MULTIOBJECTIVE DESIGN OF FAULT DETECTION FILTERS

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Abstract

In this paper, problems related to the design of fault detection filters (FDF) for dynamic systems with unknown inputs are studied. Core of our study is to consider the design problems under two different residual evaluation functions, the H_2-norm and the peak amplitude of the residual signals. The background of this study is the fault detection strategy of using multi-residual evaluation functions, which is widely adopted in practice. Based on the above-mentioned two residual evaluation functions, different design schemes for FDF are formulated as multiobjective optimization problems, which are then solved using the well-established LMI-technique.

1 Introduction and background

In this contribution, we consider fault detection (FD) problems for linear time-invariant (LTI) dynamic systems described by

$$\begin{aligned} \dot{x} &= Ax + Bu + E_d d + E_f f &(1)\\ y &= Cx + Du + F_d d + F_f f &(2) \end{aligned}$$

where $u(t) \in \mathbf{R}^{\mathbf{k_u}}$ and $y(t) \in \mathbf{R}^{\mathbf{m}}$ denote the process input and output vectors, $f(t) \in \mathbf{R}^{\mathbf{k_f}}$ fault vector that has to be detected and $d(t) \in \mathbf{R}^{\mathbf{k_d}}$ vector of unknown and bounded inputs, matrices $A, B, C, D, E_d, E_f, F_d$ and F_f are known and of appropriate dimensions.

For our purpose, the following assumptions are made throughout the paper:

A1. (C, A) is detectable;

A2. $\begin{bmatrix} A - j\omega I & E_d \\ C & F_d \end{bmatrix}$ has full row rank for all ω

A3. $\sup \|d\|_2 \le \delta_{d,2}$, $\sup_{t\ge 0} \|d(t)\| \le \delta_{d,peak}$, where

$$\begin{aligned} \|d\|_2 &= \left(\int_0^\infty d^\top(t) d(t) dt\right)^{1/2} \\ \|d(t)\| &= \left(\sum_{i=1}^{k_d} d_i^2(t)\right)^{1/2} \end{aligned}$$

and $\delta_{d,2}, \delta_{d,peak}$ are bounds on the unknown inputs.

A typical FD system consists of a residual generator and a residual evaluation stage [1], [4], [5], [6], [7], [10]. For the purpose of residual generation, we use the so-called fault detection filter (FDF) [1], [7], [10] of the form

$$\begin{aligned} \frac{d\hat{x}}{dt} &= A\hat{x} + Bu + L(y - C\hat{x} - Du) &(3)\\ r &= V(y - \hat{y}) = V(y - C\hat{x} - Du) &(4) \end{aligned}$$

where $r \in \mathbf{R}^{\mathbf{k_r}}$ is the residual signal and L, V are the design parameter matrices.

Let $e = x - \hat{x}$, then we have

$$\begin{aligned} \dot{e} &= \bar{A}e + \bar{E}_d d + \bar{E}_f f &(5)\\ r &= V\left(Ce + F_d d + F_f f\right) &(6)\\ \bar{A} &= A - LC, \bar{E}_d = E_d - LF_d, \bar{E}_f = E_f - LF_f \end{aligned}$$

To evaluate residual signal r, the H_2-norm of r is often used:

$$\|r\|_2 = \left(\int_0^\infty r^\top(t) r(t) dt\right)^{1/2} \quad (7)$$

In the past, we learnt from different industrial applications that it is very popular in practice to evaluate residual signals using multi-evaluation functions in order to reduce missing detection and false alarm rate. A typical combination is the evaluation of energy change and peak amplitude of the residual signals. The latter can be described by

$$\|r\|_{peak} := \sup_{t\geq 0} \|r(t)\|, \|r(t)\| = \left(\sum_{i=1}^{k_r} r_i^2(t) \right)^{1/2} \quad (8)$$

In this contribution, we are going to study residual evaluation schemes based on a combined use of $\|r\|_2$ and $\|r\|_{peak}$ evaluation functions.

The last step to a successful fault detection is the establishment of a decision unit. For this purpose, we first introduce two thresholds corresponding the above-defined two residual evaluation functions:

$$J_{th,2} = \sup_{f=0,d} \|r\|_2 \quad (9)$$

$$J_{th,peak} = \sup_{f=0,d} \|r\|_{peak} \quad (10)$$

Having introduced two different residual evaluation functions, we can form different decision logic depending on the requirements on the performance of fault detection systems, in order to improve the system performance from the viewpoint of a suitable trade-off between the false alarm rate and missing detection rate, for instance,

Logic 1 : if $\|r\|_2 > J_{th,2}$ and $\|r\|_{peak} > J_{th,peak}$,then alarm (a fault is detected), otherwise no alarm (no fault)

Logic 2: if $\|r\|_2 > J_{th,2}$ or $\|r\|_{peak} > J_{th,peak}$,then alarm (a fault is detected), otherwise no alarm (no fault)

Logic 3: if $w_1\|r\|_2 + w_2\|r\|_{peak} > w_1 J_{th,2} + w_2 J_{th,peak}$,then alarm (a fault is detected),otherwise no alarm (no fault), where $w_1 > 0$ and $w_2 > 0$ are known weighting factors.

It is evident that Logic 1 ensures a higher robustness to the unknown inputs and thus reduces the false alarm rate on the one side but increase the missing detection rate on the other side, while using Logic 2 the FD system becomes more sensitive to the faults and thus has a lower missing detection rate but at cost of a higher false alarm rate. By a suitable selection of weighting factors w_1 and w_2, we are able to use Logic 3 to achieve a suitable trade-off between the false alarm rate and missing detection rate, as desired by the application.

Considering that a fault detection system consists of both the residual generator and the residual evaluator including the residual evaluation functions and the thresholds, an integrated design of the residual generator and the residual evaluator is needed to achieve an optimized FD performance [2], [6]. The main objective of this paper is to approach the integrated design of FDF (3)-(4) under consideration of residual evaluation functions (7)-(8). To this end, we first introduce an approach to the integrated design of FDF under H_2-evaluation function and then give an algorithm for the calculation of $J_{th,peak}$. The main attention of this paper will be devoted to the formulation of different FDF design problems and their solutions.

2 An integrated design of FDF under H_2 evaluation function

In this section, we shall briefly describe the so-called unified solution that is used to design an FDF with H_2-norm as residual evaluation function.

Consider the following optimization problem: Given system (1)-(2) and FDF (3)-(4), find L, V such that for all ω and $\sigma_i(V(F_f + C(j\omega I - \bar{A})^{-1}\bar{E}_f)) \neq 0$

$$J \rightarrow \min, J = \frac{\left\| V(F_d + C(sI - \bar{A})^{-1}\bar{E}_d)) \right\|_\infty}{\sigma_i(V(F_f + C(j\omega I - \bar{A})^{-1}\bar{E}_f)} \quad (11)$$

where $\sigma_i(\cdot)$ denoting a non-zero singular value of a transfer function matrix. The following theorem provides a solution to the above-defined optimization problem whose proof is given in [2].

Theorem 1 *Given system (1)-(2) and suppose Assumptions A1-A2 hold, then*

$$L^* = (E_d F_d^\intercal + Y C^\intercal) Q^{-1} \quad (12)$$

$$V^* = Q^{-1/2} \quad (13)$$

solve optimization problem (11), where $Q = F_d F_d^\intercal$ *and* $Y \geq O$ *is a solution of Riccati equation*

$$\tilde{A}^T Y + Y\tilde{A} - YC^\intercal Q^{-1} CY + E_d R E_d^T = 0$$
$$\tilde{A} = A - E_d F_d^\intercal Q^{-1} C$$
$$R = I - F_d^\intercal Q^{-1} F_d$$

The optimal value is given by

$$\frac{1}{\sigma_i(V^*(F_f + C(j\omega I - A + L^*C)^{-1}(E_f - L^*F_f))}$$

Remarks

- The transfer function matrix

  $$V^*(F_d + C(sI - A + L^*C)^{-1}(E_d - L^*F_d))$$

  is a co-inner matrix [16]. Thus, $J_{th,2} = \sup_{f=0,d} \|r\|_2 = \delta_{d,2}$.

- The above solution not only gives an optimal solution to the optimization problem defined by (11) but also provides an optimal trade-off between the missing detection rate and the false alarm rate.
- The above solution is called unified solution, because the well known $H_{\infty/\infty}$- and $H_{\infty/\min}$- optimization problems [1], [5], [6] are only two special cases of the optimization problem defined by (11). This result is the basis of the following study.

3 Determination of $J_{th,peak}$

In this section, an LMI algorithm will be derived for the calculation of $J_{th,peak}$ defined by (10).

For our purpose, we first decompose system (5)-(6) into two parts:

$$\begin{aligned} r_1 &= VC(sI-\bar{A})^{-1}(\bar{E}_d d + \bar{E}_f f) \\ r_2 &= VF_d d + VF_f f \end{aligned}$$

It follows from (8) that for $f=0$

$$\begin{aligned} \|r\|_{peak} &= \sup_{t\geq 0}\|r(t)\| \leq \sup_{t\geq 0}\|r_1(t)\| + \sup_{t\geq 0}\|r_2(t)\| \\ &= \sup_{t\geq 0}\|r_1(t)\| + \sup_{t\geq 0}\|VF_d d(t)\| \\ &= \sup_{t\geq 0}\|r_1(t)\| + \sigma_{\max}(\bar{F}_d)\delta_{d,peak} \end{aligned}$$

Considering that r_1 describes the dynamic part of the FDF, it is thus reasonable to evaluate the influence of the unknown inputs at a certain energy level instead of evaluating the influence of the peak amplitude of d on r_1. Under this consideration, we now introduce the so-called generalized H_2-norm for our purpose: For a given system $y(s) = G_w(s)w(s)$ with x_w denoting the state vector, the generalized H_2-norm is defined by [14]

$$\|G_w\|_g := \sup \left\{ \begin{array}{c} \|y(T)\| : x_w(0)=0, T\geq 0 \\ \int_0^T \|w(t)\|^2\, dt \leq 1 \end{array} \right\}$$

According to this definition, we have

$$\begin{aligned} \sup_{t\geq 0}\|r_1(t)\| &= \|G_{r_1 d}\|_g\, \delta_{d,2} \\ G_{r_1 d}(s) &= VC(sI-\bar{A})^{-1}\bar{E}_d \end{aligned}$$

In order to calculate $\|G_{r_1 d}\|_g$, we now introduce the following theorem [14] which provides us with an effective algorithm for the calculation of $\|G_{r_1 d}\|_g$ and thus $J_{th,peak}$.

Theorem 2 *Given system $G_{r_1 d}(s)$ and a constant α (>0), then*

$$\|G_{r_1 d}\|_g^2 < \alpha$$

iff there exists a symmetric matrix P satisfying the following two LMI's

$$\begin{pmatrix} \bar{A}^T P + P\bar{A} & P\bar{E}_d \\ \bar{E}_d^T P & -I \end{pmatrix} < 0 \qquad (14)$$

$$\begin{pmatrix} P & C^T V^T \\ VC & \alpha I \end{pmatrix} > 0 \qquad (15)$$

To get $\|G_{r_1 d}\|_g$, we can solve the following optimization problem in an iterative way:

$$\min \alpha \text{ subject to } (14) - (15)$$

Using this solution we are able to set the threshold $J_{th,peak}$ as follows:

$$\begin{aligned} J_{th,peak} &= \sup_{f=0,d}\|r\|_{peak} \\ &= \|G_{r_1 d}\|_g\, \delta_{d,2} + \sigma_{\max}(\bar{F}_d)\delta_{d,peak} \end{aligned}$$

4 FDF design

4.1 Basic idea and problem formulation

The basic idea of the below study can be described as follows. Considering that the unified solution presented in Section 2 leads to an optimal trade-off between the false alarm rate and missing detection rate if H_2-norm is used as residual evaluation function, we shall start from this solution and modify it (slightly) such that the requirements on the residual evaluation function $\|r\|_{peak}$ could also be satisfied but without strongly affecting the system performance achieved by using the unified solution.

Suppose that L^*, V^* are the solution of the optimal FDF with H_2-norm as residual evaluation function, as given in Theorem 1. Let $L = L^* + \Delta L$. Then we have

$$\begin{aligned} \dot{e} &= (\bar{A} - \Delta LC)e + \tilde{E}_d d + \tilde{E}_f f && (16) \\ r &= \bar{C}e + \bar{F}_d d + \bar{F}_f f && (17) \\ \bar{A} &= A - L^*C, \bar{C} = V^*C, \bar{F}_d = V^*F_d \\ \bar{F}_f &= V^*F_f, \tilde{E}_d = \bar{E}_d - \Delta LF_d, \tilde{E}_f = \bar{E}_f - \Delta LF_f \\ \bar{E}_d &= E_d - L^*F_d, \bar{E}_f = E_f - L^*F_f \end{aligned}$$

Note that

$$\begin{aligned} & \bar{C}\left(sI - \bar{A} + \Delta LC\right)^{-1}\tilde{E}_d + \bar{F}_d \\ = & \left(I - \bar{C}\left(sI - \tilde{A}\right)^{-1}\Delta\bar{L}\right)\left(G_{r_1 d}(s) + \bar{F}_d\right) \\ & \bar{C}sI - \bar{A} + \Delta LC)^{-1}\tilde{E}_f + \bar{F}_f \\ = & \left(I - \bar{C}\left(sI - \tilde{A}\right)^{-1}\Delta\bar{L}\right)\left(G_{r_1 f}(s) + \bar{F}_f\right) \\ & \tilde{A} = \bar{A} - \Delta\bar{L}C, \Delta\bar{L} = \Delta LV^{*-1} \\ & G_{r_1 f}(s) = V^*C(sI-\bar{A})^{-1}\bar{E}_f \end{aligned}$$

It leads to

$$
\begin{aligned}
r &= \left(I - \bar{C}\left(sI - \tilde{A}\right)^{-1} \Delta \bar{L}\right) r^*(s) &(18)\\
r^* &= \bar{F}_d d + \bar{F}_f f + \bar{C}\left(sI - \bar{A}\right)^{-1}\left(\bar{E}_d d + \bar{E}_f f\right)
\end{aligned}
$$

where r^* is the residual signal delivered by the optimal FDF with H_2-norm as residual evaluation function. Remember that the objective of FDF design consists in the selection of the residual generator parameters that results in a suitable compromise between the evaluation functions (7) and (8). This may be well approached if we design the residual generator so that the generated residual signals $r(s)$ is not strongly different from $r^*(s)$ on the one side and the threshold established based on the evaluation function (8), $J_{th,peak}$, is smaller than a certain value on the other side. Following this idea, we formulate the following design problems:

- Design problem 1: For given constants $0 < \alpha_1 << 1$ and $\hat{\alpha}_2 > 0$, find ΔL such that the residual generator (16)-(17) is stable and

$$
\begin{aligned}
\frac{\|r - r^*\|_2}{\|r^*\|_2} &\leq \alpha_1 &(19)\\
J_{th,peak} &\leq \hat{\alpha}_2 &(20)
\end{aligned}
$$

Note that

$$
\begin{aligned}
&\frac{\|r - r^*\|_2}{\|r^*\|_2} \leq \alpha_1 \Leftrightarrow \left\|\bar{C}\left(sI - \tilde{A}\right)^{-1} \Delta \bar{L}\right\|_\infty \leq \alpha_1\\
&J_{th,peak} \leq \hat{\alpha}_2 \Rightarrow\\
&\|G_{r_1 d}\|_g \, \delta_{d,2} + \sigma_{\max}(\bar{F}_d)\delta_{d,peak} \leq \hat{\alpha}_2\\
&\Leftrightarrow \left\|\bar{C}\left(sI - \tilde{A}\right)^{-1}\left(\bar{E}_d - \Delta \bar{L}\bar{F}_d\right)\right\|_g\\
&\leq \left(\hat{\alpha}_2 - \sigma_{\max}(\bar{F}_d)\delta_{d,peak}\right)/\delta_{d,2} = \alpha_2^{1/2}
\end{aligned}
$$

Thus, this design problem can be re-formulated as finding $\Delta \bar{L}$ such that

$$
\begin{aligned}
&\left\|\bar{C}\left(sI - \tilde{A}\right)^{-1} \Delta \bar{L}\right\|_\infty \leq \alpha_1 &(21)\\
&\left\|\bar{C}\left(sI - \tilde{A}\right)^{-1}\left(\bar{E}_d - \Delta \bar{L}\bar{F}_d\right)\right\|_g^2 \leq \alpha_2 &(22)
\end{aligned}
$$

It is evident that (19) describes the requirement on the residual generator under consideration of H_2-norm evaluation function, while (20) indicates that threshold $J_{th,peak}$ should be limited to a desired value.

Depending on the requirements in applications, Design problem 1 can also be modified and re-formulated into the following two problems.

- Design problem 2: For a given constant $\alpha_2 > 0$, find $\Delta \bar{L}$ such that the residual generator (16)-(17) is stable and

$$
\begin{aligned}
&\left\|\bar{C}\left(sI - \tilde{A}\right)^{-1} \Delta \bar{L}\right\|_\infty \to \min \text{ subject to}\\
&\left\|\bar{C}\left(sI - \tilde{A}\right)^{-1}\left(\bar{E}_d - \Delta \bar{L}\bar{F}_d\right)\right\|_g^2 \leq \alpha_2
\end{aligned}
$$

- Design problem 3: For a given constant $0 < \alpha_1 << 1$, find $\Delta \bar{L}$ such that the residual generator (16)-(17) is stable and

$$
\begin{aligned}
\left\|\bar{C}\left(sI - \tilde{A}\right)^{-1}\left(\bar{E}_d - \Delta \bar{L}\bar{F}_d\right)\right\|_g^2 &\to \min\\
\text{subject to } \left\|\bar{C}\left(sI - \tilde{A}\right)^{-1} \Delta \bar{L}\right\|_\infty &\leq \alpha_1
\end{aligned}
$$

Corresponding to fault detection Logic 3 described in Section 2, we further formulate the following design problem.

- Design problem 4: Given constants $w_1 > 0, w_2 > 0$, find $\Delta \bar{L}$ such that the residual generator (16)-(17) is stable and

$$
w_1 J_{th,2} + w_2 J_{th,peak} \to \min
$$

Note that

$$
\begin{aligned}
J_{th,2} &= \sup_{f=0,d} \|r\|_2\\
&= \left\|I - \bar{C}\left(sI - \tilde{A}\right)^{-1} \Delta \bar{L}\right\|_\infty \delta_{d,2}\\
J_{th,peak} &= \left\|\bar{C}\left(sI - \tilde{A}\right)^{-1}\left(\bar{E}_d - \Delta \bar{L}\bar{F}_d\right)\right\|_g\\
&\delta_{d,2} + \sigma_{\max}(\bar{F}_d)\delta_{d,peak}
\end{aligned}
$$

Thus,

$$
\begin{aligned}
&w_1 J_{th,2} + w_2 J_{th,peak} \to \min \Rightarrow\\
&w_1 \left\|I - \bar{C}\left(sI - \tilde{A}\right)^{-1} \Delta \bar{L}\right\|_\infty + &(23)\\
&w_2 \left\|\bar{C}\left(sI - \tilde{A}\right)^{-1}\left(\bar{E}_d - \Delta \bar{L}\bar{F}_d\right)\right\|_g \to \min
\end{aligned}
$$

It is evident that reducing the threshold under fault detection Logic 3 will increase the system sensitivity to the faults and thus reduce the missing detection rate.

4.2 Solutions

In this sub-section, solutions will be derived for the design problems formulated above. For this purpose, the results of Theorem 2 and the following well-known relationship are needed.

Given system $G_w(s) = D_w + C_w(sI - A_w)^{-1}B_w$, then G_w is stable and satisfies $\|G_w\|_\infty < \alpha(> 0)$ iff there exists a symmetric matrix P with

$$\begin{pmatrix} A_w^T P + PA_w & PB_w & C_w^T \\ B_w^T P & -\alpha I & D_w^T \\ C_w & D_w & -\alpha I \end{pmatrix} < 0, P > 0 \tag{24}$$

Solution of design problem 1

Following Theorem 2 and relation (24), Design problem 1 described by (21)-(22) can be re-formulated as find $\Delta\bar{L}$ such that there exist symmetric matrices P_1, P_2 with

$$\begin{pmatrix} \tilde{A}^T P_1 + P_1\tilde{A} & P_1\Delta\bar{L} & \bar{C}^T \\ \Delta\bar{L}^T P_1 & -\alpha_1 I & O \\ \bar{C} & O & -\alpha_1 I \end{pmatrix} < 0,$$
$$\begin{pmatrix} \tilde{A}^T P_2 + P_2\tilde{A} & P_2\tilde{E}_d \\ \tilde{E}_d^T P_2 & -I \end{pmatrix} < 0$$
$$\begin{pmatrix} P_2 & \bar{C}^T \\ \bar{C} & \alpha_2 I \end{pmatrix} > 0, P_1 > 0$$

Set

$$\Delta\bar{L} = P^{-1}Y, P_1 = P_2 = P \tag{25}$$

then we have

$$\begin{pmatrix} Q & Y & \bar{C}^T \\ Y^T & -\alpha_1 I & O \\ \bar{C} & O & -\alpha_1 I \end{pmatrix} < 0 \tag{26}$$
$$\begin{pmatrix} Q & P\bar{E}_d - Y\bar{F}_d \\ \bar{E}_d^T P - \bar{F}_d^T Y^T & -I \end{pmatrix} < 0 \tag{27}$$
$$\begin{pmatrix} P & \bar{C}^T \\ \bar{C} & \alpha_2 I \end{pmatrix} > 0 \tag{28}$$
$$Q = \bar{A}^T P - \bar{C}^T Y^T + P\bar{A} - Y\bar{C}$$

As a result, we claim that Design problem 1 can be solved if there exist matrices P and Y solving LMI's (26)-(28). The solution for $\Delta\bar{L}$ is given by (25).

Solutions for Design problems 2 and 3 follow directly from the above discussion.

Solution of design problem 2 is given by solving the following optimization problem: finding matrices P and Y such that

$$\min \alpha_1 \text{ subject to}$$
$$\begin{pmatrix} Q & Y & \bar{C}^T \\ Y^T & -\alpha_1 I & O \\ \bar{C} & O & -\alpha_1 I \end{pmatrix} < 0$$
$$\begin{pmatrix} Q & P\bar{E}_d - Y\bar{F}_d \\ \bar{E}_d^T P - \bar{F}_d^T Y^T & -I \end{pmatrix} < 0$$
$$\begin{pmatrix} P & \bar{C}^T \\ \bar{C} & \alpha_2 I \end{pmatrix} > 0$$

Solution of design problem 3 is given by solving the following optimization problem: finding matrices P and Y such that

$$\min \alpha_2 \text{ subject to}$$
$$\begin{pmatrix} Q & Y & \bar{C}^T \\ Y^T & -\alpha_1 I & O \\ \bar{C} & O & -\alpha_1 I \end{pmatrix} < 0$$
$$\begin{pmatrix} Q & P\bar{E}_d - Y\bar{F}_d \\ \bar{E}_d^T P - \bar{F}_d^T Y^T & -I \end{pmatrix} < 0$$
$$\begin{pmatrix} P & \bar{C}^T \\ \bar{C} & \alpha_2 I \end{pmatrix} > 0$$

Similar to the solutions given above, the solution of the following optimization problem provides us with a solution to the Design problem 4 described by (23): Finding matrices P and Y such that

$$\min \left(w_1\alpha_1 + w_2\alpha_2^{1/2}\right) \text{ subject to}$$
$$\begin{pmatrix} Q & Y & \bar{C}^T \\ Y^T & -\alpha_1 I & -I \\ \bar{C} & -I & -\alpha_1 I \end{pmatrix} < 0$$
$$\begin{pmatrix} Q & P\bar{E}_d - Y\bar{F}_d \\ \bar{E}_d^T P - \bar{F}_d^T Y^T & -I \end{pmatrix} < 0$$
$$\begin{pmatrix} P & \bar{C}^T \\ \bar{C} & \alpha_2 I \end{pmatrix} > 0$$

5 Concluding remarks

In this contribution, we have studied the problems of designing FDF under two different residual evaluation functions. The background of this study is the observation that in practice the fault detection strategy of using multi-residual evaluation functions is widely used. The two residual evaluation functions considered in this paper are the H_2-norm, which evaluates the energy level of the residual signal, and the peak amplitude of the residual signal. Based on the two residual evaluation functions mentioned above, different design schemes for FDF are formulated as multiobjective optimization problems, which are then solved using the LMI-technique.

Different academic examples of designing FDF have been successfully solved using the methods presented in this paper. As next step, these methods will be used in different laboratory systems.

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References

[1] J. Chen and P. R. Patton, *Robust model-based fault diagnosis for dynamic systems,* Kluwer Academic Publishers, (1999).

[2] S. X. Ding, T. Jeinsch, P. M. Frank and E. L. Ding, "A unified approach to the optimization of fault detection systems", *Int. J. of Adaptive Contr. and Signal Processing*, (2000).

[3] A. Edelmayer, J. Bocor, F. Szigeti and L. Keviczky, "Robust detection filter design in the presence of time-varying system perturbations", *Automatica*, Vol.33, No.3, pp.471-475, (1997).

[4] P.M. Frank, "Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy - A survey," *Automatica*, vol-26, pp.459-474, (1990).

[5] P.M. Frank, "Enhancement of robustness in observer-based fault detection," *Int. J. Contr.*, vol.59, pp.955-981, (1994).

[6] P.M. Frank and X. Ding, "Survey of robust residual generation and evaluation methods in observer-based fault detection systems", *J of Process Control*, pp.403-424, (1997).

[7] J.J. Gertler, *Fault Detection and Diagnosis in Engineering Systems,* Marcel Dekker, (1998).

[8] M. Hou and R.J. Patton, "An LMI approach to H_-/H_∞ observers", *Proc. IEE Conf. Contr. 96*, (1996).

[9] H.H. Niemann and J. Stoustrup, "Filter design for failure detection and isolation in the presence of modeling errors and disturbances", *Proc. the 35th IEEE CDC*, p.1155-1160, (1996).

[10] R.J. Patton, P.M. Frank and R.N. Clark, (Eds.) *Fault Diagnosis in Dynamic Systems, Theory and Application*, Prentice Hall, (1989).

[11] Z. Qui and J. Gertler, "Robust FDI systems and H_∞ optimization", *Proc. the 32nd IEEE CDC*, p.1710-1715, (1993).

[12] M.A. Sadrnia, R.J. Patton and J. Chen, "Robust H_∞/μ observer-based residual generation for fault diagnosis", *Proc. SAFEPROCESS '97*, p.147-153, (1997).

[13] D. Sauter, F. Rambeaux and F. Hamelin, "Robust fault diagnosis in an H_∞ setting", *Proc. SAFEPROCESS '97*, p.879-884, (1997).

[14] C. Scherer, P. Gahinet and M. Chilali, "Multiobjective output-feedback control via LMI optimization", *IEEE Trans. on AC,* Vol. 42, No.7, pp.896-911, (1997).

[15] M. Tadjine, H. Berkhedda and R.J. Patton, "Frequency domain trade-off robust fault detection: towards an integrated approach", *Proc. SAFEPROCESS '97*, p.166-170, (1997).

[16] K. Zhou, J. C. Doyle and K. Glover, *Robust and optimal control*, Prentice Hall, (1995).