FLIGHT CONTROL SYSTEM VALIDATION USING GLOBAL NONLINEAR OPTIMISATION ALGORITHMS

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Abstract

In this paper the results from a validation of the functionality of Maneuver Load Limiter in the longitudinal control system for the generic aircraft simulation model ADMIRE, using global optimisation algorithms are presented. The analysis is based upon the reformulation of the nonlinear time domain simulation-based stability criterion Clonk, into a global optimisation problem. Results from both a traditional grid-based search for the worst uncertainty parameter combination and optimisation-based search with algorithms based upon a Genetic Algorithm and Adaptive Simulated Annealing are presented.

1 Introduction

This study was conducted as a continuation of the work done previously within the GARTEUR project *FM(AG-11) New Analysis Techniques for Clearance of Flight Control Laws*. The aim of this study was to validate the robust functionality of the envelope protection system, or *Maneuver Load Limiter (MLL)*, implemented in the Flight Control System *(FCS)* of the nonlinear closed-loop simulation *ADMIRE*. The work presented here is a based upon previous work reported in [1], [2] and [3].

The part of the validation of the FCS within a Flight Clearance process conducted here, is based upon the application of the *Clonk Criterion*, [4], which was developed by SAAB in order to assess the proneness for departure of the Gripen aircraft, on the ADMIRE.

2 Analysis Method

The system that we will study here is defined through a set of ordinary nonlinear differential equations

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{p}), \quad \boldsymbol{x}(\boldsymbol{p}, t_0) = \boldsymbol{x}_0(\boldsymbol{p}) \tag{1}$$

defined by the functions $\boldsymbol{f}$, together with a set of output functionals

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{p}) \tag{2}$$

depending on the state vector $\boldsymbol{x} \in \mathcal{X}$, and a constant parameter vector $\boldsymbol{p}$ belonging to the admissible parameter space $\mathcal{P}$.

Given an output signal y from $\boldsymbol{y}(\boldsymbol{x}, \boldsymbol{p})$ the FCS validation analysis should reveal if there exists any parameter vector $\boldsymbol{p}^* \in \mathcal{P}$ and $\boldsymbol{x}^* \in \mathcal{X}$ such that

$$y^* = y(\boldsymbol{x}^*, \boldsymbol{p}^*) \geq y_{limit} \tag{3}$$

which can be regarded as a robust stability/performance problem for the system described by Eq. 1. The maximum value of y can then be found by solving

$$y^* = \max_{\boldsymbol{p} \in \mathcal{P}, \boldsymbol{x} \in \mathcal{X}} y(\boldsymbol{x}, \boldsymbol{p}) \tag{4}$$

together with the corresponding parameter vector $\boldsymbol{p}^*$ and the state vector $\boldsymbol{x}^*$, which means that the robust stability problem has been reformulated into a global nonlinear optimisation program.

In order to solve the transformed stability problem for the case study, described below, two different optimisation algorithms were used as search algorithms. *Genetic Algorithm (GA)* and *Adaptive Simulated Annealing (ASA)* are both heuristic optimisation algorithms that are capable of finding approximate solutions to a global optimisation problem depending upon cost function evaluations only.

GA is based upon ideas found within the evolutionary biology, and is an attempt to mimic what is called *the survival of the fittest*, i.e. the principle of natural selection hypothesised by Darwin. The optimisation is initiated with an initial population consisting of an uncertainty parameter vector $\boldsymbol{p}_i \in \mathcal{P}$. Here, the initial population $\boldsymbol{p}_i$ was randomly distributed over the admissible parameter space. Then genetic operators *(GO)* that are applied on the population perform the basic search algorithm. The GO that are used to generate new generations are *crossover* and *mutation*. Crossover uses two individuals and mutation acts on a single individual in order to create new members of the population. A probabilistic selection is performed, so that the better ones have a higher probability to be selected. The specific algorithm used here and its implementation in MATLAB/Simulink are described in [5]. A more general description of genetic algorithms and its application to different problems can be found in [6] and [7].

As the name of ASA indicates, the algorithm is designed to simulate the annealing process in a gas. The algorithm is initiated with a set of ”high energy” particles that are sampled randomly from the parametric uncertainty space $\mathcal{P}$. As the ”temperature” is decreased the possibility of the particles to move

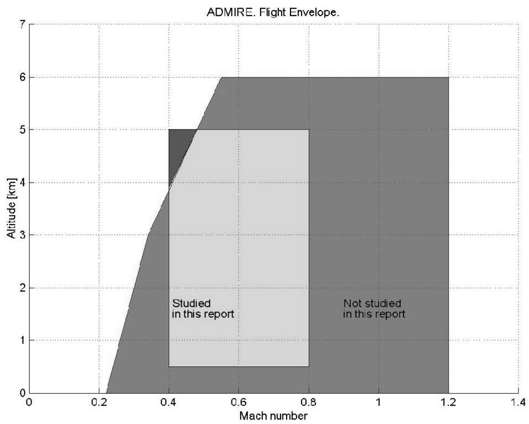

Figure 1: Analysed Flight Envelope (FE).

from one local minimum to another is reduced. This forms the principal search algorithm. ASA permits an annealing schedule for the "temperature" decreasing exponentially with time. The introduction of "re-annealing" also permits adaptation to changing sensitivities in the multi-dimensional parameter space. The algorithm is described in more detail in [8].

For a general overview of new heuristic optimisation algorithms and thier application to different problems, see [9].

3 Case Study

As a case study the closed loop nonlinear simulation model of a generic fighter aircraft, ADMIRE, was selected. A detailed description of the model and its implementation can be found in [10]. In Fig. 1, the part of the model FCS design envelope that was analysed is shown.

In order to have a MLL functionality of the ADMIRE, the original longitudinal FCS was replaced by a new controller developed in a thesis work at LiTH, [11]. In order to be able to assess the robustness of the controller, a set of parametric model uncertainties and their corresponding limits where defined, see Tab. 1.

Parameter	Limits	Description
M	$[0.4; 0.8]$	Mach number
h	$[500; 5000]$	Altitude [m]
$\delta mass$	± 0.1	Var.in aircraft mass [%]
δx_{cg}	± 0.075	Var. in pos. of the centre of mass [m]
$\delta Cm_{\delta e}$	± 0.005	Unc. in pitch mom. due to elev. defl. [1/rad]
$\delta Cl_{\delta a}$	± 0.005	Unc. in roll mom. due to aileron defl. [1/rad]
δCm_{α}	± 0.05	Unc. in pitch. mom. due to AoA [1/rad]
δI_{yy}	± 0.025	Var. in aircraft inertia around y-axis. [%]
δI_{xz}	± 0.01	Var. in product of inertia. [%]
δCm_{q}	± 0.05	Unc. in pitch. mom. due to pitch rate. [-]
δCl_{β}	± 0.02	Unc. in roll mom. due to side-slip. [1/rad]
δCl_{p}	± 0.05	Unc. in roll mom. due to roll rate. [-]
δCn_{β}	± 0.02	Unc. in yaw mom. due to side-slip. [1/rad]
δCn_{p}	± 0.05	Unc. in yaw mom. due to roll rate. [-]
δM_{err}	± 0.04	Error in Mach number sensor. [-]
$\delta \alpha_{err}$	± 0.02	Error in angle of attack sensor [rad]

Table 1: Model Uncertainty Parameters.

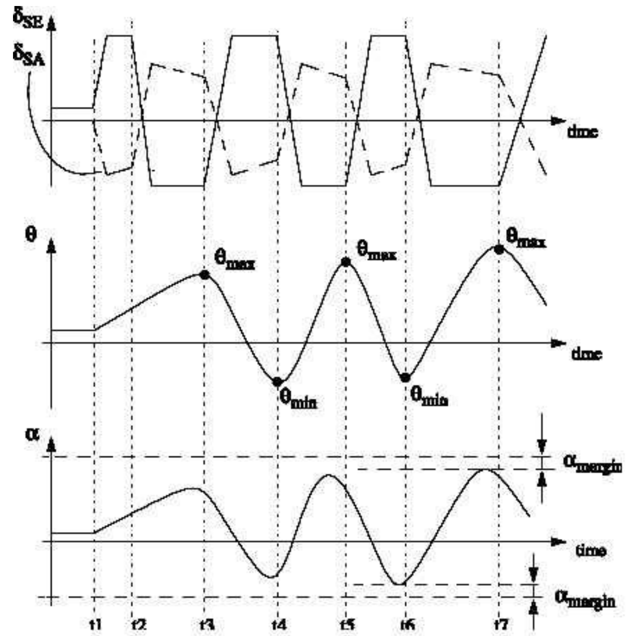

Figure 2: Description of time domain based Clonk Criterion.

As a criterion for the validation of the MLL functionality of the FCS in ADMIRE, the Clonk Criterion, developed by SAAB, was selected. The criterion is a nonlinear closed loop simulation-based, time domain stability test, with the dynamics of both the lateral and longitudinal modes of the aircraft involved, see [4]. The Clonk Criterion is applied, for a given altitude and Mach number, as a combination of lateral and longitudinal stick displacements, see Fig. 2, during the simulation, whereupon the maximum angle of attack (α) and load factor (n_z) are checked against predefined limits.

In accordance with the optimisation-based analysis method described above, it is possible to reformulate the nonlinear stability analysis based upon the Clonk Criterion into a Nonlinear Program. The question to be answered is, in which part of the FE and for what combinations of uncertainty parameters will α and n_{z} be maximised?

Two different problems were defined for the optimisation-based validation of the MLL functionality in the FCS of the model ADMIRE.

$$n_{\mathrm{z}}^* = \max_{\substack{\boldsymbol{p}\in\mathcal{P} \\ M,h\in\mathcal{M},\mathcal{H}}} n_{\mathrm{z}}(M,h,\boldsymbol{p}) \tag{5}$$

and

$$\alpha^* = \max_{\substack{\boldsymbol{p}\in\mathcal{P} \\ M,h\in\mathcal{M},\mathcal{H}}} \alpha(M,h,\boldsymbol{p}) \tag{6}$$

where the functions α and n_{z} are the output signals from the integration of the nonlinear simulation ADMIRE over the time $t \in [0..15\,\mathrm{s}]$ within the Flight Envelope defined by $\mathcal{M},\mathcal{H}$, described in Fig. 1.

The analysis has been performed for three different cases; in the first case (*nominal*) only M and h were altered, the second case (*reduced*) involved only the first five parameters defined in Tab. 1, plus M and h. Finally, the full set of uncertainty parameters were used in the third case (*complete*).

4 Results

The two different problems were solved for three different sets of uncertainty parameters, a *nominal*, a *reduced* and the *complete* set of parameters using the two different search algorithms, GA and ASA, plus a traditional search based upon grid points, referred to as the *Baseline Solution* (BS). This resulted in fourteen different cases which are put together in Tab. 2.

It can be seen that for the nominal model, cases 1-6, none of the search methods has found any flight condition (M, h) where the limits in in α and n_{z} are exceeded, although GA and ASA have located solutions that are closer to the limits than BS. The BS was conducted by the use of grid-points in FE; in h $(500, 1000, 2000, 3000, 4000, 5000\,\mathrm{m})$ and M $(0.4, 0.5, 0.6, 0.7, 0.8)$. The two different solutions found by GA and ASA are approximately the same.

In the reduced cases, see cases 7-12 in Tab. 2, M and h were augmented by the first five uncertainty parameters: $\delta mass$, δx_{cg}, $\delta Cm_{\delta e}$, $\delta Cl_{\delta a}$ and δCm_{α}, defined in Tab. 1. The BS, cases 7-8, was conducted in a similar way as for cases 1-2, except that the involved uncertainties were applied using their extreme values in permutation. In Tab. 2, cases 9-12, it can be seen that n_{z} has a maximum in the region $0.4 < M < 0.5$ and altitude $1680\,\mathrm{m}$ and a maximum α at an altitude $2200\,\mathrm{m}$. The reduced set analysis reveals that $\alpha_{\max}$ has a maximum when $\delta mass$, δx_{cg}, $\delta Cm_{\delta e}$ and δCm_{α} are maximised and $\delta Cl_{\delta a}$ is at its minimum. The highest value of n_{z} will occur when $\delta mass$ and $\delta Cl_{\delta a}$ are minimised and δx_{cg}, $\delta Cm_{\delta e}$ and δCm_{α} are maximised.

In the complete cases, only the ASA algorithm was used, due to the fact that ASA is implemented in C-code and GA is entirely implemented in MATLAB, which affects the required execution time. The execution time required to solve the different optimisation problems can be found in Tab. 3. The effects of the implementation can be seen from the fact that the GA requires more than the double time of ASA to solve a specific case.

No analysis using BS were conducted for the same reason. For the complete analysis, case 13 and 14, all uncertainties plus M and h were allowed to vary within the prescribed limits. It can be noted that for case 13, the value of $\alpha_{\max}$ is $38.0°$ compared to $33.2°$ in case 11. The finding indicates that the FCS of the closed loop simulation ADMIRE is specially sensitive to errors in air-data sensors. The same result can be noted for $n_{\mathrm{z}_{\max}}$ at 11.10 (case 14) in comparison with 9.84 for case 12. During the process of evaluation it was evident that the largest influences on the result were from the change in δM_{err}. In Figs 3 and 4 the time histories of the cases 12 and 14 are shown.

Case	Alg.	Mach	Alt [m]	$\alpha_{\max}$	$n_{\mathrm{z}_{\max}}$	$\delta mass$	δx_{cg}	$\delta Cm_{\delta e}$	$\delta Cl_{\delta a}$	δCm_{α}	δI_{yy}	δI_{xz}	δCm_q	δCl_{β}	δCl_p	δCn_{β}	δCn_p	δM_{err}	$\delta \alpha_{err}$
						±0.1	±0.075	±0.005	±0.005	±0.05	±0.025	±0.01	±0.05	±0.02	±0.05	±0.02	±0.05	±0.04	±0.02
1	BS	0.4	3000	22.53															
2		0.7	1000		8.12														
3	GA	0.4272	2550	22.85															
4		0.4664	1607		8.47														
5	ASA	0.4277	2545	22.85															
6		0.4676	1657		8.47														
7	BS	0.4	3000	30.13		0.1	0.075	0.005	-0.005	0.05									
8		0.7	2000		9.50	-0.1	0.075	0.005	-0.005	0.05									
9	GA	0.4435	2231	33.31		0.1	0.075	0.005	-0.005	0.05									
10		0.4685	1690		9.88	-0.1	0.075	0.005	-0.005	0.0499									
11	ASA	0.4455	2170	33.15		0.095	0.075	0.0049	-0.005	0.0499									
12		0.4687	1679		9.84	-0.094	0.0738	0.005	-0.002	0.05									
13	ASA	0.4002	2143	38.03		0.0941	0.0748	0.0049	-0.0005	0.0499	-0.0192	-0.0036	-0.0359	-0.0141	-0.0335	0.017	0.0319	-0.0389	-0.0175
14		0.5008	2413		11.10	-0.0991	0.0657	0.0046	0.0038	0.0497	-0.0110	-0.0090	0.0222	-0.0004	0.0256	-0.0104	-0.0047	-0.0395	0.0015

Table 2: Additional Combination of uncertainty parameters.

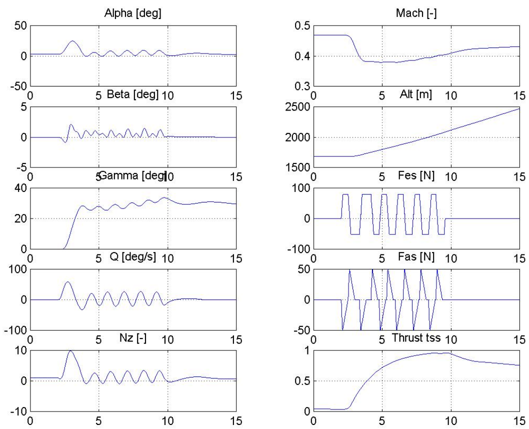

Figure 3: Seven uncertainty parameters, case 12.

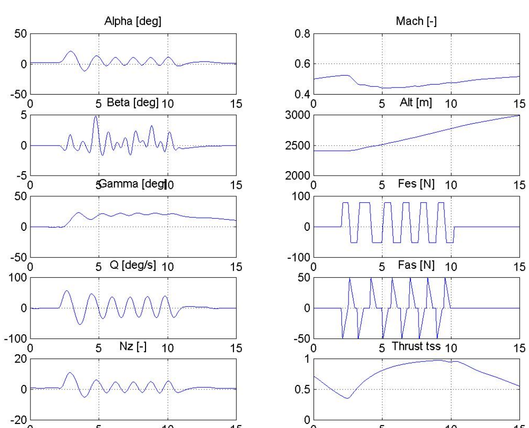

Figure 4: Sixteen uncertainty parameters, case 14.

Case	Alg.	No. par.	Time [s]	no. of it.
1	BS	2	~ 600	30
2			~ 600	30
3	GA		15592	731
4			15505	732
5	ASA		6073	317
6			6329	317
7	BS	7	~ 43000	2160
8			~ 43000	2160
9	GA		17430	737
10			15924	722
11	ASA		7345	392
12			7455	392
13	ASA	16	13633	716
14			12709	716

Table 3: Time required to solve the Flight Clearance problem.

5 Conclusions

The results presented here indicate the advantage of using optimisation-based search instead of traditional grid-based search while conducting validation of Flight Control Laws. The use of the proposed analysis method is particularly useful when applied to criteria based upon nonlinear simulations in the time domain. Some details in the results should be noticed:

- The result obtained by the use of optimisation-based search are located between the grid-points of the traditional search, which indicates that the proposed analysis method has a higher reliability.

- The difference in the time required for the algorithms to converge into a solution, is explained by the fact that ASA is implemented i C-code and called from MATLAB, while GA is implemented entirely in MATLAB.

- Both optimisation-based algorithms found solutions that are equivalent.

- The time required to perform a grid-based search increases dramatically with the number of uncertainty parameters. Here, it was only possible to perform the grid-based search for the nominal and reduced model.

- The advantage of optimisation-based search compared to a grid-based will increase as the size of the problem increases.

Our analysis has shown that nonlinear optimisation algorithms can be successfully employed to perform the robust stability analysis of a class of nonlinear simulation-based multi-axis stability criteria. Although no global search algorithm can guarantee that the approximative solution found is global and not just a local extreme point, the algorithms like GA and ASA have a much higher reliability than traditional local algorithms, e.g. gradient-based, though to a price of involving higher computational costs.

The method has also a potential to search for worst-case combinations of generic (or parametric) piece-wise continuous pilot input signals, where the optimisation algorithm is allowed to vary the duration and rate of the different stick inputs. This could produce general series of signals that are potentially dangerous to the system. Finally, the proposed optimisation-based analysis method described here can alternatively be used, given a suitable metric of the problem, for sensitivity analysis of dynamical systems.

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