NEURO-MECHANICAL MODELLING AND CONTROL OF WINDING PROCESSES

P.Kabore, H.Wang, W. Hamad and H. Jaafar

Control Systems Centre, UMIST, Manchester, M60 1QD, UK, email:-hong.wang@umist.ac.uk

Keywords: Partial differential equation, neural network, dynamic model, nonlinear system, winding processes, Kalman observer.

Abstract

This paper presents a novel modelling and control algorithm for systems which are governed by partial differential equations. Different from existing distributed parameter systems so far discussed in literature, the control input of the system considered in this paper appears only inside the boundary conditions. The purpose is to control the boundary condition so as to realize the total distribution control of the system output. For this purpose, an orthogonal B-spline neural network model is used to approximate the solution of the system. By substituting this B-spline model into the original partial differential equation and letting the residual orthogonal to all the selected basis functions, the dynamics of the weights of the B-spline model are obtained. Such a dynamics relates the input to the weights of the B-spline model through an ordinary differential equation, where the controller designed only needs to be focused on the control of the weights because all the basis functions are fixed. This leads to the design of an observer based controller which realizes the profile control of the system output with guaranteed stability. The control algorithm is applied to a winding process in the paper making industry, where the model of the winding process is developed and the corresponding controller is derived. Some simulations are provided to show the effectiveness of the control algorithm.

1.Introduction

Control of infinite dimensional systems has long been regarded as an important area of research in control engineering and practice([3, 5, 2]). This is simply because there are many systems whose dynamic model can only be expressed as a set of partial differential equations (PDE). Examples are information flow, transmission process, power transmission systems and 3D temperature control systems. The model of the system is represented by a set of partial differential equations that link dynamically the system state or output to a control input vector. A set of boundary conditions are also required to define the system. In most existing techniques([3, 5, 2]),control input appears directly in the partial differential equation that describes the dynamic behaviour of the system. For such models, techniques developed for ordinary differential equation (ODE) systems have been used. For example, Galerkin's method has been used to formulate a finite-dimensional system that describes the dominant dynamics of the original PDE system for parabolic PDE systems ([5]), where Lyapunov design can be directly used. Recently, H^∞ method has been generalized to characterize the 2-norm for both the signal in space and signal in time for LTI spatially distributed systems ([12]), where sensors and actuators have been treated as the part of the controller. In this approach, the control design obtained is similar to those used in H^∞ design for ODE systems.

However, in some practical systems, the control input only applies to the boundary condition. Examples are the winding processes ([15]) in the paper, printing and film industries. For these types of systems, it is necessary to develop effective modelling and control strategies.
Indeed, in winding processes the roll quality depends on variables such as winding tension of the web, the speed and acceleration of winding. These variables influence the internal stresses that are expressed by a 4-D variable which depends on the 3-D position inside the roll and is also time dependent ([15]). A good roll should have a uniform distribution of the internal stress. As can be seen in figure 1, during winding the incoming web is added as new layers with an input tension (i.e., the control input signal). This tension acts only on the edge of the roll and appears inside a boundary condition.

In this study, we consider a class of distributed parameter systems that are represented by a linear PDE with a boundary condition that contains the control input. The model is extracted from surface winding processes ([7, 15, 11]). The proposed method consists of combining the physical model and a neural network where an orthogonal B-spline neural network ([6]) is used to provide an approximate solution to the model. In this context, the initial model of the system is transformed into a finite dimensional linear time-varying model, where the boundary conditions are expressed in terms of constraints. This is then followed by the design of an observer based controller. The paper is organized as follows. In section 2, the problem formulation and the theoretical results are presented. In section 3 the design of a controller for winding processes is addressed. In section 4, the modelling details of winding processes are given. Also, the control problem is derived following the methodology presented in section 2 and 3. In section 5, a simulation case study is proposed to show the effectiveness of the method.

2.Some preliminaries

In this paper, we consider systems which are described by the following Partial Differential Equations (PDE):

$$\frac{\partial^2 v}{\partial t^2} = \alpha_1(t,r)\frac{\partial^2 v}{\partial r^2} + \alpha_2(t,r)\frac{\partial v}{\partial r} + \alpha_3(t,r)v(r,t) \quad (1)$$

where $r \in [r_0, r_1]$ is a variable (e.g., in winding processes, r is the radius of the roll), α_1, α_2 and α_3 are known continuous nonlinear functions with respect to (r,t), $v(t,r)$ is the output of the system whose distribution along r is to be controlled through the following boundary conditions:

$$\beta_1(t)\frac{\partial v}{\partial r}(r_1,t) + \beta_2(t)v(r_1,t) = u(t) \quad (2)$$

$$v(r_0,t) = \int_{r_0}^{r_1}(\delta_1(t,r)\frac{\partial v}{\partial r} + \delta_2(t,r)v(r,t))dr \quad (3)$$

where $u(t) \in \mathbb{R}^1$ is the control input of the system, $\beta_1(t)$ and $\beta_2(t)$ are the known time-varying parameters, δ_1, δ_2 are known continuous functions with respect to (r,t). It can be seen that the system represented by (1)-(3) belongs to distributed parameter systems [3]. However, different from all the existing studies on the distributed parameter systems ([3, 12, 5]), the control input $u(t)$ only appears explicitly in the boundary conditions of the system as shown in equation (2) rather than in equation (1). It will be shown that the winding processes to be discussed can be represented by such systems.

This systems falls into the study of parameter nonlinear infinite dimensional systems. Although linear infinite dimensional systems have been thoroughly studied [3, 12, 5], the focus here is on the use of a B-spline neural network as an alternative way to model such systems. For this purpose, consider a solution to (1) as

$$V(r,t) = \sum_{i=1}^{N} x_i(t)B_i(r) \quad (4)$$

where $B_i(r)$, $(1 \le i \le N)$ are the pre-specified and uniformly bounded basis functions defined on $[r_0, r_1]$, the $x_i(t)$ are the associated weights of the neural network. N is the number of basis functions which, when being selected properly, can ensure that $V(r,t)$ will represent a good approximation of $v(r,t)$ (see also [10]). In this context, the following assumption is made.

A1: $\forall \epsilon > 0$; $\exists N > 0$ so that $\forall t$,

$$\|v(r,t) - \sum_{i=1}^{N} x_i(t)B_i(r)\|^2 =$$

$$\int_{r_0}^{r_1} p(r)(v(r,t) - V(r,t))^2 dr < \epsilon$$

where $p(r) > 0$ is a pre-specified weighting function.

By substituting $V(r,t)$ into the original system (1), a dynamical relationship between $x_i(t)$ and $u(t)$ can be obtained. Without the loss of generality, it is assumed that N is large enough and all the basis functions $B_i(r)$ are orthogonal. By substituting $V(r,t)$ into (1), the following residual can be obtained:

$$\begin{aligned} R(r,t) &= \sum_{i=1}^{N}(\ddot{x}_i(t)B_i(r) - x_i(t)(\alpha_1(r,t)\frac{d^2 B_i}{dr^2}(r) + \\ &\quad \alpha_2(r,t)\frac{dB_i}{dr}(r) + \alpha_3(r,t)B_i(r))) \end{aligned} \quad (5)$$

A good approximation of $V(r,t)$ to $v(r,t)$ means that $R(r,t)$ should be made as small as possible. This can be realized by making $R(r,t)$ orthogonal to all the basis function, leading to the following condition.

$$\int_{r_0}^{r_1} p(r)R(r,t)B_j(r)dr = 0, \quad j = 1, \ldots, N \quad (6)$$

This condition is in line with the Gallerkin orthogonality property of $R(r,t)$ ([4]) in the functional space spanned by the basis functions $B_j(r)$. By applying condition (6) to equation (5), the dynamic equation for the weights $x_i(t)$ can be formulated to give

$$\begin{aligned} \ddot{x}_i(t) &= \frac{1}{\|B_i\|^2}\sum_{l=1}^{N} x_l(t)b_{lj}(t) \\ b_{lj}(t) &= \int_{r_0}^{r_1} p(r)(\alpha_1(r,t)\frac{d^2 B_l}{dr^2}(r) + \alpha_2(r,t)\frac{dB_l}{dr}(r) + \\ &\quad \alpha_3(r,t)B_l(r))B_j(r)dr \end{aligned} \quad (7)$$

Rearranging this equation into a state space format leads to the following system:

$$\dot{X} = \begin{pmatrix} \dot{X}_1 \\ \dot{X}_2 \end{pmatrix} = AX, A = \begin{pmatrix} 0_{N\times N} & I_{N\times N} \\ A_1 & 0_{N\times N} \end{pmatrix} \quad (8)$$

where $A_1 = [a_{lj}(t)]_{1\le j,l \le N}$, $a_{lj}(t) = \frac{1}{\|B_i\|^2}b_{lj}(t)$ and $X_1 = [x_1, \ldots, x_N]$ $X_2 = \dot{X}_1$. Simimarly, substituting $V(r,t)$ into the boundary conditions (2)-(3) leads to the following approximations:

$$\begin{aligned} u(t) &= C_1(t)X(t) = \bar{C}_1 X_1, 0 = C_2(t)X(t) = \bar{C}_2 X_1 \\ C_1(t) &= [C_{11} \ \ldots \ C_{1N} \ \ 0 \ \ldots \ 0] = [\bar{C}_1 \ \ 0_{N\times N}] \end{aligned} \quad (9)$$

where

$$\bar{C}_{1i}(t) = C_{1i}(t) = \beta_1(t)\frac{dB_i(r)}{dr}|_{r=r_1} + \beta_2(t)B_i(r_1) \quad (10)$$

$$C_2(t) = [C_{21} \ \ldots \ C_{2N} \ \ 0 \ \ldots \ \ 0] = [\bar{C}_2 \ \ 0_{N\times N}] \quad (11)$$

$$\begin{aligned} \bar{C}_{2i}(t) = C_{2i}(t) = B_i(r_0) - \int_{r_0}^{r_1}(\delta_1(r,t)\frac{dB_i}{dr} + \\ \delta_2(r,t)B_i(r))dr \ \ i = 1, \ldots, N \end{aligned} \quad (12)$$

In general, the boundary value of $v(r_0,t)$ is measurable through $y(t) = \gamma(t)v(r_0,t)$, where $\gamma(t)$ is the known gain of the sensor. Replacing $v(r_0,t)$ by its approximate $V(r_0,t)$ leads to

$$\begin{aligned} Y_3(t) &= C_3(t)X(t) \\ C_3(t) &= \gamma(t)[B_1(r_0) \ \ \ldots \ \ B_N(r_0) \ \ 0_{N\times N}] \end{aligned} \quad (13)$$

This implies that the dynamics of the weights of the B-spline model should satisfy

$$\begin{cases} \dot{X} = AX \\ Y = \begin{pmatrix} u \\ 0 \\ Y_3 \end{pmatrix} = CX = \begin{pmatrix} C_1 X \\ C_2 X \\ C_3 X \end{pmatrix} \end{cases} \quad (14)$$

This indicates that using orthogonal basis functions and the Gallerkin orthogonality property of $R(r,t)$ [4], equations (1)-(3) can be transferred into an ordinary differential equation (14), where the weights of the B-spline model are directly related to the control input. Since all the basis functions are fixed, the weights determine the shape of $v(r,t)$. This means that it is possible to control the shape of $v(r,t)$ by controlling the weights vector $X(t)$. In this context, the task is to design a controller for system (14) so that $v(r,t)$ is made as close as possible to a given pre-specified $V^*(r,t)$.

The following lemma gives the relationship between $v(r,t)$ and the weights vector $X_1(t)$:

Lemma Suppose that assumption A1) holds, then the solution $v(r,t)$ of (1) and its approximation $V(r,t)$ satisfy:

$$\lambda_{max}(B)\|X_1\|^2 \leq \|v(r,t)\|^2 < \epsilon + \lambda_{min}(B)\|X_1\|^2 \quad (15)$$

where $\epsilon > 0$ is given in A1), $B = Diag(\|B_1\|^2, \ldots, \|B_N\|^2)$, $\lambda_{min}(B) = min\{\|B_i\|^2, \ i = 1, \ldots, N\}$ $\lambda_{max}(B) = max\{\|B_i\|^2, \ i = 1, \ldots, N\}$.

Proof To prove this lemma, we use the fact that $0 \leq \|v(r,t) - \sum_{i=1}^{N} x_i(t)B_i(r)\|^2 < \epsilon$ as provided by assumption A1). By calculating $\|v(r,t) - \sum_{i=1}^{N} x_i(t)B_i(r)\|^2$, it can be obtained that:

$$\begin{aligned}
&\int_{r_0}^{r_1} p(r)(v(r,t) - V(r,t))^2 dr = \\
&\int_{r_0}^{r_1} p(r)v^2(r,t)dr - 2\sum_{i=1}^{N} x_i(t)\int_{r_0}^{r_1} p(r)v(r,t)B_i(r)dr \\
&+ \int_{r_0}^{r_1} p(r)(\sum_{i=1}^{N} x_i(t)B_i(r))^2 dr + \|v(r,t)\|^2 \\
&-2\sum_{i=1}^{N} x_i(t)\int_{r_0}^{r_1} p(r)v(r,t)B_i(r)dr + \\
&\sum_{i=1}^{N} x_i^2(t)\|B_i(r)\|^2 \quad (16)
\end{aligned}$$

The last equality is obtained by using the orthogonality of the $B_i(r)$. This leads to:

$$\begin{aligned}
&\int_{r_0}^{r_1} p(r)(v(r,t) - V(r,t))^2 dr = \|v(r,t)\|^2 \quad (17)\\
&- \sum_{i=1}^{N} x_i^2(t)\|B_i(r)\|^2 \\
&= \|v(r,t)\|^2 - X_1^T B X_1
\end{aligned}$$

Therefore, the following inequality can be obtained

$$X_1^T B X_1 < \|v(r,t)\|^2 < \epsilon + X_1^T B X_1 \quad (18)$$

and inequalities (15) hold.

3.Controller design

The purpose of controller design is to find $u(t)$ so that $v(r,t)$ is made to follow a pre-specified distribution $V^*(r,t)$. Since the control input does not appear directly in the dynamics of X in (14), an observer is needed for system (14) so that the available measurement from the process can be used to estimate the actual ”equivalent” weights of B-spline neural network. This means that the adaptation of the weights of the neural network is provided by the observer. One possible choice of such an observer for system (14) is the following Kalman observer [1]

$$\begin{aligned}
\dot{\hat{X}} &= \tilde{A}(t)\hat{X} + K(t)\begin{pmatrix} u(t) \\ 0 \\ y(t) \end{pmatrix} \quad (19)\\
\tilde{A}(t) &= A(t) - K(t)C(t) \quad (20)
\end{aligned}$$

where K is the gain of the observer (see Appendix and [1]). If this system converges exponentially towards the approximate system (14), it remains to verify that under an appropriate control input $u(t)$, the observer will converge towards the real system. Using the same group of the basis functions, the target distribution $V^*(r,t)$ can also be expressed as $V^*(r,t) = C_r(r)X^*(t)$, where $C_r(r) = [C_{r1}(r), 0_N] \in R^{2N}$ and $C_{r1}(r) = [B_1(r), \cdots, B_N(r)]$. $X^*(t)$ is the desired weight vector that should satisfy:

$$\begin{cases} \dot{X}^*(t) = A(t)X^*(t) \\ Y^*(t) = \begin{pmatrix} u^* \\ 0 \\ y^* \end{pmatrix} = C(t)X^* = \begin{pmatrix} C_1X^* \\ C_2X^* \\ C_3X^* \end{pmatrix} \end{cases} \quad (21)$$

Indeed, the first N target weights, X_1^*, can be calculated from

$$X_1^*(t) = M^{-1}\int_{r_0}^{r_1} C_{r1}^T(r)V^*(r,t)dr \quad (22)$$

where the matrix

$$M = \int_{r_0}^{r_1} C_{r1}(r)C_{r1}^T(r)dr \quad (23)$$

is invertible because the basis functions are orthogonal. Since all the basis functions are uniformly bounded, matrices $C(t)$, $C_r(r)$ and $A(t)$ are also uniformly bounded. Using these preliminaries, the main result can be summarized by the following theorem.

Theorem Suppose assumption A1) is satisfied, then the following controller:

$$\begin{cases} \dot{\xi}(t) = \tilde{A}(t)\xi(t) + K(t)\begin{pmatrix} u^*(t) \\ 0 \\ y^*(t) \end{pmatrix} \\ u(t) = C_1(t)\xi(t) \end{cases} \quad (24)$$

together with the observer

$$\dot{\hat{X}}(t) = \tilde{A}(t)\hat{X}(t) + K(t)\begin{pmatrix} u(t) \\ 0 \\ y(t) \end{pmatrix} \quad (25)$$

realizes

$$\lim_{t\to\infty} \|v(r,t) - v^*(r,t)\| \leq \varrho\epsilon$$

where $u^*(t) = C_1(t)M^{-1}\int_{r_0}^{r_1} C_{r1}^T(r)V^*(r,t)dr$, $y^*(t) = C_3(t)M^{-1}\int_{r_0}^{r_1} C_{r1}^T(r)V^*(r,t)dr$, $\varrho > 0$ and ϵ is the error of approximation given in assumption A1.

Proof In order to prove the theorem, the error vectors are defined as $e_1(t) = \hat{X}(t) - X(t)$, $e_2(t) = \hat{X}(t) - \xi(t)$ and $e_3(t) = \xi(t) - X^*(t)$. By taking the observer gain as $K = S_\theta^{-1}C^T$ ([1]), the first order derivatives of e_1, e_2 and e_3 become:

$$\dot{e}_1(t) = (A - S_\theta^{-1}C^TC)e_1 - S_\theta^{-1}(C_1^TC_1(-e_1+e_2) + C_3^T(C_3X - y)) \quad (26)$$

$$\dot{e}_2(t) = (A - S_\theta^{-1}C^TC)e_2 - S_\theta^{-1}((C_1^TC_1 + C_3^TC_3)e_3 + C_3^TC_3e_2 - C_3^TC_3e_1 + C_3^T(y - C_3X)) \quad (27)$$

$$\dot{e}_3(t) = (A - S_\theta^{-1}C^TC)e_3 - S_\theta^{-1}(\bar{C}_1^T\bar{C}_1 + \bar{C}_3^T\bar{C}_3)(\bar{X}_1^* - X_1^*) \quad (28)$$

where X_1^* is defined in equation (22) and $\bar{X}_1^* = M^{-1}(t)\int_a^b C_r^T(r)V^*(r,t)dr$. As such, equation (28) becomes:

$$\dot{e}_3(t) = (A - S_\theta^{-1}C^TC)e_3 - S_\theta^{-1}(\bar{C}_1^T\bar{C}_1 + \bar{C}_3^T\bar{C}_3)M^{-1}(t)\int_a^b C_r^T(r)(v^*(r,t) - V^*(r,t))dr \quad (29)$$

By setting $L_i = e_i^TS_\theta e_i,\ \ i = 1,2,3$ and define $L = L_1 + L_2 + L_3$ as a global Lyapunov function, it can be shown that

$$\dot{L}_1(t) \leq -\theta V_1 + k_1\|e_1\|^2 + k_2\|e_2\|^2 + k_3\epsilon \quad (30)$$

where $k_1,\ k_2,\ k_3 > 0$. Similar to the above formulation, we have

$$\dot{L}_2(t) =\leq -\theta L_2 + \kappa_1\|e_2\|^2 + \kappa_2\|e_3\|^2 + \kappa_3\epsilon \quad (31)$$

and

$$\dot{L}_3(t) \leq -\theta L_3 + \gamma_1\|e_3\|^2 + \gamma_2\epsilon \quad (32)$$

with κ_i and γ_j being positive numbers. Using equations (30)-(32), it can be seen that there exist ι_i (i = 1, 2, 3, 4) such that

$$\dot{L} \leq -\theta L + \iota_1\|e_2\|^1 + \iota_2\|e_2\|^2 + \iota_3\|e_3\|^2 + \iota_4\epsilon \quad (33)$$

Using the fact that $C(t)$ and $A(t)$ are uniformly bounded, it can be shown that [1]:

$$\exists\theta_0;\ \ \forall S_\theta(0) > 0;\ \ \forall\theta > \theta_0;\ \ \exists\eta_1 > 0;\ \ \exists\eta_2 > 0:\ \ \ \forall t > t_0$$

$$\eta_1 I \leq S_\theta(t) \leq \eta_2 I$$

where I is the identity matrix. As such, replacing this inequality into (33) leads to:

$$\frac{d}{dt}(L - \frac{\iota_4\eta_2}{\theta\eta_1 - \iota}\epsilon) \leq -\frac{\theta\eta_1 - \iota}{\eta_2}(L - \frac{\iota_4\eta_2}{\theta\eta_1 - \iota}\epsilon) \quad (34)$$

or

$$\lim_{t\to\infty}\|X - X^*\|^2 \leq \lim_{t\to\infty}(\|e_1\|^2 + \|e_2\|^2 + \|e_3\|^2) \leq \frac{\iota_4\eta_2}{(\theta\eta_1 - \iota)\eta_1}\epsilon \quad (35)$$

As a result, it can be shown that: $\lim_{t\to\infty}\|v(r,t) - v^*(r,t)\| = \lim_{t\to\infty}\|v(r,t) - C_r(r,t)X + C_r(r,t)X - C_r(r,t)X^* + C_r(r,t)X^* - v^*(r,t)\| \leq \varrho\epsilon$ •

4.Modelling and control of winding processes

In this section, the above controller is applied to a winding process in the film and paper industries. The aim here is to find a control strategy that helps optimize the wound roll structure. A winding process consists of a web, a continuous flexible and thin material such as paper, plastic film, metal foil and textile etc., which is wrapped onto a core by successive layers as the roll rotates. The aim of such a process is to obtain a compact roll which allows to keep the quality of the wound material during storage, transportation and unwinding. For instance, a roll which is wound lose will be difficult to handle and transport. On the other hand, a roll which is wound too tight or non-uniform will generate internal defects such as wrinkles, buckles or local breaks due to the excess of in-roll stress generated during winding. There are different types of stresses taking place in such processes. Stresses that are orthogonal to the radial direction are said to be *tangential stresses*. While the focus of the study here is on both stresses, other stresses such as shear stresses exist. The fundamental material on elasticity used in this section can be found in [14] and [8]. The modelling methodology starts by considering an element of material located inside the roll being wound. After a detailed analysis and the use of some physical principles, the stress model of this element can be written as follows:

$$\begin{cases} \rho\dfrac{\partial^2 u_\theta}{\partial t^2} = \dfrac{1}{r}\dfrac{\partial\sigma_\theta}{\partial\theta} + \dfrac{2\tau_{\theta r}}{r} + \dfrac{\partial\tau_{\theta r}}{\partial r} + \rho g sin\theta + \dfrac{\partial\tau_{\theta z}}{\partial z} \\ \rho\dfrac{\partial^2 v(r,t)}{\partial t^2} = \dfrac{\partial v(r,t)}{\partial r} + \dfrac{1}{r}(v(r,t) - \sigma_\theta) + \dfrac{\partial\tau_{rz}}{\partial z} \\ +\dfrac{1}{r}\dfrac{\partial\tau_{\theta r}}{\partial\theta} - \rho g cos\theta \\ \rho\dfrac{\partial^2 u_z}{\partial t^2} = \dfrac{\partial\sigma_z}{\partial z} + \dfrac{\partial\tau_{rz}}{\partial r} + \dfrac{\tau_{rz}}{r} + \dfrac{1}{r}\dfrac{\partial\tau_{\theta z}}{\partial\theta} \end{cases} \quad (36)$$

where $v(r,t)$, σ_θ and σ_z are the radial, tangential and the z-direction stresses, respectively. $\tau_{\theta r}$, τ_{rz} and $\tau_{\theta z}$ are the shear stresses. $\dot{u}_\theta = r\omega$ and ω is the angular speed. The above system describes the internal stress distribution behavior in three dimensions. In addition to the above equations, a mass balance should be added:

$$\frac{dm}{dt} = nHG \quad (37)$$

where m is the mass of roll, n the speed of the incoming web and G the grammage of the web defined as the mass per meter square. This equation allows to calculate the radial growth of the roll with time. In this paper, only the radial description of the process will be considered. It is assumed that the roll has an axi-symmetric structure, with cylindrical anisothropy. This leads to the following equation:

$$\rho\frac{\partial^2 v(r,t)}{\partial t^2} = \frac{\partial v(r,t)}{\partial r} + \frac{1}{r}(v(r,t) - \sigma_\theta) \qquad (38)$$

where the elasticity properties of the material are related to the stresses by the Hooke's relations on strain and stress [14] in the following way:

$$v(r,t) = \frac{E_r}{1-\mu_r\mu_\theta}(\epsilon_r + \mu_\theta\epsilon_\theta) \quad \sigma_\theta = \frac{E_\theta}{1-\mu_r\mu_\theta}(\epsilon_\theta + \mu_r\epsilon_r) \qquad (39)$$

. where ϵ_r and ϵ_θ are the radial and the tangential residual strains, respectively. μ_r and μ_θ are the Poisson ratio in the r and θ directions, which expresses the radial (respectively tangential) expansion due to a radial (respectively tangential) stress. E_r (respectively E_θ) is the radial (respectively tangential) modulus of the material. The strains are related to the displacement by the following relations:

$$\epsilon_r = \frac{\partial v(r,t)}{\partial r}, \quad \epsilon_\theta = \frac{v(r,t)}{r}.$$

Intensive investigations have been carried out to model the radial modulus [11, 7, 15]. In fact, the wound layers can be regarded as a stack of material on which a pressure is exerted where Pfeiffer has derived a pressure-strain relation $P = -K_1 + K_1 e^{K_2\epsilon}$ with P being the pressure applied and ϵ being the resulting compression strain. K_1 and K_2 are parameters defined by the experiment. Therefore the radial modulus in (26) can be defined as:

$$E_r = \frac{\partial P}{\partial \epsilon} = K_2(K_1 + P) \qquad (40)$$

Equation (40) was later extended to a polynomial expression by Hakiel, [7] and Willett and Poesch [15]. However, the expression of the radial modulus is affected by friction (or the number of surfaces in contact), moisture, the creep properties of the wound roll. For all these reasons, here we consider $E_r(r,t)$ as depending on time and radius. In a more recent work [9] it has been shown, using a series of different stacks of paper, that $E_r(r,t)$ depends not only on the pressure but also on the number of layers in the stack. By integrating equations (39) and the strain expression in (38), it can be obtained that:

$$\frac{\partial^2 v(r,t)}{\partial r^2} + (\frac{\lambda_1}{r} + \frac{1}{E_r}\frac{\partial E_r}{\partial r})\frac{\partial v(r,t)}{\partial r} + (\frac{\lambda_2}{r^2} + \frac{\mu_\theta}{rE_r}\frac{\partial E_r}{\partial r})v(r,t) = \frac{\rho(1-\mu_\theta\mu_r)}{E_r}\frac{\partial^2 v(r,t)}{\partial t^2} \qquad (41)$$

where $\lambda_2 = -\frac{E_\theta}{E_r}$ and $\lambda_1 = 1 = 1 + \mu_\theta + \lambda_2$. The above equation expresses the wave evolution in a resistant medium represented by the layers. In the rest of the paper, it is assumed that E_r depends on r and the pressure.

By using the approximation $V(r,t) = \sum_{i=1}^{N} x_i(t)B_i(r)$ and the Gallerkin orthogonality property of $R(r,t)$, an equation of the form (7) can be obtained, where

$$\bar{b}_{ij}(t) = \int_{r_c}^{r_e} \frac{E_r(t,r)}{\rho(1-\mu_\theta\mu_r)}(\frac{d^2B_i(r)}{dr^2} + (\frac{E_r(t,r)(\mu_\theta+1) - E_\theta}{r\rho(1-\mu_\theta\mu_r)} + \frac{1}{\rho(1-\mu_\theta\mu_r)}\frac{\partial E_r}{\partial r})\frac{dB_i(r)}{dr} + (-\frac{E_\theta}{r^2\rho(1-\mu_\theta\mu_r)} + \frac{\mu_\theta}{\rho r(1-\mu_\theta\mu_r)}\frac{\partial E_r}{\partial r})B_i(r))B_l(r)dr \qquad (42)$$

In order to define the target distribution, equation (39) is re-

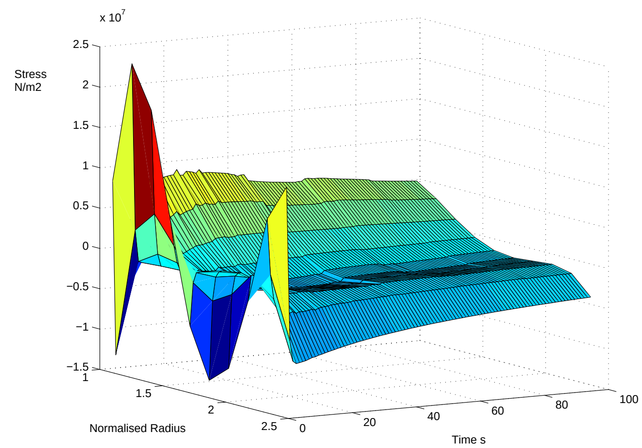

Figure 1: Radial stress.

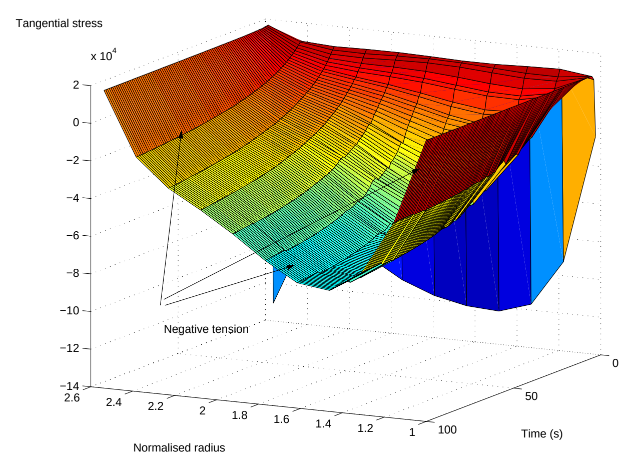

Figure 2: Tangential stress.

used to obtain the equivalent displacement reference:

$$V^*(r,t) = r(\frac{v^*}{E_r} - \mu_r\frac{\sigma^*_\theta}{E_\theta}) \qquad (43)$$

where σ_θ^* and $v*$ are the desired tangential and radial stress profiles. Also, to avoid internal layers being damaged, the tangential stress should not exceed a certain value. For example, it can be shown using a tensile testing on a newsprint paper sample that, up to 2% of strain (elongation), there is no damage and the characteristic can be approximated by a linear curve. In addition, the tangential stress must be maintained positive while the roll in being wound. A negative tangential stress means a presence of buckling: the circumference of the affected layers are longer the allocated space. A pressure profile of the form $(1/r^2)$ ([13]) has been used to avoid inter-layer slippage. For the winding model (28), the control objective is to control $v(r,t)$ to a desired displacement profile $V^*(r,t)$. Simulations

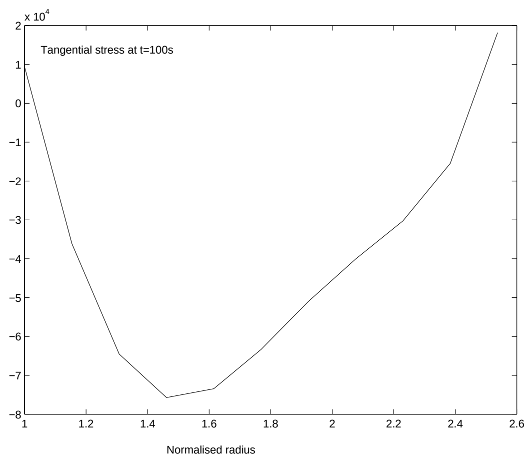

Figure 3: Tangential stress at t=100 s.

were carried out where the results are shown in Figure 1 - 3, it can be seen that the tension of the internal layers which are initially positive (wound with a tension of 750 psi) becomes negative.

5.Conclusions

In this paper, a neuro-mechanical approach is proposed for the modelling and control of a class of parameter distributed systems whose control input appears in the boundary conditions of the system equation. This type of system represents commonly used winding processes in the film and paper industries. Using the physical model derived from the process, a B-spline neural network is used to approximate the system so that a controller can be designed. An observer-based controller has been proposed and its stability proved. The application to winding processes is then developed, where some simulations are provided to show the effectiveness of the proposed method.

Future works include the extension of the modelling and control algorithm to a 2D and a 3D cases. Also, the implementation of the control strategy as well as its use for on-line monitoring should be considered.

Acknowledgement This work is funded by the current EPSRC grant with GR/L87743. This is gratefully acknowledged.

2ex0.5ex

*References

[1] G. Besancon, and H.Hammouri, ”Observer synthesis for a class of nonlinear systems”, *European Journal of control*, **2**, pp. 176-192, (1998).

[2] D. J. Costello, and P.J. Gawthrop, ”Physical-model based control:experiments with a stirred-tank heater”, *Technical Report CSC-95003*, Centre for systems and Control, Univ. of Glasgow, (1995).

[3] R.F. Curtain, and H. J. Zwart, ”An introduction to infinite dimensional linear systems theory”, *Springer Verlag*, (1995).

[4] K. Eriksson, D. Estep, P.Hansbo, and C.Johnson, ”Computatinal Diffrential equations” *Cambridge University Press*, (1996).

[5] N. H. EI-Farra, A. Armaou, and P. Christofides, ”Output feedback control of parabolic PDF Systems with input constraints”,*The Proceedings of the 40th IEEE Conference on Decension and Control*, pp. 541 - 546, (2001).

[6] K. Gawthrop, G. W. Irwin, K.J.Hunt,”Neural networks for control and systems, *Peter Peregrinus*, London, (1992).

[7] Z. Hakiel, ”Nonlinear model for wound roll stresses”, *Tappi Journal*, pp. 113-117, (1987).

[8] W. Hamad,”Winding of Anisothropic Materials”, *Pira Technology Series*, (1998).

[9] P. Kabore, H. Jaafar, W. Hamad, and H. Wang, ”Determining the radial modulus profile in wound rolls”,*Internal Report, Paper Science department, UMIST, Manchester, UK*, (2000).

[10] E. Kreyszig, ”Introduction to functional analysis with applications”, *Wiley and Sons*, New York, (1989).

[11] J. D. Pfeiffer, ”Prediction of roll defects from roll structure formulas”, *Tappi Journal*, **62**, pp. 83-85, (1979).

[12] J. Reinschke, and M. C. Smith, ”Feedback control of LTI spatially distributed systems: an input-output approach”, *The Proceedings of the 38th IEEE Conference on Decension and Control*, pp. 1101 - 1106, (2001).

[13] R.D. Roisum, ”Mechanics of Winding”, *Tappi Press*, Atlanta, (1994).

[14] S.P. Timoshenko, and J.N. Goodier, ”Theory of Elasticity-3rd Edition”, *McGraw Hill*, New York, (1987).

[15] M.S. Willett, and W.L. Poesch, ”Determining the stress distribution in wound reels of megnetic tape using a nonlinear finite diffrence approach” *Journal of Applied Mechanics*, **55**, pp.365-371, (1988).