CONTROL STRUCTURE SELECTION FOR OPEN-LOOP UNSTABLE PLANT WITH PURE INTEGRATORS USING MULTI-PROJECTED SYSTEMS

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Abstract

Selection of inputs and outputs for stabilizing an open-loop unstable process is discussed in this paper. The procedure includes decomposition of the process into multiple projections in order to isolate its unstable poles. Several projected subsystems are produced in state-space form indicating pure integrators, slow and fast modes as their only poles. By examining the numerical values of these state-space matrices, one can identify the outputs and inputs, which have strong connections with unstable pole(s). Tennessee-Eastman test-bed problem [4] has been chosen for the case study due to its umpteen I/O combinations. The process, linearized around its base case operating point, shows six unstable poles (including two pure integrators), which are separated as three antistable projections of the overall process. Analysis shows that by closing three loops, it is possible to stabilize the entire plant. PI controllers are designed for the TE plant and simulation results confirm the closed-loop stability of the plant.

1 Introduction

Control structure selection has been a challenging research topic among control engineers during the last decade. Plantwide processes are usually nonsquare and they comprise of an excess number of manipulated and measured variables. Therefore, structural decisions *i.e.* the choice of inputs and outputs in the control structure affects the performance, complexity and costs of the resulting control system. Due to the combinatorial nature of selection problem, systematic methods are needed to complement one's intuition, experience and physical insight. Van de Wal and Jager [12] reviewed the currently known I/O selection methods, which aids the control engineer in picking a suitable method for the problem at hand.

However, the most available selection methods do not explicitly address the issue of feedback stability of open-loop unstable processes. Sometimes, closed loop stabilization is performed based on a plant model and a proposed set of candidate actuators and sensors [9]. Reasons for not using all the available devices could be the reduction of control system complexity. These decisions are made on the basis of engineering understanding while quantitative justifications are seldom available.

A technique is proposed in this work to identify the particular unit operation(s) of the process which is contributing unstable poles to the entire processes. The method involves isolation of unstable poles through decomposition of entire process into multiple projections of smaller subsystems. Useful information can be obtained from the state space matrices of these subsystems which indicate the inputs and outputs having greatest influence on the unstable poles. Tennessee-Eastman test-bed problem [4] has been chosen for a case study due to its umpteen I/O combinations (41 measurements out of which 22 are potential outputs; 12 manipulated variables). Effort has been made to stabilize the process with less control effort than given by previous researchers [9]. The paper has been organized as follows. Section 2 provides the theoretical background of the multi-projected systems including its relation with pole direction method as well as its usage to calculate steady state gain of the process. Section 3 presents brief description about the Tennessee-Eastman test bed problem. Simulation and data analysis is done in the Section 4, whereas design and implementation of stabilizing PID controller is discussed in section 5. Finally the conclusion is reached in the section 6.

2 Theoretical background

A transfer function is anti-stable if it has no poles in the closed left half plane. A transfer function G which may have poles both on the open left and right half planes can be decomposed as $G = G_s + G_u$ where G_s and G_u are called stable and anti-stable projections of G. Necessary and sufficient condition for robust stability to G is the stabilization of its anti-stable projection [5]. It is possible to decompose anti-stable G_u further into several projections according to the unstable pole locations, such as

$$G = G_s + G_{i1} + G_{i2} + ... + G_{ip} + G_{u1} + G_{u2} + ... + G_{uq} \qquad (1)$$

where $G_{ij}\{j = 1,2,...p\}$ and $G_{uj}\{j = 1,2,...q\}$ are state space realization of projections on p pure integrators and q non-zero unstable poles (pole pairs) respectively. Algorithms for these projections can be found elsewhere [11].

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A subsystem $G^* := (A, B, C)$, having m inputs and n outputs, will have the following state space form (A=0, for pure integrators):

$$\dot{x} = Ax + B \begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_m \end{bmatrix}; \quad \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{bmatrix} = Cx \qquad (2)$$

$$\text{where } C = \begin{bmatrix} c_1 \\ c_2 \\ \dots \\ c_n \end{bmatrix}; B = \begin{bmatrix} b_1 & b_2 & \dots & b_m \end{bmatrix}$$

Relative importance of the inputs and outputs, in terms of influence over the pole associated with G^*, can be quantified by input and output effectiveness [2][3] calculated as $\eta_I^2 = diag(B^+ B)$ and $\eta_O^2 = diag(CC^+)$ where $(\bullet)^+$ denotes the pseudo inverse [6] and η_I^2 and η_O^2 are $m \times 1$ and $n \times 1$ vectors respectively, containing square of input and output effectiveness as their elements. Higher value indicates greater influence.

2.1 Relation between projected subsystem and pole direction method

Havre [7] suggested a method for computation of pole directions for multivariable systems in terms of standard eigenvalue problem. Pole direction can be used as a controllability measure for control structure selection. For a system on state-space form with a pole located at $s = p$, the input and output pole vectors are computed as $u_p = B^H x_{pi}$ and $y_p = C x_{po}$ respectively, where $x_{pi}, x_{po} \in C^n$ are normalized eigenvectors corresponding to the two eigenvalue problems $x_{pi}^H A = p x_{pi}^H$ and $A x_{po} = p x_{po}$. The corresponding pole directions are obtained by normalizing the pole vectors,

$$u_{pd} = \begin{cases} u_p / \|u_p\|_2 & if \quad \|u_p\|_2 \neq 0 \\ 0 & if \quad \|u_p\|_2 = 0 \end{cases} \quad and$$

$$y_{pd} = \begin{cases} y_P / \|y_P\|_2 & if \quad \|y_P\|_2 \neq 0 \\ 0 & if \quad \|y_P\|_2 = 0 \end{cases} \qquad (3)$$

The above computation method takes the whole system into account *i.e.* all the poles are taken into consideration simultaneously.

The demerits of this method are

- The computations are relatively vast *i.e.* calculations involving certain poles (such as stable poles) could be unnecessary.
- The effect of certain input(s) or output(s) towards a particular pole is not understood explicitly.

On the other hand, method of multiple projection can isolate poles in several independent subsystems where dimensions of

x_{pi} and x_{po} would be one (or two for complex poles). In other words, projected subsystem of a particular pole will contain the characteristic information of that pole *only*. Thus the pole direction of the projected subsystem will be equal to the pole direction of the entire system. This is confirmed by the following theorem.

Assume $G = G_1 + G_2$, where $G := (A, B, C)$, and $G_i := (A_i, B_i, C_i)$. Then

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$$

Let p be a pole of G and G_1 and its pole directions linking with G and G_1 be (x_{pi}, x_{po}, u_p, y_p) and ($x_{1pi}, x_{1po}, u_{1p}, y_{1p}$) respectively. Then the following theorem is ready to be proved.

Theorem 1: Pole directions associated with the same pole of G and G_1 have the following relations:

$$x_{pi} = \begin{bmatrix} x_{1pi} \\ 0 \end{bmatrix}, x_{po} = \begin{bmatrix} x_{1po} \\ 0 \end{bmatrix}, u_p = u_{1p}, \text{ and } y_p = y_{1p}.$$

If p is a real distinct pole of G, *i.e.* p is the only pole of G_1, then the pole directions of p are $x_{1pi} = x_{1po} = 1$, $u_{1p} = B_1^H$ and $y_{1p} = C_1$.

In cases of complex poles and repeated poles, pole subspaces of a group of such poles are more meaningful than single pole directions. In the multiple projection method, each subsystem contains only a real distinct pole, or a pair of distinct poles, or a group of identical poles/ pole pairs. Therefore, the input pole subspace of the poles in a subsystem, G_i is spanned by the rows of B_i whilst the output pole subspace is spanned by the columns of C_i.

As a result of the above theorem, input(s) and output(s) affecting that particular pole can be explicitly identified through suitable controllability measures such as input (or output) effectiveness factors. Analysis will certainly differ in presence of repeated poles. However, it would still be confined within a certain pole space. The implication of this fact is that attention can be focused only on the *troublesome* poles while analyzing the process without losing important information.

2.2 Calculation of steady state gain in presence of integrating poles

Multiple projected subsystems can be useful to compute steady state gains for processes with pure integrators. Usually steady state gains of state space model can be calculated by transforming the model into transfer function form as

$$G(s) = \frac{y(s)}{u(s)} = C(sI - A)^{-1} B \qquad (4)$$

and setting $s = 0$, to get

$$G(0) = -CA^{-1}B \qquad (5)$$

However, if there are integrators in the process, A becomes singular and A^{-1} does not exist. Arkun and Downs [1] presented an approach where gains are calculated for rate of change of these integrating variables through singular value decomposition of matrix A. McAvoy [8] successfully implemented this approach for Tennessee-Eastman test bed problem. Following their approach $G(s)$ can be reformatted with separate transfer functions for integrating and non-integrating terms as $G(s) = G_i(s) + G_{ni}(s)$. Unlike Arkun and Downs [1], steady state values of the above two terms can be directly calculated from the state space matrices of projected subsystems, preventing any extra computation effort for the same. The steady state gain for $G_{ni}(s)$ can be calculated as

$G_{ni}(0) = -C_{ni}A_{ni}^{-1}B_{ni}$. However, for integrating process, $G_i(s) = s^{-1}C_iB_i$. The rate of change of integrating variable can be expressed as
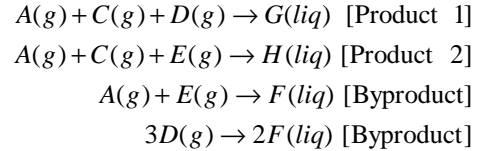
$$\dot{y}_i(s) = sy_i(s) = sG_i(s)u(s) = C_iB_iu(s) = G_i'(s)u(s) \qquad (6).$$

Hence, $G_i'(0) = C_iB_i$. The final gain matrix, K, to be used for interaction analysis consists of rows of $G_{ni}(0)$ and $G_i'(0)$ appropriately placed in K.

3   Tennessee-Eastman Process

The Tennessee Eastman test-bed problem [4] involves the control of five unit operations: an exothermic two-phase reactor, a water-cooled condenser, a centrifugal compressor, a flash drum and a reboiler stripper. The simulated plant has 41 process variables and 12 manipulated variables as illustrated in Figure 1, which are modeled with 50 state variables. The twelve manipulated variables (Fig. 1) are the four feed rates, the purge rate, the agitation rate, steam rate, condenser coolant rate, reactor coolant rate, compressor recycle, flash drum discharge rate and the stripper production rate. Out of 41 process variables there are 22 controllable outputs including level, pressure, temperature, flow and 19 composition indicators as illustrated in Fig. 1.

The chemical reactions are irreversible and occur in the vapor space of the reactor. The chemical reactions are:

$$A(g) + C(g) + D(g) \rightarrow G(liq) \text{ [Product 1]}$$
$$A(g) + C(g) + E(g) \rightarrow H(liq) \text{ [Product 2]}$$
$$A(g) + E(g) \rightarrow F(liq) \text{ [Byproduct]}$$
$$3D(g) \rightarrow 2F(liq) \text{ [Byproduct]}$$

The formation of an inert byproduct, F, is undesirable. The products G and H accumulate in the reactor. Product may thus only be removed via the vapor stream leaving for the condenser. The rate of heat removal owing to the exothermic reactions is dictated by the agitation speed and the cooling water flow rate to the cooling coils in the reactor. Should the
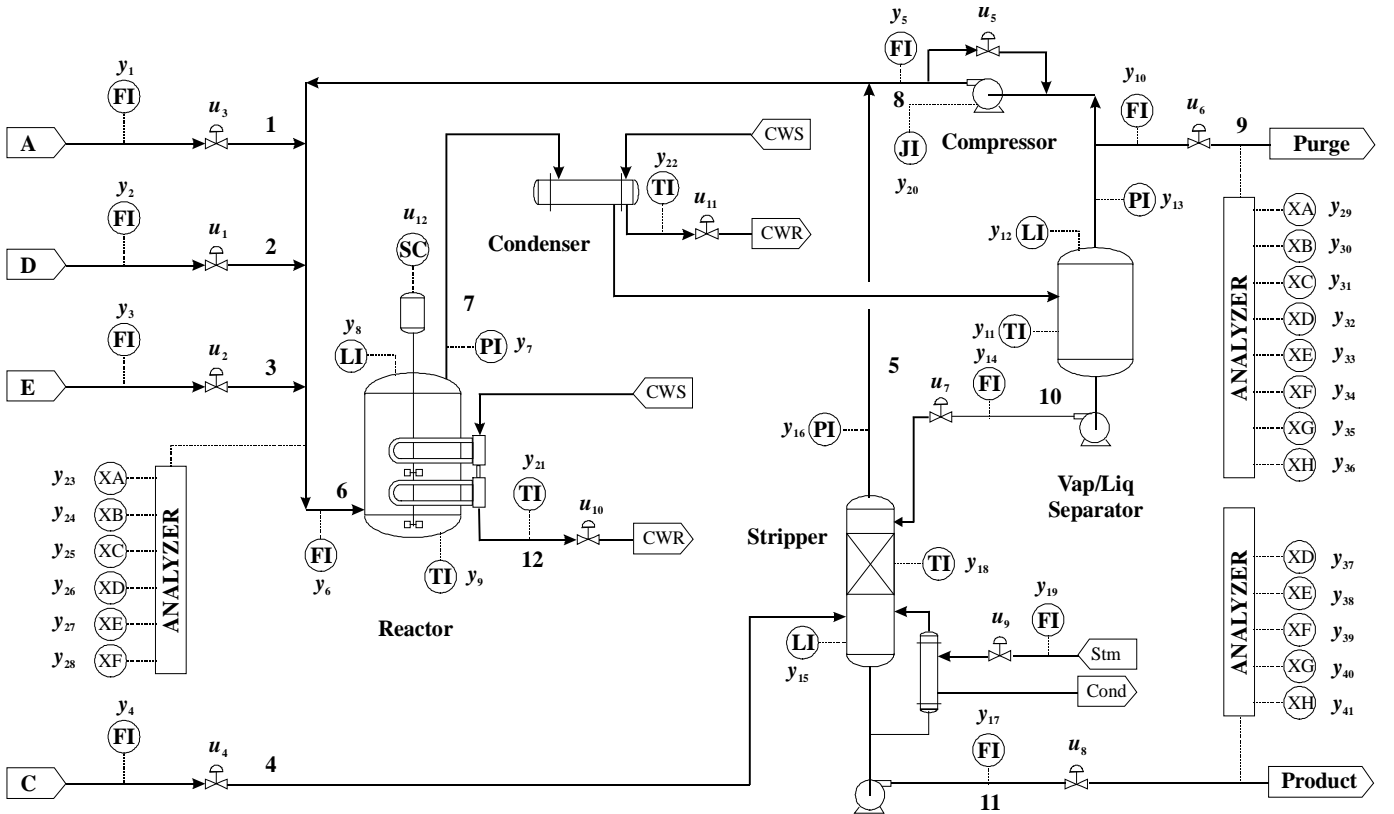

Figure 1: Schematic diagram of Tennessee-Eastman process

liquid level in the reactor fall below 50% the loss of heat-transfer surface area for cooling becomes pronounced. For the reactor pressure, Ricker [10] has indicated that the optimal steady state conditions for the various operating modes are in near proximity to the upper shutdown limit of 3000 kPa. The ability to operate in close proximity to the upper shutdown limit is dictated by the ability of the controller to maintain high performance in the presence of disturbances.

The vapor discharge from the reactor is fed to a partial condenser. The flash drum serves to separate the liquid and vapor phase fed from the condenser. The liquid fraction is fed to the reboiled stripper and the vapor fraction is returned to the reactor via a centrifugal compressor as a recycle stream. The liquid feed to the stripper is distilled to remove impurities in the bottoms product and the vapor stream is recycled to the reactor. A purge is necessary to prevent the unacceptable accumulation of the inert B.

In this paper the desired set point is 50% G and 50% H on a mass basis. By-product F may be present in the product with 97.5% of the product being composed of G and H. A model of the process is generated in Simulink® software. Details of the process data are available elsewhere[4]. Open loop simulation of model indicates that the process is unstable in nature. The control objective is to identify the unit operation(s) as well as the inputs & outputs associated with them which are

responsible for the instability and finally to stabilize the complete system.

4 Simulation and Data Analysis

The entire analysis has been carried out through MATLAB® software. The process is first linearized around its base case operating point and resulting state space linear model is obtained through *LINMOD* function. Due to wide range of the numerical values of the base case I/O variables, proper scaling is essential for reliable analysis. The manipulated inputs are already scaled between 0 to 100%, therefore no further scaling is required. On the other hand, outputs are scaled by dividing by their respective nominal steady state values. Resulting scaled process is decomposed into stable and antistable subsystems using *STABPROJ* subroutine of the Robust Control Toolbox. The antistable subsystem has two pairs of positive complex poles, $3.0648 \pm 5.0837i$ & $0.024973 \pm 0.15521i$ and two pure integrators (numerical value is of the order of 10^{-9}). The antistable subsystem is further decomposed into three subsystems using *SLOWFAST* subroutine. Resulting subsystems are

$G_i := (A_i, B_i, C_i)$ containing   pure   integrator s

$G_{u1} := (A_{u1}, B_{u1}, C_{u1})$ containing poles $0.02497 \pm 0.15521i$

$G_{u2} := (A_{u2}, B_{u2}, C_{u2})$ containing pole s $3.0648 \pm 5.0837i$
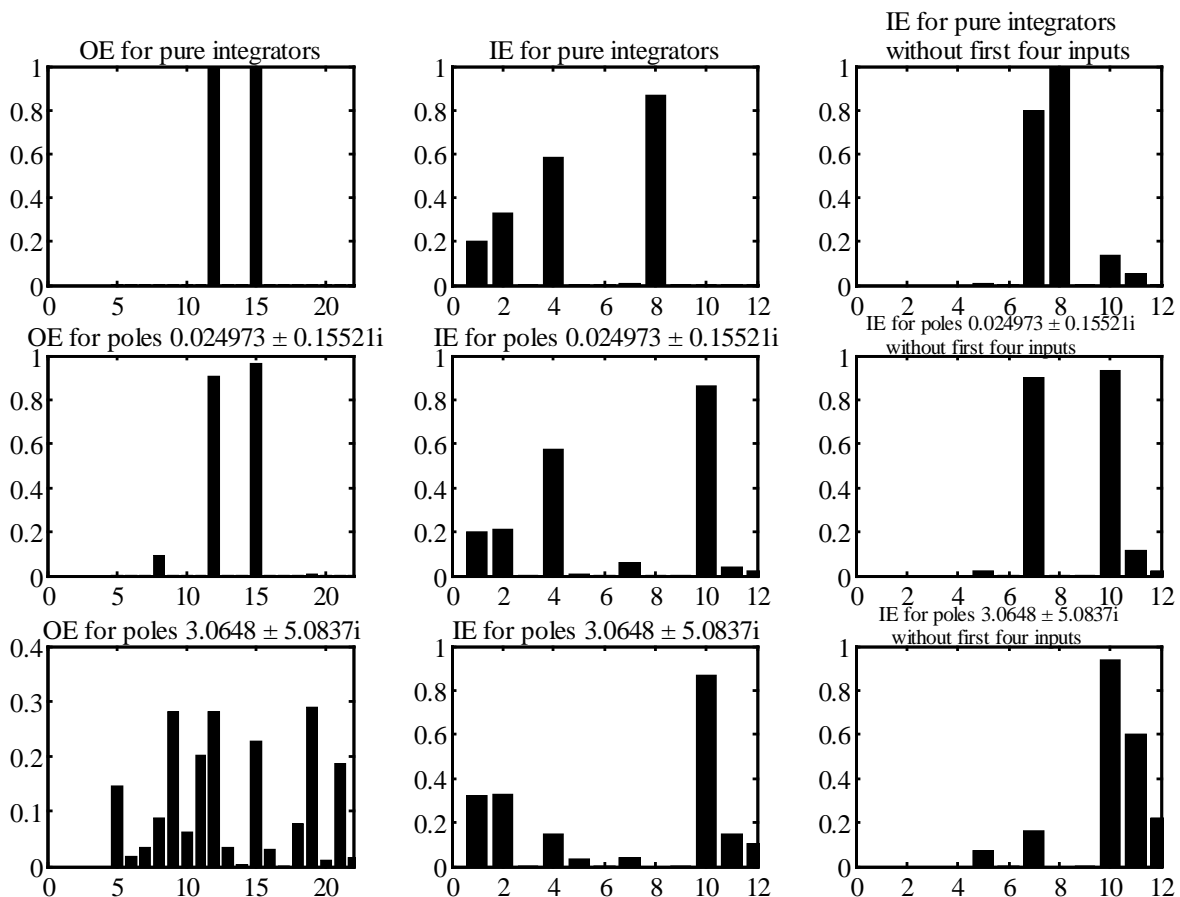

Figure 2: Input effectiveness and Output effectiveness of the TE process analyzed for unstable pole including pure integrators.

Following the theory in section 2, the numerical values of B & C matrices of the above subsystems are calculated. The input and output effectiveness factors are calculated and drawn in a bar chart in Figure 2. It clearly indicates that outputs 12 & 15, *i.e.* the liquid levels inside separator and stripper pots, are responsible for contributing four non-negative poles (two pure integrators and the pair of low value complex poles) out of total six. However, similar conclusive statement cannot be made for the case of remaining positive poles just by analyzing the data because they are contributed jointly by the outputs 9, 12, and 19. Nevertheless, it will be wise not to choose output 19 since it generates a RHP zero in the closed loop system. Thus output 9, *viz.* Reactor temperature, is selected as the third controlled variable. By closing three loops, it is possible to shift all the poles to the left hand side of real axis in the closed loop plant. For input selection, the graph (second column of Fig 2) suggests that inputs, feeds D, A & C, contribute significantly for generating the non-negative poles. However, further analysis suggests that inputs 1-4 generates right half plane zeros (non-minimum phase behaviour), if chosen as manipulated variables. Thus if we omit the effects of feed streams from the graphs, then the next possible candidates are inputs 7, 8 and 10 *viz.*, separator pot liquid flow, stripper liquid product flow and reactor cooling water flow because these three inputs are showing prominent influence on all the poles as shown in the third column of Fig

2. From the engineering understanding, it is quite justified as these inputs are having direct influence on outputs 12, 15 & 9 respectively.

5 Controller design

The graphical user interface *SISOTOOL* is a design tool in MATLAB[®] software which allows one to design single-input/single-output (SISO) compensators by interacting with the root locus, Bode, and Nichols plots of the open-loop system. PI controllers are designed for TE process using this tool by closing one loop at a time and tabulated below.

Loops	Controller settings	
	Gain	Reset
Output 12 vs. Input 7	-0.1	10
Output 15 vs. Input 8	-0.1	10
Output 9 vs. Input 10	-7.2	720

Table 1: Controller settings for TE plant

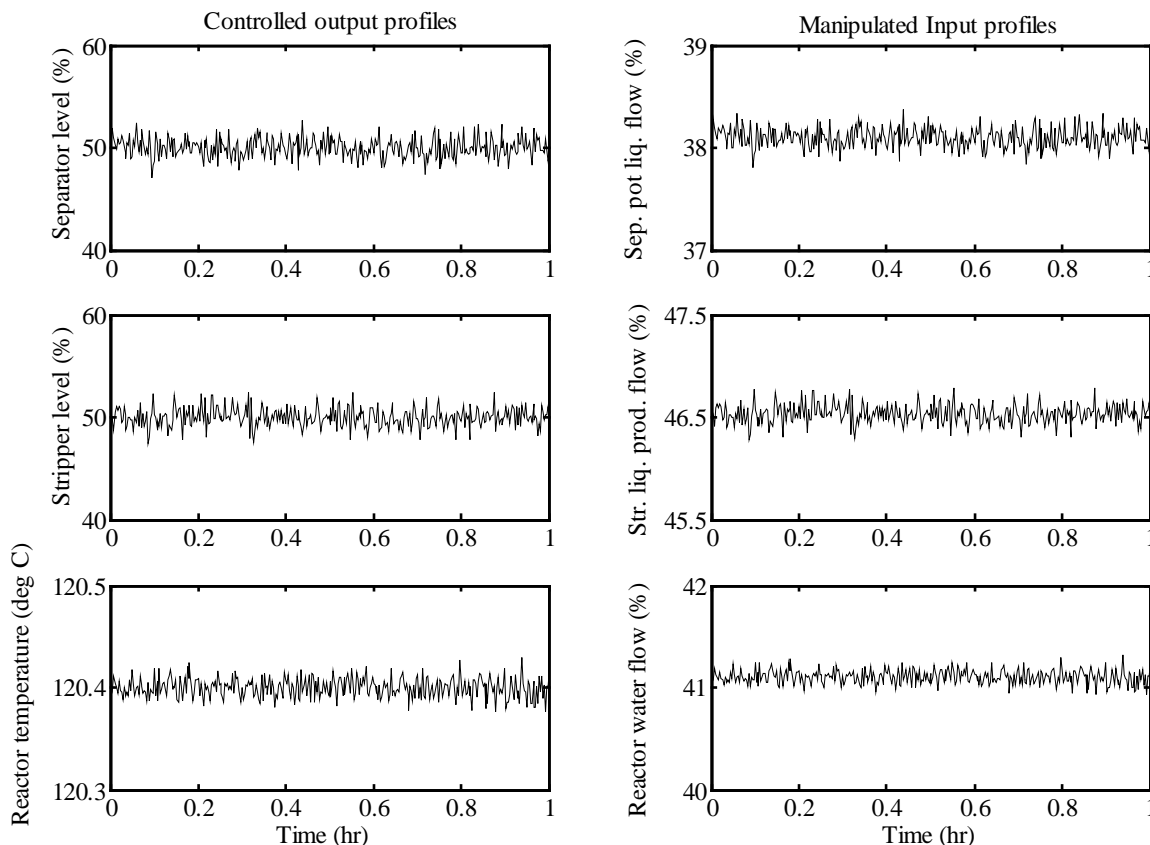The controller renders the desired stability (Figure 3).

Figure 3: Closed loop response of TE plant using stabilizing PI controller

6 Conclusion

It is possible to identify the unit operations and associated inputs and outputs in an open-loop unstable process which are principle contributors to the non-negative poles of the process by using multi-projected subsystems. Simulation results with Tennessee-Eastman test-bed problem supports this theory. Control structure with only three closed loops, selected using this theory, is able to provide the desired stability to the system.

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