MULTIRATE SAMPLED-DATA STABILIZATION OF NONLINEAR SYSTEMS

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Abstract

The problem of state feedback sampled-data stabilization of nonlinear systems is considered under the "low measurement rate" constraint and in presence of (not necessarily small) time-delay in the measurement channel. A multirate control scheme is proposed which utilizes a numerical integration scheme to approximately predict the current state from the delayed measurements and to reconstruct state trajectories between samples. For both the controller emulation approach and the approach based on approximate discrete-time model of the system, we show that under the standard assumptions borrowed from [8, 10] the closed-loop multirate sampled data system is asymptotically stable in the semiglobal practical sense. An illustrative example of sampled-data control of VTOL aircraft is presented that demonstrates the advantages of the proposed scheme.

1 Introduction

Since modern control systems usually employ digital technology for controller implementation, the study of sampled-data control systems [2, 1] becomes an important part of control science. In the recent years, essential progress has been made in the area of nonlinear sampled-data control systems (see [7], and the bibliography therein). In particular, in the survey paper [7] two different approaches to sampled-data controller design for nonlinear systems were described. The first one, so-called controller emulation, involves digital implementation of a continuous-time stabilizing control law at a sufficiently high sampling rate. The second approach consists of discretizing the plant model and then proceed to design a discrete-time control law. The main difficulty of the second approach is that an exact discrete-time model of a continuous-time nonlinear system is usually not available. This problem has been successfully overcome in [8] by providing a set of sufficient conditions that ensure that a controller which stabilizes an approximate model of the system, will also stabilize the exact discrete-time model. Note that both of these approaches are essentially single-rate, i.e. sampling rates of the input and the measurement channels are assumed to be equal.

In practical applications, however, hardware restrictions on input and measurement sampling rates can be essentially different. For example, the D/A converters are generally faster than the A/D converters, so the input sampling rate can be made higher than the measurement one. Moreover, in some important cases, for instance, in visual servo control systems [6, 3], the sensor channel contains complex image processing blocks which cause essential slowing down of the measurement sampling rate together with delays in the measurement process. Note that, even in the absence of the measurement delay, none of the mentioned approaches directly leads to controllers that can successfully operate under the essential restrictions on measurement sampling rate. Indeed, for the controller emulation approach, the closed-loop sampled data system inherits the stability properties of the corresponding continuous time system only if sampling rate is sufficiently fast. On the other hand, for a discrete-time model of a continuous-time nonlinear system with sufficiently large sampling period a stabilizing controller may not exist or can lead to strong deterioration of control performance.

In this paper another scenario is considered, namely, the use of a multirate control scheme. We address the problem of sampled-data stabilization of nonlinear systems under the "low measurement rate" constraint and in the presence of delay in the measurement channel. The idea behind our approach is as follows. We propose a multirate controller that contains a "fast" numerical integration scheme that represents an approximate discrete-time model of the plant and allows us to reconstruct approximately state trajectories between samples. The state of the model is updated periodically using the "low-rate delayed" measurements of the actual state. The control law, in turn, depends on the state of the model rather than the actual state of the plant. We show that, using this scheme, one can successfully overcome the difficulties that arise if the measurement sampling rate is low and/or time delay exists in the measurement channel. More precisely, we address the design of multirate controllers based on the controller emulation approach as well as on the approach that utilizes the properties of approximate discrete-time models. In both cases we show that under the standard assumptions borrowed from [8, 10] , for any measurement delay and for an arbitrary slow output measurement sampling rate the proposed multirate scheme makes the system asymptotically stable in the semiglobal practical sense.

The paper is organized as follows. In section 2, the general statement of the problem is presented. In section 3, we address the multirate scheme based on the controller emulation approach, while in section 4 the multirate scheme based on the properties of approximate discrete-time models is considered. An illustrative example of sampled-data hovering control of vertical take-off and landing (VTOL) aircraft is presented in section 5. Some concluding remarks are given in section 6.

The following standard notations and definitions will be used throughout the paper. Denote $R^+ := [0, +\infty)$. A continuous function $\alpha: R^+ \to R^+$ is said to belong to class $\mathcal{K}$ ($\alpha \in \mathcal{K}$) if $\alpha(0) = 0$ and it is strictly increasing. A continuous function $\beta: R^+ \times R^+ \to R^+$ is said to belong to class $\mathcal{KL}$ ($\beta \in \mathcal{KL}$), if for each fixed $t \geq 0$, $\beta(\cdot, t)$ belongs to class $\mathcal{K}$ and for each fixed $s \geq 0$, $\beta(s, t)$ decreases to zero as $t \to +\infty$. Also, a closed ball of radius $\Delta \geq 0$ centered at 0 in R^n will be denoted by $\bar{B}^n(\Delta)$.

2 Statement of the problem

Consider a nonlinear system of the form

$$\dot{x} = f(x, u), \tag{1}$$

where $x \in R^n$ is a state, $u \in R^m$ is input, and the right-hand side $f(\cdot, \cdot)$ is assumed to be locally Lipschitz in both arguments.

In sampled-data control systems, a continuous-time plant is connected with digital controller via the analog-discrete (A/D) and discrete-analog (D/A) converters. In this paper we will consider a practically important case when essential hardware restrictions are imposed on the "measurement-A/D conversion" process. More precisely, we address a problem of multi-rate sampled data state feedback stabilization of the system (1) under "low measurement rate" constraint and in the presence of delay in the measurement channel, described as follows. We assume that the minimal output sampling period $T_m^* > 0$ as well as the minimal measurement delay $\tau^* \geq 0$ are given, so that the "measurement-A/D conversion" process is described by the following "ideal sampler + delay" equation

$$y(i) := x(iT_m - \tau), \quad \text{for each } i \in \{0, 1, \ldots\}, \tag{2}$$

where the measurement sampling period T_m and the delay τ can be chosen by designer but must satisfy the constraints

$$\begin{aligned} T_m &\geq T_m^*, \\ \tau &\geq \tau^*. \end{aligned} \tag{3}$$

Informally speaking, we address the case when the constants T_m^*, τ^* are large enough, i.e. the measurement sampling rate is low, and the measurement delay is large. In this case, the single rate design methods presented in [7] may lead to unstable closed-loop sampled data system (see example in section 5).

Let the digital controller be described by difference equations of the form

$$\begin{aligned} x_c(k+1) = F\,(&x_c(k), x_c(k-1), \ldots, x_c(k-q_c), \\ &y(i), y(i-1), \ldots y(i-q))\,, \end{aligned} \tag{4}$$

$$\begin{aligned} v(j) = G\,(&x_c(k), x_c(k-1), \ldots, x_c(k-q_c), \\ &y(i), y(i-1), \ldots y(i-q)) \end{aligned} \tag{5}$$

where q, q_c are some nonnegative integers.

Finally, the D/A converter is described as a zero-order hold of the form

$$u(t) = v(j) \quad \text{for } t \in [jT_i, (j+1)T_i)\,,$$

where $T_i > 0$ is input sampling period. In the following, the input sampling period $T_i > 0$ is assumed either to be arbitrarily chosen by designer (section 3), or fixed but such that the single-rate discrete-time model of the system corresponding to sampling period T_i satisfies some set of assumptions (section 4).

We endeavour to solve the following problem: for any given minimal measurement sampling period $T_m^* > 0$ and any given minimal measurement delay $\tau^* > 0$ find a controller of the form (4),(5), which makes (possibly after choosing $T_i > 0$ appropriately small) the closed loop system asymptotically stable with the prescribed (finite) restriction and the prescribed (nonzero) offset.

3 Multirate design based on continuous-time model

In this section we consider a multi-rate control scheme which is based on the knowledge of the solution of continuous-time stabilization problem for the system (1). More precisely, we address the problem formulated in the previous section under the following two assumptions.

Assumption 1. There exists a state feedback locally Lipschitz control law γ: $R^n \to R^m$ such that the equilibrium $x = 0$ of the closed-loop continuous-time system

$$\dot{x} = f(x, \gamma(x)) \tag{6}$$

is globally asymptotically stable.

Assumption 2. The value of input sampling period $T_i > 0$ can be assigned arbitrarily.

Now, given $h > 0$, let us denote

$$f_h(x,u) := x + hf(x,u). \tag{7}$$

Thus,

$$x(k+1) = f_h(x(k), u(k)) \tag{8}$$

is the discrete-time Euler approximation of the continuous-time system (1) corresponding to integration period $h > 0$. In the following, we will utilize the approximation (8) in the construction of our multirate controller. The value of integration period $h > 0$ will always be taken from a set $\mathcal{H}$ defined as follows. First, without loss of generality we assume that the measurement sampling period T_m and the delay τ, which satisfy the constraints (3), are chosen such that

$$\frac{\tau}{T_m} = \frac{p_1}{p_2},$$

for some integers $p_1 \geq 0$, $p_2 > 0$. Now, let the set $\mathcal{H}$ be defined as follows

$$\mathcal{H} = \left\{ \frac{T_m}{k_h p_2},\ k_h = 1, 2, \ldots \right\}. \tag{9}$$

Note that, if $h \in \mathcal{H}$, then T_m, τ, and h are related as follows

$$T_m = k_h p_2 h, \quad \tau = k_h p_1 h,$$

where $k_h \in \{1, 2, \ldots\}$. Now, let us denote the sequence of "low-rate" delayed measurements as follows

$$y(j) = x(jT_m - \tau) = x(hk_h(jp_2 - p_1)). \tag{10}$$

We propose a digital controller of the following structure

$$\begin{aligned} & x_c(i+1) \\ &= \begin{cases} f_h(\tilde{x}(i), \gamma(x_c(i))) & \text{for } i = jk_hp_2 - 1,\ j = 0, 1, \ldots, \\ f_h(x_c(i), \gamma(x_c(i))) & \text{otherwise,} \end{cases} \end{aligned} \tag{11}$$

$$v(i) = \gamma(x_c(i)), \tag{12}$$

where $\tilde{x}(i)$ is the Euler approximate estimate of the state $x(ih)$ that is calculated for each $i = jk_hp_2 - 1$, $j = 0, 1, \ldots$ using measurements (10) by the formula

$$\begin{aligned} \tilde{x}(i) = f_h(\ldots(f_h(y(j), \gamma(x_c(i+1-k_hp_1))), \\ \gamma(x_c(i+2-k_hp_1))), \ldots, \gamma(x_c(i-1))). \end{aligned} \tag{13}$$

Finally, using assumption 2, we set $T_i = h$, so that

$$u(t) = v(i) \quad \text{for } t \in [iT_i, (i+1)T_i). \tag{14}$$

Remark 1. The main idea behind the proposed controller (11), (12) is to use Euler approximate discrete-time model of a continuous time system to obtain an approximation of the system's current state based on the delayed low-rate measurements. Note that, due to measurement delay $\tau = k_hp_1h$, each $y(j) = x(hk_h(jp_2 - p_1))$ becomes available to the controller at the (jp_2k_h)-th step.

To formulate our result, we need to identify a state of the closed loop sampled-data system (1), (10), (11), (12), (14). In general, a closed-loop sampled data system with delay in the measurement channel can be described by functional differential equations [10]. A state of the closed loop sampled data system for each $t \in [ih, (i+1)h)$ can be defined as

$$\mathbf{x}(t) := \left\{ [x(\cdot)]_{t-\tau}^{t}, [x_c(\cdot)]_{i-k_hp_1}^{i} \right\}, \tag{15}$$

where $[x(\cdot)]_a^b$ $(a \leq b)$ is a piece of continuous-time trajectory $x(t)$ restricted on interval $[a, b]$, while $[x_c(\cdot)]_{i_1}^{i_2}$ $(i_1 \leq i_2)$ is a sequence $\{x_c(i_1), x_c(i_1+1), \ldots, x_c(i_2)\}$. Thus, the state $\mathbf{x}(t)$ consists of pieces of trajectories of the continuous-time system (1) and the discrete-time controller (11). Let us define a norm of $\mathbf{x}(t)$, where $t \in [ih, (i+1)h)$, as follows

$$\|\mathbf{x}(t)\| := \max \left\{ \max_{s \in [t-\tau, t]} |x(t)|, \max_{j \in \{i-k_hp_1, i\}} |x_c(j)| \right\}. \tag{16}$$

The main result of this section is presented in the following theorem.

Theorem 1. Consider the system (1), (10), (11), (12), (14). Under Assumptions 1,2 there exists $\beta \in \mathcal{KL}$ such that the following holds. Given $T_m \geq T_m^*$, $\tau > \tau^*$, $\Delta > 0$, $\delta > 0$, there exists $h_{\max} > 0$ such that if $h \in \mathcal{H} \cap (0, h_{\max}]$, then all trajectories with initial conditions $\|\mathbf{x}(0)\| \leq \Delta$ satisfy

$$\begin{aligned} |x(t)| &\leq \beta(|x(0)|, t) + \delta \quad && \text{for all } t \geq 0, \\ |x_c(i)| &\leq \beta(|x(0)|, ih) + \delta \quad && \text{for all } i \in \{0, 1, \ldots\}. \end{aligned}$$

4 Multirate design based on discrete-time model

In this section, we address an alternative approach to design of multi-rate sampled-data systems, which utilizes the knowledge of some properties of the set of approximate discrete-time models. Consider a system (1). Let the following discrete-time system

$$x(i+1) = F\left(x(i), u(i)\right) \tag{17}$$

be the exact discrete-time model of the continuous-time system (1) corresponding to given sampling period $T > 0$. Throughout this section we assume that the input sampling period T_i is fixed equal to the sampling period of the model (17), i.e. $T_i = T$, and any of T, T_i will be called "sampling period". Following the general statement of our problem (see section 2), we assume that the measurement sampling period $T_m > 0$ and the measurement delay $\tau \geq 0$ satisfy the constraints (3), where the corresponding lower bounds $T_m^* > 0$ and $\tau^* \geq 0$ are given. Without loss of generality we assume that both T_m and τ are multiples of the input sampling period T, i.e.

$$\tau = l_1 T, \quad T_m = l_2 T, \tag{18}$$

for some integers $l_1 \geq 0$, $l_2 > 0$. The sequence of measurements $y(\cdot)$ will be numbered as follows

$$y\,(j) = x(jl_2 - l_1), \quad j = 0, 1, \ldots. \tag{19}$$

Since the measurement signal $y(\cdot)$ comes with delay l steps, we understand that $y(j)$ becomes available for controller at jl_2-th step.

It is worth noting (see [8]) that for the nonlinear system of the form (1) its exact discrete-time model (17) is usually unknown. Instead, following [8], it is assumed that a family of approximate discrete-time models of the system (1) corresponding to sampling period $T > 0$ is available parameterized by modelling parameter $h > 0$

$$x(i+1) = F_h\left(x(i), u(i)\right). \tag{20}$$

Usually, the parameter $h > 0$ is the integration period of the numerical integration scheme which is used to generate the family of approximate discrete-time models.

In the following, we assume that the family of approximate models $F_h(\cdot,\cdot)$ satisfies the following assumptions (which are close to assumptions used in [8]).

Assumption 3. (Equi-global asymptotic stabilizability of approximate models with equi-Lipschitz Lyapunov functions) There exist $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$, $\alpha_3 \in \mathcal{K}$, and $h^* > 0$ such that for each $h \in (0, h^*]$ there exist a function $V_h\colon R^n \to R^+$ and a control law $\gamma_h(\cdot)$ with the following properties:

i) $\alpha_1(|x|) \leq V_h(x) \leq \alpha_2(|x|)$ for each $h \in (0, h^*]$;

ii) for each $D > 0$ there exist $M > 0$ and $h_1^* \in (0, h^*]$ such that for each $x_1, x_2 \in \bar{B}(D)$ and each $h \in (0, h_1^*]$

$$|V_h(x_1) - V_h(x_2)| \leq M\,|x_1 - x_2|\,;$$

iii) for any $\Delta_1 > 0$ there exists $h_2^* \in (0, h^*]$ such that

$$\sup_{\substack{h \in (0, h_2^*] \\ x \in \bar{B}(\Delta_1)}} |\gamma_h(x)| < \infty; \tag{21}$$

iiii) for each $x \in R^n$ and each $h \in (0, h^*]$

$$V_h\left(F_h(x, \gamma_h(x)\right) - V_h(x) \leq -\alpha_3(|x|).$$

Assumption 4. (uniform local Lipschitz property of the approximate models) For any $\Delta_1, \Delta_2 \geq 0$ there exist $L > 0$ and $h^* > 0$ such that for any $x, z \in \bar{B}\left(\Delta_1\right)$, any $u \in \bar{B}(\Delta_2)$, and any $h \in (0, h^*]$

$$|F_h(x,u) - F_h(z,u)| \leq L\,|x - z|\,. \tag{22}$$

Assumption 5. (consistency of the approximation scheme) For any $\Delta_1, \Delta_2 \geq 0$ there exists a $\mathcal{K}$-class function $\rho(\cdot)$ such that for any $x \in \bar{B}\left(\Delta_1\right)$ and any $u \in \bar{B}(\Delta_2)$

$$|F(x,u) - F_h(x,u)| \leq \rho(h). \tag{23}$$

To stabilize the system (17) subject to the measurements (19), we will use a multi-rate control strategy similar to one presented in section 3. The proposed multi-rate digital controller is described as follows

$$\begin{aligned} &x_c(i+1) \\ &= \begin{cases} F_h\left(\tilde{x}(i), \gamma_h\left(x_c(i)\right)\right) & \text{for } i = l_2 j - 1,\ j = 0,1,2,\ldots, \\ F_h\left(x_c(i), \gamma_h\left(x_c(i)\right)\right) & \text{otherwise,} \end{cases} \end{aligned} \tag{24}$$

$$u(i) = \gamma_h\left(x_c(i)\right) \qquad \text{for } i \in \{0, 1, \ldots\}\,, \tag{25}$$

where $\tilde{x}(i)$ is an approximate estimate of the state $x(i)$ that is calculated for each $i = l_2 j - 1$, $j = 0, 1, 2, \ldots$, based on measurements $y(j)$ using the following formula

$$\begin{aligned} \tilde{x}(i) := F_h\left(\ldots\left(F_h\left(y(j), \gamma_h\left(x_c(i+1-l_1)\right)\right),\right.\right. \\ \left.\left.\gamma_h\left(x_c(i+2-l_1)\right)\right), \ldots, \gamma_h\left(x_c(i-1)\right)\right). \end{aligned} \tag{26}$$

The state of the closed loop system (17), (19), (24), (25) (26) at the i-th step is the following "lifted" signal

$$\mathbf{x}(i) := \left\{[x(\cdot)]_{i-l_1}^{i}, [x_c(\cdot)]_{i-l_1}^{i-1}\right\},$$

where $[x(\cdot)]_{i_1}^{i_2}$ $(i_1 \leq i_2)$ is a sequence $\{x(i_1), x(i_1+1), \ldots, x(i_2)\}$, and so is $[x_c(\cdot)]_{i_1}^{i_2}$. Again, a norm of the state $\mathbf{x}(i)$ can be defined as follows

$$\|\mathbf{x}(i)\| := \max\left\{\max_{k\in\{i-l_1,i\}} |x(k)|, \max_{j\in\{i-l_1,i-1\}} |x_c(j)|\right\}.$$

The main result of this section is given by the following theorem.

Theorem 2. Consider a system (17) with sampling period $T > 0$. Suppose a family of approximate models (20) is available that satisfies the Assumptions 3-5. Then there exists $\beta \in \mathcal{KL}$ such that the following holds. Given $\Delta > 0$, $\delta > 0$, $\tau = l_1 T$, where $l_1 \in \{0, 1, \ldots\}$, and $T_m = l_2 T$, where $l_2 \in \{1, 2, \ldots\}$, there exists $h_{\max} > 0$ such that if $h \in (0, h_{\max}]$, then all trajectories of the closed loop system (17), (19), (24), (25) (26) with initial conditions $\|\mathbf{x}(0)\| \leq \Delta$ satisfy

$$\max\{|x(i)|, |x_c(i)|\} \leq \beta(|x(0)|, iT) + \delta$$

for all $i \in \{0, 1, 2, \ldots\}$.

5 Example: VTOL aircraft

A possible area of application of the multirate schemes proposed above is visual based control of autonomous vehicles. Indeed, the presence of a visual processing block in the measurement channel usually results in essentially larger measurement sampling period together with delay in the measurement process. In this case, the stability and performance of the sampled-data system may be improved drastically using the multirate scheme proposed. To illustrate this statement, consider the problem of hovering control of the vertical take off and landing (VTOL) aircraft. The following simplified mathematical model borrowed from [5] describes the motion of VTOL aircraft in the vertical (x,y)-plane

$$\begin{aligned} \ddot{x} &= -u_1 \sin\theta + \epsilon u_2 \cos\theta, \\ \ddot{y} &= u_1 \cos\theta + \epsilon u_2 \sin\theta - 1, \\ \ddot{\theta} &= u_2. \end{aligned} \tag{27}$$

Here, x, y are the position coordinates of the aircraft in the vertical-lateral plane, θ is the roll angle, and the control inputs u_1, u_2 are the thrust and the rolling moment respectively. Also, ϵ is a small positive coefficient which represents the coupling between the rolling moment and the lateral acceleration of the aircraft. In the simulations below, we put $\epsilon = 0.01$.

As shown in [9], the origin $(x, y, \theta) = (0, 0, 0)$ of the system (27) can be stabilized by the following continuous time control law

$$\begin{aligned} u_1 &= \sqrt{v_1^2(x,\dot{x}) + (v_2(y,\dot{y})+1)^2}, \\ u_2 &= -k^2\left(\theta - \tan^{-1}\left(\frac{-v_1(x,\dot{x})}{v_2(y,\dot{y})+1}\right)\right) - k\dot{\theta}, \end{aligned} \tag{28}$$

where

$$\begin{aligned} v_1(x,\dot{x}) &= -k_{11}x - k_{12}\dot{x}, \\ v_2(y,\dot{y}) &= -k_{21}y - k_{22}\dot{y}, \end{aligned}$$

$k_{11}, k_{12}, k_{21}, k_{22} > 0$. Throughout this section, we put $k = 10$, $k_{11} = k_{12} = k_{21} = k_{22} = 1$, and assume the initial conditions are $x(0) = 5$, $y(0) = 0$, $\theta(0) = 0$. The corresponding transient process for x-coordinate of the system (27) with continuous time control law (28) is shown in figure 1.

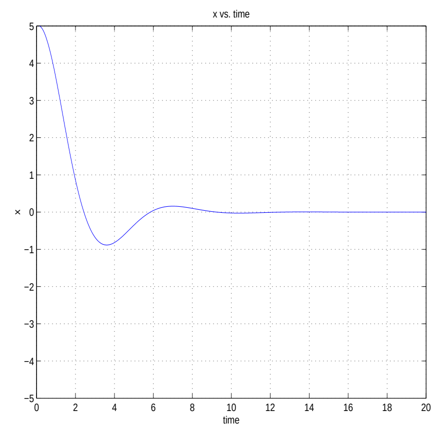

Figure 1: Continuous-time system.

As the second step, we consider a single-rate sampled data system obtained from (27), (28) using controller emulation. Our simulation shows that, even without measurement delay, this single-rate sampled-data controller stabilizes the origin of the system only for small values of sampling period $T < 0.2$ sec. Example of the simulations for sampling period $T = 0.2$ sec is shown in figure 2; one can see that corresponding sampled-data system is unstable.

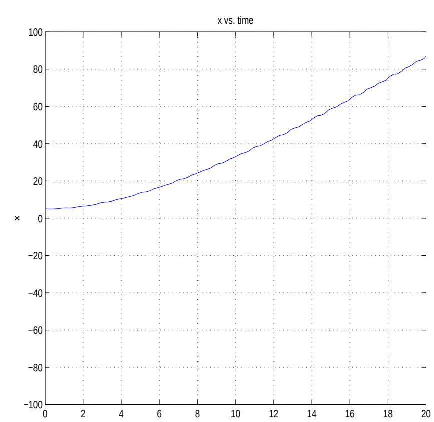

Figure 2: Single-rate scheme with $T_s = 0.2$ sec.

An example of simulations of multirate sampled-data system is shown in figure 3. In this set of simulations, we investigate the system with low measurement sampling rate $T_m = 1$ sec and in presence of measurement delay $\tau = 1$ sec. We put the input sampling period $T_i = 0.1$ sec, and the integration step of the controller $h = 0.01$ sec. One can see that the proposed multirate controller successfully stabilizes the system. Moreover, the form of transient response curve is almost equivalent to the one obtained using continuous time controller.

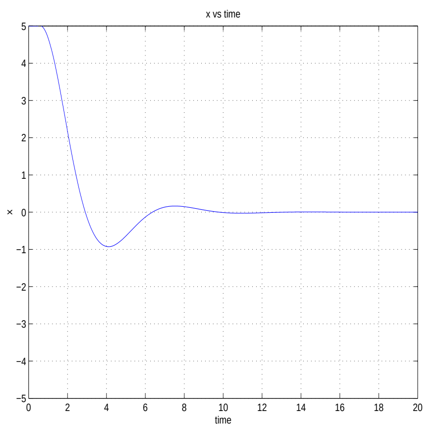

Figure 3: Multirate scheme with $T_m = 1$ sec, $\tau = 1$ sec, $T_i = 0.1$ sec, and $h = 0.01$ sec.

6 Conclusions

In this paper we have addressed the problem of sampled-data stabilization of a nonlinear system under "low measurement rate" constraints and in presence of (not necessarily small) delay in the measurement channel. Our approach to the solution of this problem employs a multirate control law. The main feature of the scheme is that an approximate discrete-time model of the system is included into the controller, and the control action depends on the state of this model. The state of the model is, in turn, corrected from time to time using the "low rate" delayed measurements of the actual state of the plant. It is worth noting that, for linear systems and in absence of measurement delay, some close ideas were used in [3, 4] to design high-performance multirate servo systems. Since the open-loop estimator is used to obtain approximations of state trajectories between samples, the robustness properties of the proposed scheme need further investigations. This topic will be a subject of future research.

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