\le -k_i z_i^2 + |z_i|\left(|z_{i+1}| + q_{i,i}|z_i| + \sum_{j=1}^{i-1} k_j^{i-j} q_{i,j}|z_j| + d_i\right).\end{aligned}$$

Using now (16), we can majorize $\dot{V}_i$ as

$$\begin{aligned}\dot{V}_i \le -|z_i|\Big[&\Big(k_i - q_{i,i} - \sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j}\alpha_{j,i}\Big)|z_i| \\ &- |z_{i+1}| - \sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j}\beta_{j,i} - d_i\Big].\end{aligned} \tag{17}$$

From this equation it follows that

$$|z_i| \le \alpha_{i,i+1}|z_{i+1}| + \beta_{i,i+1} \tag{18}$$

where the parameters

$$\alpha_{i,i+1} = \left(k_i - q_{i,i} - \sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j}\alpha_{j,i}\right)^{-1}$$

and $\beta_{i,i+1} = \alpha_{i,i+1}\left(\sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j}\beta_{j,i} - d_i\right), \quad i = 4, ..., n-1$

are positive if the condition

$$k_i > q_{i,i} + \sum_{j=1}^{i-1} k_j^{(i-j)} q_{i,j}\alpha_{j,i} \tag{20}$$

holds. Substitution of (18) in (16) gives the following set of inequalities for the subspace $(z_1, z_2, ..., z_{i-2}, z_{i-1}, z_i, z_{i+1})$:

$$\begin{aligned}|z_1| &\le \alpha_{1,i+1}|z_i| + \beta_{1,i+1} \\ |z_2| &\le \alpha_{2,i+1}|z_i| + \beta_{2,i+1} \\ &\vdots \\ |z_{i-1}| &\le \alpha_{i-1,i+1}|z_i| + \beta_{i-1,i+1} \\ |z_i| &\le \alpha_{i,i+1}|z_{i+1}| + \beta_{i,i+1}\end{aligned} \tag{21}$$

where $\alpha_{j,i+1} = \alpha_{j,i}\alpha_{i,i+1}$ and $\beta_{j,i+1} = \alpha_{j,i}\beta_{i,i+1} + \beta_{j,i}$, $j = 1, ..., i$, $i = 4, ..., n-1$.

At the last step we have the domain of convergence in the subspace $(z_1, z_2, ..., z_{n-1})$ defined by the following inequalities:

$$|z_i| \le \alpha_{i,n-1}|z_{n-1}| + \beta_{i,n-1},\ i = 1, ..., n-2 .$$

These expressions are used to evaluate the derivative of the Lyapunov function candidate $V_{n-1} = \frac{1}{2} z_{n-1}^2$ along the trajectories of (11c), that is

$$\begin{aligned}\dot{V}_{n-1} &= -k_{n-1} z_{n-1}^2 + z_{n-1} g_{n-1}(z_1, ..., z_{n-1})w \\ &\le -k_{n-1} z_{n-1}^2 + |z_{n-1}| \left(q_{n-1,n-1} + \sum_{j=1}^{r-2} k_j^{n-1-j} q_{r-1,j}|z_j| + d_{n-1}\right) \\ &\le -\left(k_{n-1} - q_{r-1,n-1} - \sum_{j=1}^{r-2} k_j^{(n-1-j)} q_{r-1,j}\alpha_{j,n-1}\right) |z_{n-1}|^2 \\ &\quad + \left(\sum_{j=1}^{r-2} k_j^{(n-1-j)} q_{r-1,j}\beta_{j,n-1} + d_{r-1}\right) |z_{n-1}|\end{aligned}$$

If k_{n-1} is chosen such that the condition

$$k_{n-1} > q_{n-1,n-1} + \sum_{j=1}^{r-2} k_j^{(n-1-j)} q_{n-1,j}\alpha_{j,n-1} \tag{22}$$

holds, then we obtain

$$\dot{V}_{n-1} = -2\alpha_{n-1}V_{r-1} + \beta_{n-1}\sqrt{2V_{n-1}}$$

with positive

$$\alpha_{n-1} = k_{n-1} - q_{n-1,n-1} - \sum_{j=1}^{n-2} k_j^{(n-1-j)} q_{n-1,j}\alpha_{j,n-1}$$

and $\beta_{n-1} = \sum_{j=1}^{n-2} k_j^{(n-1-j)} qn_{r-1,j}\beta_{j,n-1} + d_{n-1}$.

By the Comparison Lemma (Khalil, 1996), we have

$$\left|\mathbf{z}_{n-1}(t)\right| \le \gamma_{n-1,n-1} \exp\left[\tfrac{1}{2}\,\alpha_{n-1}(t-t_0)\right] + h_{n-1} \qquad (23)$$

where $\gamma_{n-1,n-1} = \left|z_{n-1}(t_0)\right| - h_{n-1}$, and $h_{n-1} = \dfrac{\beta_{n-1}}{\alpha_{n-1}}$. Thus

$$\lim_{t\to\infty} \sup \left|z_{n-1}(t)\right| \le h_{n-1} \qquad (24)$$

Therefore, using the obtained upper (23) and ultimate (24) bounds on the solution $z_{n-1}(t)$, and the inequalities (21), and going back, from the $(n-1)^{th}$ block to the first block of (11a)-(11c), we can find step-by-step upper estimations and ultimate bounds on the solutions $z_{n-2}(t),\ z_{n-3}(t), \ldots,\ z_1(t)$.

In order to reduce the effect of the unknown disturbances action in (8), that is, ensure inequalities (15), (20) and (22) for given bounds (12a)-(12d), and, respectively, increase the region of the sliding mode stability (10), it is needed to increase the value of the controller gain k_n in (9).

This high gain, however, can produce "chattering" due to some imperfections in the control devices. To solve this problem we propose to adjust the value of the gain k_n depending on the value of s and the distance d_n defined as

$$d_n = \left(z_1^2 + \cdots + z_{n-1}^2\right)^{\frac{1}{2}}$$

It can be done by using the fuzzy logic scheme. For the case of a bounded control, the value of k_n begins with $k_n = k_{n,\max}$ and then, as s tends to zero, the value of k_n decreases smoothly up to $k_n = k_{n,\min}$, avoiding "chattering".

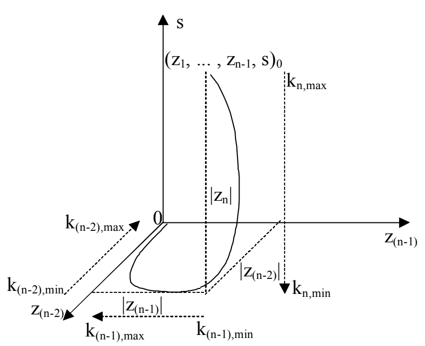

Fig 1: Evolution of *s* and *k*

A schematic diagram of evolution of s and k_n is shown in Figure 1. The block diagram of the closed-loop system with block transformation and sliding mode fuzzy logic controller (SMFLC_{BC}) is presented in the Figure 2.

The diagram consists of the following parts:

Block Control part. This block transforms state $\mathbf{x}$ in the new coordinate $\mathbf{z}$

$$T_{\text{BC}} : \mathbf{x} \to \mathbf{z}\text{, such that } \mathbf{z} = T_{\text{BC}}(\mathbf{x})$$

where the map T_{BC} is defined by (6a)-(6d), and computes the value of the following distances:

$$s = z_n \quad \text{and} \quad d = \left(z_1^2 + \cdots + z_{n-1}^2\right)^{\frac{1}{2}}.$$

Slope Change block: Using fuzzy logic, the gains $k_1, \cdots, k_{n-1}$ are modified by this block when the sliding surface $s = 0$ is reached. Opposed to k_n changes, the gains $k_1, \cdots, k_{n-1}$ (slope gains) are incremented from minimal to maximal values, resulting on an increasing of the sliding mode motion rate in (11) and of the stability region.

Fuzzy Controller block: This block uses two inputs: $In_1 = s$ and $In_2 = d$, instead of all state $\mathbf{x}$, and it determines the gain k_n depending on the magnitude of the inputs such that to satisfy the stability condition (10). The block consists of the following parts:

Normalization Input part scales (normalizes) inputs such that the sliding surface is reached in a smooth and fast way.

Fuzzification part transforms the crisp input values in fuzzified values

$$F:\ In \to LIn \ \text{ such that } F(In_i) = LIn(i, j)$$

where $In_i \in In$ is a crisp input value defined on the discourse universe *In*, and *Lin(i, j)* is a corresponding fuzzified input value also named as membership degree.

Inference Mechanism part uses the following type rule:

Rule m: If (Lin(1, j) and Lin(2, k)) Then CR(j, k)=Cout(l)

where *CR(j, k)* is the corresponding value in the rule consequent, and *COut(l)* is the *l-th* central value of the output set.

Defuzzification part based on the weighted mean

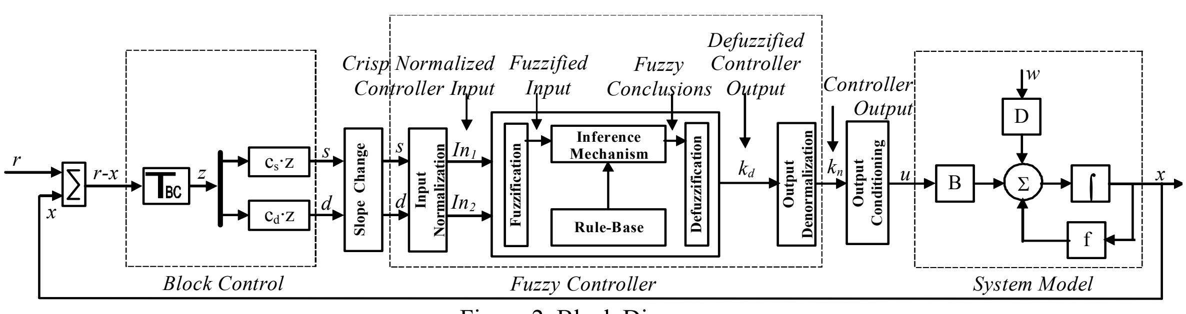

Figure 2. Block Diagram

defuzzification method (Driankov, et al., 1996) produces a scalar value k_d calculated as

$$k_d = \frac{\sum_{j=1}^{ne_1}\sum_{k=1}^{ne_2} LAnt(j,k)\ CR(j,k)}{\sum_{j=1}^{ne_1}\sum_{k=1}^{ne_2} LAnt(j,k)} \tag{25}$$

where $LAnt(j,k) = \min(LIn(1,j),\ LIn(2,k))$ is the premise quantification of the active rule, and ne_i for $i=1,2,$ is the fuzzy set size.

<u>Denormalization part</u> multiplicities normalized fuzzy controller output with denormalization factor (*scale*), $k_n = k_d \cdot scale$, such that the system (8) stays stable.

<u>Output Conditioning block:</u> This block verifies the constrains on the control in order to preserve stability conditions, and control limitation. Finally, the control u is obtained.

Figure 3 shows the gain k_d (25) computation for hypothetical inputs (In_1 and In_2).

3 Stepper Motor Control

In this section, we apply the proposed scheme to control a permanent magnet stepper motor. Its mathematical model is given by

$$\begin{aligned}
\frac{d\theta}{dt} &= \omega \\
\frac{d\omega}{dt} &= \frac{1}{J}[-K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta) - B_v\omega - \tau_l] \\
\frac{di_a}{dt} &= \frac{1}{L}[-Ri_a - K_m \cos(N_r\theta) + u_a] \\
\frac{di_b}{dt} &= \frac{1}{L}[-Ri_b + K_m \cos(N_r\theta) + u_b)]
\end{aligned} \tag{26}$$

where, θ is the angular position; ω is the shaft speed; i_a and i_b are is the currents in phases *A* and *B* respectively; u_a and u_b are is the voltages in phases *A* and *B,* respectively; J is the moment of inertia; *R* and *L* are the resistance and inductance in each of the phase windings, N_r is the number of rotor teeth, K_m. Is the motor torque constant, B_v is the viscous friction and τ_l presents the loud torque perturbation.

Selecting the following state variables, $x_1 = \theta$, $x_2 = \omega$, $x_3 = i_b$, $x_4 = i_a$, the system (26) is represented as block controllable system consisting of tree blocks, and subject to unknown disturbance, $\tau_l = w_1$.

$$\begin{aligned}
&[\dot{x}_1] = [x_2] \\
&[\dot{x}_2] = [-a_2x_2 + b_1(x_1)x_3 - b_2(x_1)x_4] - d_2w_1 \\
&\begin{bmatrix}\dot{x}_3 \\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}-a_3x_3 - c_1b_1(x_1)x_2 \\ -a_4x_4 + c_1b_2(x_1)x_2\end{bmatrix} + b_0\begin{bmatrix}u_1 \\ u_2\end{bmatrix}
\end{aligned} \tag{27}$$

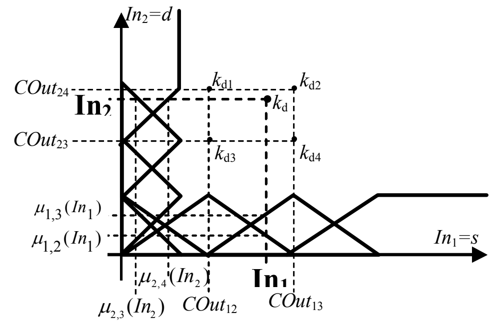

Figure 3 Fuzzy Gain k_d

where $a_2 = \frac{B}{J}$, $b_1(x) = \frac{K_m}{J}\cos(N_r\theta)$,

$b_2(x_1) = \frac{K_m}{J}\sin(N_r\theta)$, $d_1 = \frac{1}{J}$, $a_3 = a_4 = \frac{R}{L}$, $c_1 = \frac{J}{L}$.

Suppose that the output $y = x_1$ is required to follow the reference signal x_{1ref}. Following the block transformation procedure, first we define the tracking error as $z_1 = x_1 - x_{1ref}$, and

$$\dot{z}_1 = x_2 - \dot{x}_{1ref}. \tag{28}$$

Then a desired dynamics for z_1 is introduced as

$$\dot{z}_1 = -k_1z_1 + z_2. \tag{29}$$

Solving (28) and (29) for z_2, we obtain

$$z_2 = k_1x_1 + x_2 - k_1x_{1ref} - \dot{x}_{1ref}$$

Then

$$\begin{aligned}\dot{z}_2 = (k_1 - a_2)x_2 &+ b_1(x_1)x_3 - b_2(x_1)x_4 \\ - k_1\dot{x}_{1ref} - \ddot{x}_{1ref} &- d_2w_1\end{aligned} \tag{30}$$

From this equation and the desired dynamics

$$\dot{z}_2 = -k_2z_2 + z_3 - d_2w_1 \tag{30}$$

we have

$$z_3 = f_3(x_1, x_2) + b_1(x_1)x_3 - b_2(x_1)x_4 + \varphi(t)$$

where z_3 is a new variable, $f_3 = k_1k_2x_1 + (k_1 + k_2 - a_2)x_2$, $\varphi(t) = -k_1k_2x_{1ref} - (k_1 + k_2)\dot{x}_{1ref} - \ddot{x}_{1ref}$. In order to have a nonsingular transformation we introduce a new variable z_4

$$z_4 = -b_2(x_1)x_3 - b_1(x_1)x_4$$

such that the matrix $\mathbf{B}_2 = \begin{bmatrix} b_1(x_1) & -b_2(x_1) \\ -b_2(x_1) & -b_1(x_1)\end{bmatrix}$ has full rank.

In order to obtain the control action, first we define the switching functions as

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} f_3(x_1, x_2) \\ 0 \end{bmatrix} + \begin{bmatrix} b_1(x_1) & -b_2(x_1) \\ -b_2(x_1) & -b_1(x_1) \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \varphi(t) \\ 0 \end{bmatrix}$$

Then the projection motion on the subspace s_1, s_2 is governed by

$$\begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} \bar{f}_3(\mathbf{x}) \\ \bar{f}_4(\mathbf{x}) \end{bmatrix} + b_0 \mathbf{B}_2 \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} \dot{\varphi}(t) \\ 0 \end{bmatrix}$$

where $\mathbf{x} = (x_1, x_2, x_3, x_4)^T$; $\bar{f}_3(\mathbf{x})$ and $\bar{f}_4(\mathbf{x})$ are continuous functions. The control strategy is selected of the form

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -\mathbf{B}_2^{-1} \begin{bmatrix} k_3 sign(s_1) \\ k_4 sign(s_2) \end{bmatrix}$$

and the sliding mode stability conditions are

$$b_0 k_3 > \left| \bar{f}_3(\mathbf{x}) + \dot{\varphi}(t) \right| \text{ and } b_0 k_4 > \left| \bar{f}_4(\mathbf{x}) \right| .$$

Under these conditions the state converges to the sliding manifold $s_1 = 0$, $s_2 = 0$, and when this manifold is reached the sliding mode motion is described by the second order system with unknown nonvanishing perturbation

$$\dot{z}_1 = -k_1 z_1 + z_2$$
$$\dot{z}_2 = -k_2 z_2 + w_1 .$$

In order to reduce the disturbances influence, we apply the described in the section 2 sliding mode fuzzy logic control scheme for adjusting of the controller gains k_1, k_2, k_3 and k_4 such that $k_3 \le u_0$ and $k_4 \le u_0$ with $u_0 > 0$.

4 Simulation Results

In this section, simulation results are presented for the Permanent Magnet Stepper Motor with parameters: $L = 10\,mH$, $R = 8.4\,\Omega$, $J = 3.6 \times 10^{-6}\ Nms^2/rad$, $k_m = 0.05\,Vs/rad$, $N_r = 50$, $B = 1 \times 10^{-4}\ Nms/rad$. The maximal supplied voltage is $u_0 = 2\ V$. In all Figures it is shown the behavior of the state (x_1, x_2, x_3, x_4) as well as new state (z_1, z_2, z_3, z_4), and the gains k_i i=1,2,3, the control u, and disturbance $w_1 = \tau_l$.

Figure 4 displays Block Control Tracking (BCT) with Fuzzy Logic (FL) results using the proposed approach, where smooth variation of k_i gain is observed, and the chattering is reduced.

5 Conclusions

As it can be seen, the proposed scheme performance is very encouraging. Simulations assume that the disturbance cannotbe measured, which is an extreme situation. However, the proposed hierarchical sliding mode control approach with the fuzzy logic control, improves the system behavior, reducing chattering and guaranteeing stability.

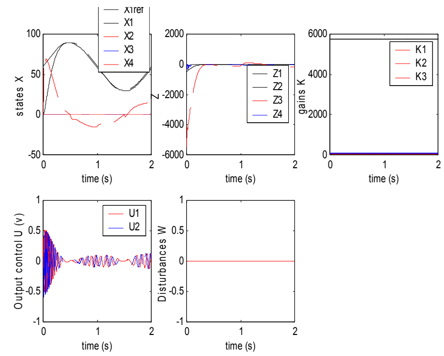

Figure 4: BCT with FLC and disturbances

6 References

Dimiter Driankov, Hans Hellendoorn, and, Michael Reinfrank (1996). *"An Introduction to Fuzzy Control"*, Springer-Verlag, USA.

Drajenovic B. (1969). "The invariance conditions in variable structure systems", *Automatica*, , 5; 287-295.

Hassan K., Khalil (1996), "*Nonlinear systems"*, 2nd. Ed., Prentice-Hall, New-Jersey.

L. K. Wong, Frank H. F. Leung, and, Peter K. S. Tam (2001). "A Fuzzy Sliding Controller for Nonlinear Systems", *IEEE Trans. Ind. Electron*., vol. 48, No. 1, pp. 32-37.

Luigi Scibile and Basil Kouvaritakis (2001). "A Discrete Adaptive Near-Time Optimum Control for the Plasma Vertical Position in a Tokamak", *IEEE Trans. Contr. Syst. Technol*., vol. 9, No.1., pp. 148-162.

A.G. Loukianov, (1998). "Nonlinear Block Control with Sliding Mode", *Automation and Remote Control*, Vol. 59, No. 7, pp. 916-933.

M. Alexík and J. Vittek (1994). "Adaptive Sliding Mode Control of Position Servo System", *IFAC Smolenice*, Slovakia, September 7-10, pp. 278-283.

Okyay Kaynak, Kemalettin Erbatur, and, Meliksah Ertugrul (2001). "The Fusion of Computationally Intelligent Methodologies and Sliding-Mode Control-A Survey*",* *IEEE Trans. Ind. Electron*., vol. 48, pp. 4-17, No. 1, February 2001.

Q. P. Ha, Q. H. Nguyen, D. C. Rye, and, H. F. Durrant-Whyte (2001). "Fuzzy Sliding-Mode Controllers with Applications", *IEEE Trans. Ind. Electron*., vol. 48, pp. 38-46, No. 1.

Rainer Palm, Dimiter Driankov, and, Hans Hellendoorn . (1997). *"Model Based Fuzzy Control - Fuzzy Gain Schedulers and Sliding Mode Fuzzy Controllers"*, Springer-Verlag, Germany.

V. I. Utkin (1992). *"Sliding Modes in Control Optimization"*, Springer-Verlag, USA.

Vadim I. Utkin, Jürgen Guldner, and, Jingxin Shi (1999). *"Sliding Mode Control in Electromechanical Systems"*, Taylor and Francis.