

# A VARIABLE STRUCTURE APPROACH TO ENERGY SHAPING

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## Abstract

The port-Hamiltonian formalism is a very powerful tool for describing dynamical systems and their interconnections and for designing control laws with specified energetic properties. In this paper, in particular, it is shown how a variable structure control can be designed in this general framework in order to achieve a passive systems with, additionally, the robust properties obtainable with variable structure systems. Simulation results obtained with a 2-dof manipulator are reported and discussed in order to validate the proposed approach.

## 1 Introduction

Output regulation is one of the main challenges in controlling nonlinear dynamical systems, and several different methodologies have been proposed in the literature to tackle this problem, among many others see [3, 4]. Very often, control techniques validity depends on the kind of dynamical system to be controlled, and various model structures have been studied, e.g. Lagrangian models [7] and models affine in the input [3].

A powerful modeling language is the port Hamiltonian formalism, [13], that allows to model any *physical* system *explicitly* showing both energetic properties and dynamical invariants (Casimir functions). Moreover, a port Hamiltonian model of a system immediately reveals its passivity properties. Therefore, from the control point of view, global stability can be easily achieved by means of passivity-based control techniques, such as damping injection, [13, 9], and intrinsically stable control schemes both for telemanipulation, [10], and for haptic devices, [11, 2] have been developed. Furthermore, the port Hamiltonian formalism has been recently extended to infinite-dimensional systems, [6].

A useful control technique for the output regulation of port Hamiltonian systems is the so-called *energy shaping*. This technique can be fruitfully applied to systems described in this form since their energy properties appear explicitly in the model expression. It is well known that any physical system, with no forcing action, assumes a configuration in which its total energy function assumes a (possibly local) minimum value. Unfortunately, very rarely the minimum of the potential energy coincides with the desired configuration. The energy shaping

control technique overcomes this limitation: loosely speaking, it consists of two steps. In the first one, the total energy of the plant is *shaped* by means of a proper state feedback law in order to assign a minimum in the desired configuration (energy shaping), then dissipation effects are introduced in order to have a (globally) asymptotically stable equilibrium point (damping injection).

A possible drawback of the energy shaping technique is that an exact knowledge of the system physical parameters is needed to correctly shape the energy function: this is not always true in practical applications. A consequence is that the energy function does not assume its minimum in the desired configuration and some regulation errors are introduced. The aim of this paper is to overcome this drawback by using variable structure control techniques [12], which are intrinsically robust with respect to model uncertainties. Furthermore, the passivity (and therefore the stable behavior) of the overall system will be preserved and a possible saturation of the actuators will be taken into account by properly designing the controller.

This paper is organized as follows: in Sec. 2, a background on energy shaping techniques for port Hamiltonian systems is introduced and applied to an  $n$ -dof, fully actuated mechanical system. The well known PD + gravity compensation (PD + g) controller results to be a particular case of this control technique. In Sec. 3, it is shown how the saturation of the actuators can be taken into account without losing the passivity property of the proposed control scheme. Then, in Sec. 4, a variable structure algorithm that properly shapes the energy of the closed-loop systems and assures optimal regulation performances even in presence of model errors is presented. Simulation results are presented in Sec. 5 in order to prove the validity of the proposed control scheme, while comments and indications about future work are discussed in Sec. 6.

## 2 Background

Consider a port Hamiltonian system with dissipation (PHD system)

$$\begin{cases} \dot{x} &= [J(x) - R(x)] \frac{\partial H}{\partial x} + G(x)u \\ y &= G^T(x) \frac{\partial H}{\partial x} \end{cases} \quad (1)$$

with  $x \in \mathcal{X}$ ,  $u \in \mathcal{U} \subset \mathbb{R}^m$ ,  $y \in \mathcal{Y} \equiv \mathcal{U}^*$ , being  $\mathcal{U}^*$  the dual of  $\mathcal{U}$ , and  $H : \mathcal{X} \rightarrow \mathbb{R}$  the Hamiltonian (energy) function. Moreover, suppose that  $J(x) = -J^T(x)$  and that  $R(x) = R^T(x) \geq 0$ ,  $\forall x \in \mathcal{X}$ ;  $J(x)$  and  $R(x)$  are, respectively, the interconnection and the damping matrices.

Let  $x_d \in \mathcal{X}$  be a desired configuration in the state space. It is well known that, if it is possible to find a state feedback law  $u = \beta(x)$  such that the dynamics of the resulting closed-loop system is given by

$$\dot{x} = [J_d(x) - R_d(x)] \frac{\partial H_d}{\partial x}$$

where  $J_d(x)$  and  $R_d(x) > 0$  are *desired* interconnection and damping matrices, then the system can be (globally) regulated to  $x_d$  in a passive way if the *desired* energy function  $H_d(x)$  assumes a (global) minimum on  $\mathcal{X}$ . This procedure is called *Interconnection and Damping Assignment (IDA)*; more details can be found in [8].

Consider an  $n$ -dof fully-actuated mechanical system with generalized coordinates  $q \in \mathcal{Q}$ . If  $p = M(q)\dot{q} \in T^*\mathcal{Q}$  are the generalized momenta, with  $M(q)$  the inertia matrix, a PHD representation of this system can be obtained by assuming in (1)  $\dim(\mathcal{X}) = 2n$  and  $m = n$ , then defining  $x := [q \ p]^T$ ,  $H(q, p) := \frac{1}{2}p^T M^{-1}(q)p + V(q)$ , where  $V(q)$  is the potential energy, and, finally,

$$J = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \quad R = \begin{bmatrix} 0 & 0 \\ 0 & D(q, p) \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ B(q) \end{bmatrix}$$

with  $D(q, p) = D^T(q, p) \geq 0$  taking into account the dissipation effects. Moreover, assume  $\text{rank}(G) = n$ , since the mechanical system is fully actuated. These considerations lead to the following model

$$\begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & -D \end{bmatrix} \begin{bmatrix} \partial_q H \\ \partial_p H \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u \\ y = B^T \partial_p H \end{cases} \quad (2)$$

Suppose that  $q_d \in \mathcal{Q}$  is a desired configuration in the joint space. If the IDA control technique is applied, one of the possible feedback law that allows to regulate the mechanical system in  $q_d$  assumes the following form:

$$u = B^{-1} \left[ \frac{\partial V}{\partial q} - \frac{\partial H_a}{\partial q} - K_D \frac{\partial H}{\partial p} \right] \quad (3)$$

where  $H_a : \mathcal{Q} \rightarrow \mathbb{R}$  is chosen in order to have a (global) minimum in  $q_d$  and  $K_D = K_D^T \geq 0$  in order to have  $D + K_D > 0$ . Then, the closed-loop dynamics of (2) with input given by (3) becomes

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ -I_n & -(D + K_D) \end{bmatrix} \begin{bmatrix} \partial_q H_d \\ \partial_p H_d \end{bmatrix} \quad (4)$$

with

$$H_d(q, p) = \frac{1}{2}p^T M^{-1}(q)p + H_a(q) \quad (5)$$

The state feedback law (3) shapes the total energy by compensating the effect of the potential  $V(q)$  and by introducing a new potential  $H_a(q)$  with a minimum in the desired configuration; then, some damping is added in order to dissipate energy so that the new minimum can be reached. The most critical point in the

implementation of this control technique is that a perfect compensation of the original potential contribution is necessary in order to have a steady state regulation error equal to zero. This compensation requires a perfect knowledge of all the robot's parameters; if this is not the case, the mechanical system stops, but not in the desired configuration, as discussed in Sec. 4.

If in (3) it is assumed

$$H_a(q) = \frac{1}{2}(q - q_d)^T K_P (q - q_d) \quad (6)$$

with  $K_P = K_P^T > 0$ , then

$$u(q, p) = B^{-1} \left[ \frac{\partial V}{\partial q} - K_P (q - q_d) - K_D \dot{q} \right]$$

that is the well-known PD + g controller, [1, 5]. Its action can be interpreted as the effect of a set of  $n$  linear springs acting in the joint space with center of stiffness in  $q_d$ .

### 3 Saturated springs

As already pointed out at the end of Sec. 2, the PD + g controller can be interpreted as a set of  $n$  linear springs with center of stiffness in  $q_d$ . In order to extend this controller to take into account the saturation of each actuator, some non-linearity in the energy function of the springs can be introduced.

It is well known that a spring is an element storing potential energy whose behavior can be described by Fig. 1. The input

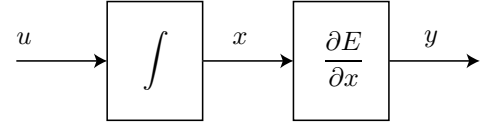


Figure 1: Energy storing element behavior

$u$  is the deformation rate of the extremum of the spring,  $x$  is the state associated to the spring and  $E(x)$  is a lower bounded function representing the stored energy. The output  $y$  is the force applied by the spring.

The simplest springs are the linear ones, i.e. springs whose energy function are quadratic:

$$E(x) = \frac{1}{2}x^T K x \quad (7)$$

where  $K = K^T > 0$  represents the *stiffness*. The force applied by the springs turns out to be:

$$f = \frac{\partial E}{\partial x} = K x$$

In the case of mechanical systems (e.g. robots), each component of the force is applied to the plant by means of an actuator. Intuitively speaking, if the amount of stored energy increases too much, then the force generated by the springs, that is the force that the actuators have to apply, can be greater than the

physical limits of the actuators themselves. If the robot is controlled by means of the PD + g controller, this situation can happen e.g. if the initial error is sufficiently high.

For simplicity, assume that  $K = \text{diag}(k_1, \dots, k_n)$ , that is the spring energy in (7) can be written as

$$E(x) = \sum_{i=1}^n E_i(x_i) = \frac{1}{2} \sum_{i=1}^n k_i x_i^2$$

Then, suppose that each actuator is limited, i.e.  $f_{i,m} \leq f_i \leq f_{i,M}$ ,  $i = 1, \dots, n$ . Consider  $x_M = (x_{1,M}, \dots, x_{n,M})$  and  $x_m = (x_{1,m}, \dots, x_{n,m})$  such that  $f_{i,M} = k_i x_{i,M}$  and  $f_{i,m} = k_i x_{i,m}$ . The saturation of each actuator can be taken into account if the following energy function is introduced:

$$E_s(x) = E_{1,s} + \dots + E_{n,s} \quad (8)$$

where

$$E_{i,s}(x_i) = \begin{cases} f_{i,m} [x_i - \frac{1}{2}x_{i,m}], & \text{if } x_i < x_{i,m} \\ \frac{1}{2}k_i x_i^2, & \text{if } x_{i,m} \leq x_i \leq x_{i,M} \\ f_{i,M} [x_i - \frac{1}{2}x_{i,M}], & \text{if } x_i > x_{i,M} \end{cases} \quad (9)$$

The passivity properties of the spring are preserved since the proposed energy function is  $C^1$  and bounded from below.

The energy function of a 1-dimensional spring and the relative force in function of the state are represented in Fig. 2 both for the non-saturated and saturated case. Clearly, the energy

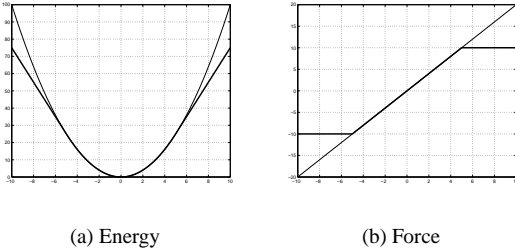


Figure 2: Energy and force of non saturated (continuous) and saturated (dashed) spring.

functions start to differ in the saturation zone. When the stored energy becomes infinite, the force generated by a non-saturated spring increases to infinity, while it remains limited for the saturated case.

The saturation of each actuator can be taken into account in the (passive) control of a robot if in (3) it is assumed

$$H_a(q) = \sum_{i=1}^n E_{i,s}(q_i - q_{i,d}) \quad (10)$$

where  $E_{i,s}(\cdot)$  is defined as in (9),  $k_i > 0$  can be freely assigned, and  $f_{i,m}, f_{i,M}$  depend on the characteristics of the  $i$ -th actuator. Since  $H_a(\cdot)$  is characterized by a (global) minimum in  $q_d$ , the control action (3) still assures the (global) stability of this configuration.

## 4 A variable structure algorithm for energy shaping

As already pointed out in Sec. 2, the closed loop dynamics of system (2) with input (3) is given by (4), where  $H_d$  is a function bounded from below. It is well known that

$$\frac{dH_d}{dt} = -\frac{\partial^T H_d}{\partial p} (D + K_D) \frac{\partial H_d}{\partial p} < 0 \quad (11)$$

until  $\frac{\partial H_d}{\partial p} = \dot{q} \neq 0$ . Since  $H_d$  is bounded from below, necessarily  $\dot{q}(t) = 0$  for some  $t = \bar{t}$ ; moreover, the possible configurations in which the robot stops are clearly given by the solutions of the following equation:

$$\left. \frac{\partial H_d}{\partial q}(q, p) \right|_{p=0} = 0 \quad (12)$$

or, in the case of perfect compensation of the potential  $V(q)$ , by:

$$\frac{\partial H_a}{\partial q}(q) = 0 \quad (13)$$

If  $H_a$  is characterized by a global minimum in  $q = q_d$ , e.g. as in (6) or in (10), then the robot reaches the desired configuration  $q_d$ .

The key point is that a perfect compensation of the original potential energy of the robot has to be implemented. If this is not the case, then some regulation errors will be present. Suppose that  $\hat{V}(q)$  is an estimate of the potential term in  $H(q, p)$ ; then, (3) becomes

$$u = B^{-1} \left[ \frac{\partial \hat{V}}{\partial q} - \frac{\partial H_a}{\partial q} - K_D \frac{\partial H}{\partial p} \right] \quad (14)$$

and the resulting closed loop dynamics is still given by (4), but with  $H_d$  now given by

$$H_d(q, p) = \frac{1}{2} p^T M^{-1}(q) p + H_a(q) - \Delta V(q) \quad (15)$$

where  $\Delta V(q) = \hat{V}(q) - V(q)$ . Since (11) holds, the final configurations the robot can assume are still solutions of (12), or, equivalently, of

$$\frac{\partial H_a}{\partial q} = \frac{\partial \Delta V}{\partial q} \quad (16)$$

Even if  $H_a$  is characterized by a (global) minimum in  $q_d$ , it is not sure that this configuration can be reached.

In order to make the control law (3~14) robust also in terms of performances with respect to unknown parameters,  $H_a$ , which is freely assignable, can be chosen with a variable structure. For example, assume

$$H_a(q) = \frac{1}{2} \sum_{i=1}^n k_i [q_i - q_{i,d} + \text{sign}(q_i - q_{i,d}) \bar{q}_i]^2 \quad (17)$$

where  $k_i > 0$  and  $\bar{q}_i > 0$ , with  $i = 1, \dots, n$ . It is possible to prove that, if

$$\left| \frac{\partial \Delta V}{\partial q} \right| \leq M < \infty \quad (18)$$

and if  $\bar{q}_i$ ,  $i = 1, \dots, n$ , are properly chosen, then the control law (14) with  $H_a$  given by (17), can drive the system in  $q = q_d$ . The proof is immediate in the case that a perfect compensation of the potential  $V(q)$  is possible, that is if  $\Delta V(q) = 0$ : in fact, in this situation,  $H_d$  is characterized by a global minimum in  $(q_d, 0)$ .

Suppose, then, that  $\Delta V(q) \neq 0$  and, in particular, that (18) holds and consider a generic initial condition  $(q_0, p_0)$ . Define  $\sigma := [\sigma_1, \dots, \sigma_n]$ , where

$$\sigma_i = \begin{cases} 1 & \text{if } q_{i,0} - q_{i,d} \geq 0 \\ -1 & \text{if } q_{i,0} - q_{i,d} < 0 \end{cases}$$

only depends on the initial condition. Then, assume that the control input  $u$  is given by (14), but with  $H_a$  given by:

$$H_a(q) = \frac{1}{2} \sum_{i=1}^n k_i (q_i - q_{i,d} + \sigma_i \bar{q}_i)^2 \quad (19)$$

If, with a proper choice of  $\bar{q}$ , this continuum control input can drive the robot in a final configuration  $q^*$  such that

$$\begin{cases} q^* - q_{i,d} < 0 & \text{if } q_{i,0} - q_{i,d} > 0 & (\sigma_i = 1) \\ q^* - q_{i,d} > 0 & \text{if } q_{i,0} - q_{i,d} < 0 & (\sigma_i = -1) \end{cases} \quad (20)$$

then an instant  $\bar{t}$  such that  $q(\bar{t}) - q_d = 0$  has to exist. Consequently, the variable structure controller resulting from (14) and (17) makes the configuration  $q = q_d$  globally attractive and, clearly, globally stable.

The final configuration  $q^*$  assumed if  $u$  is given by (14~19) are solution of (16), that is

$$q_i^* - q_{i,d} + \sigma_i \bar{q}_i = \frac{1}{k_i} \frac{\partial \Delta V}{\partial q}(q^*) \quad (21)$$

Since the values  $\bar{q}_i$ ,  $i = 1, \dots, n$  have to be chosen according to (20), it can be deduced that

$$\bar{q}_i > \frac{M}{k_i}, \text{ with } i = 1, \dots, n \quad (22)$$

With this choice, the configuration  $q = q_d$  is globally attractive and stable. In Fig. 3, the behavior of the proposed controller is presented.

The actuator saturation can be taken into account by introducing the saturated springs of Sec. 3. If  $E_{i,s}$  is the energy function of a saturated spring as reported in (9), suppose that

$$H_a(q) = \sum_{i=1}^n E_{i,s}[q_i - q_{i,d} + \text{sign}(q_i - q_{i,d}) \bar{q}_i] \quad (23)$$

Then, the final configurations the robot can assume if  $u$  is given by (14) are the solutions of (16). If

$$|\max(f_{i,m}, f_{i,M})| > M \left( \geq \frac{\partial \Delta V}{\partial q} \right) \quad (24)$$

$i = 1, \dots, n$ , then in the steady state configuration none of the actuators is in saturation. A consequence is that, if  $\bar{q}_i$ ,  $i =$

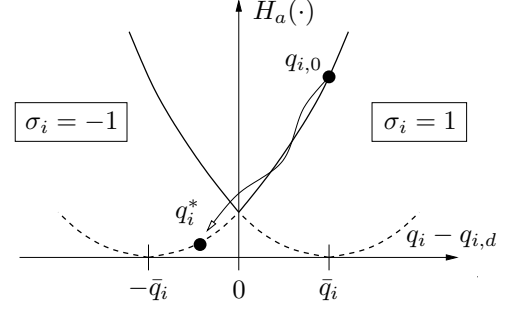


Figure 3: Behavior of the proposed controller. The initial error  $q_{i,0} - q_{i,d}$  is greater than 0, but  $\bar{q}_i$  is chosen in such a way that all the possible steady state configurations  $q^*$ , if  $H_a$  is given by (19), satisfy  $q_i^* - q_{i,d} < 0$ . If the variable structure of the controller deriving from (17) is adopted, then the system is *constrained* in  $q_d$ .

$L_1 = L_2 = 1 \text{ m}$	Links lengths
$L_{g1} = L_{g2} = 0.5 \text{ m}$	Center of mass
$M_1 = M_2 = 20 \text{ Kg}$	Links mass
$I_1 = I_2 = 5 \text{ Kg m}^2$	Links inertia
$D_1 = D_2 = 0 \text{ N m s}$	Viscous friction
$g = 9.81 \text{ m/s}^2$	Gravity acceleration

Table 1: Parameters of the considered manipulator.

$1, \dots, n$ , are chosen according to (22), then the controller is able to regulate the robot in  $q_d$  that will be an asymptotically stable configuration.

In conclusion, even in presence of modeling uncertainties, a variable structure passive controller (14), with  $H_a$  given by (17) or (23), if the saturation of the actuators is taken into account, is able to drive the system in the desired configuration.

## 5 Case study

In order to test the control algorithm presented in Sec. 3 and in Sec. 4, a 2 dof planar manipulator has been considered. The main parameters of the manipulator are reported in Tab. 1. The manipulator is subject to gravity force active in the negative  $y$  direction. In the following, some simulation results are reported in order to show the features of the proposed controller in comparison with the classical PD + gravity compensation regulator. As a reference case, a simulation with the PD + g compensation controller is reported. A fixed set-point  $p = (1.75, 0.1)$  has been assigned as desired goal for the tip of the manipulator, corresponding to joint positions  $q_1 = -0.4453 \text{ rad}$  and  $q_2 = 1.0048 \text{ rad}$ . In this case, the dynamic parameters are supposed to be perfectly known. As expected, the errors nicely tend to zero, as shown in Fig. 4. In this case, the control parameters are  $K_p = \text{diag}(6000, 6000)$ ,  $K_d = \text{diag}(1100, 1100)$ . The final errors are  $e_x = -0.0001$ ,  $e_y = 0.000064 \text{ (m)}$  corresponding to  $e_{q_1} = 0.0047$  and

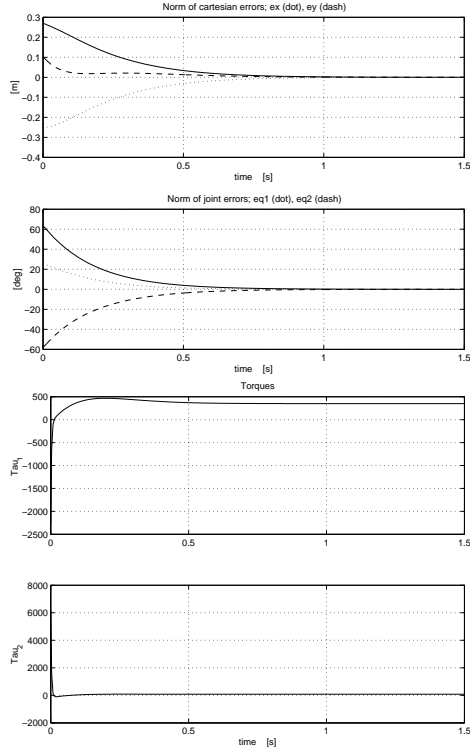


Figure 4: Simulation results with PD+g(q): errors and torques.

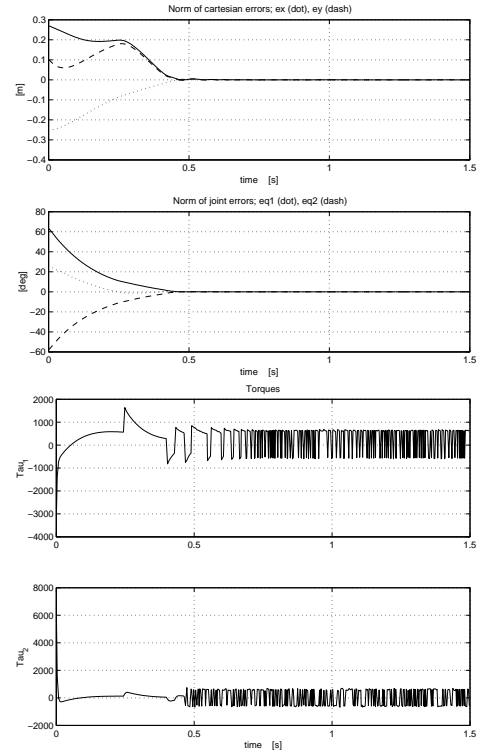


Figure 5: Simulation results with the proposed control scheme: errors and joint torques.

$$e_{q_2} = -0.0141 \text{ (deg)}.$$

Results obtained with the proposed controller are reported in Fig. 5, with the same control parameters as in the previous case for the PD part, i.e.  $K_p = \text{diag}(6000, 6000)$ ,  $K_d = \text{diag}(1100, 1100)$  while  $\bar{q} = \text{diag}(0.1, 0.1)$ . Also in this case, the desired configuration is reached without errors. Note the behavior of the torques: after a transient, when the errors are null, a switching behavior takes place in order to constrain the state on the desired configuration (corresponding to the minimum of  $H_a$ ). The final errors in this case are  $e_x = 1.1433e - 005$  and  $e_y = -1.9248e - 006$  (m), corresponding to  $e_{q_1} = -0.0006$  and  $e_{q_2} = 0.0013$  (deg).

If the robot parameters are not perfectly known, the PD + gravity compensation scheme is not able to reach the desired configuration. This case is shown in Fig. 6, where, as limit case, it is assumed that the parameters  $m_1$  and  $m_2$  are not known at all (i.e. the values  $m_1 = m_2 = 0$  are assumed). As expected, the robot reaches a different final configuration and the final errors are not null:  $e_x = -0.0098$ ,  $e_y = 0.1134$  (m),  $e_{q_1} = -3.2861$  and  $e_{q_2} = -0.84212$  (deg). The corresponding simulation with the proposed controller is shown in Fig. 6. Note that in this case the desired configuration is reached without errors. In this case, the final errors are  $e_x = -1.2406e - 006$ ,  $e_y = 0.000017$  (m),  $q_1 = -0.0005$  and  $q_2 = -0.00003$  (deg).

Finally, the case of saturation has been considered. A saturation value of  $800 \text{ Nm}$  has been considered for the actuators. Results obtained with the proposed controller (and no knowledge of

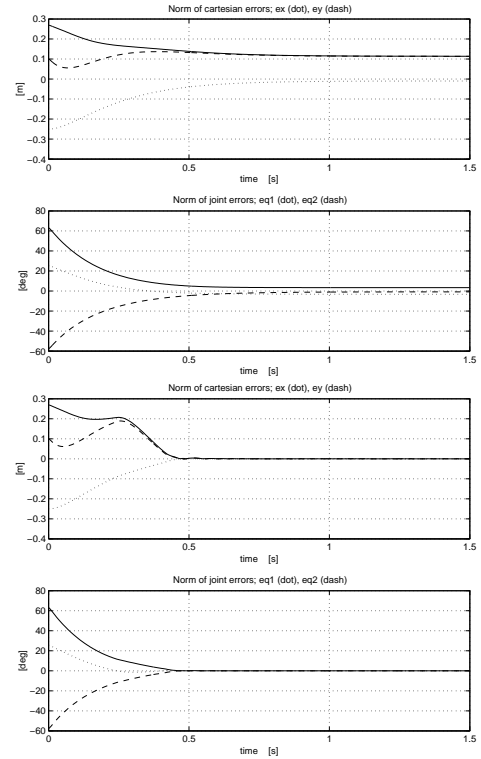


Figure 6: Simulation results with partial knowledge of mass parameters: PD + g controller (top) and proposed controller (bottom).

the parameters  $m_1$  and  $m_2$ ) are reported in Fig. 7. Errors in

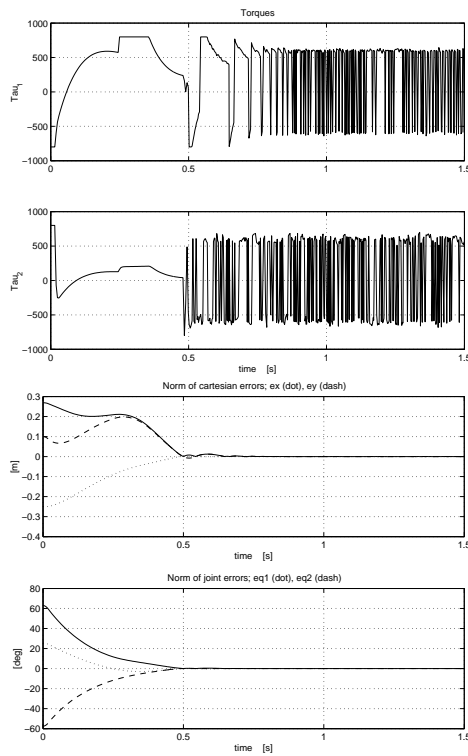


Figure 7: Simulation results with partial knowledge of mass parameters and saturation: torques and errors.

this case are  $e_x = -9.0561e - 006$ ,  $e_y = 1.2897e - 005$  (m),  $q_1 = 0.000056$  and  $q_2 = -0.00098$  (deg).

## 6 Conclusions and future work

In this paper, a variable structure control based on passivity and energy shaping considerations has been presented. Simulation results, obtained with a two-dof planar manipulator, show the validity of the proposed approach. Future work, besides a comparison with existing VS control law, will address the experimental verification of the proposed control technique on an industrial robot, as well as the application to the tracking problem.

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