SYNTHESIS OF ROBUST PID CONTROLLERS FOR TIME DELAY SYSTEMS

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Abstract

Tuning rules are frequently used to choose the parameters of PID controllers. However, they are often based on heuristics, are limited to systems of a certain class, use dead time approximations and ignore parameter uncertainties in the modeled system. This work develops a systematic, universal and transparent method to design a robust PID controller based on the parameter space approach, which is extended to cope with quasipolynomials.

1 Introduction

PID is the most common controller in industrial practice [2]. But a high percentage of PID control systems seems to be tuned badly [6]. One major reason may be that today's tuning methods are limited to very restricted conditions on the plant (concerning model order, pole and zero location, neglected parameter uncertainty) [12]. Time delay systems, especially with uncertain and immeasurable dead time, present one of the most challenging problems for tuning a PID controller.

This paper develops a PID tuning method based on the parameter space approach [1]. So far, in [3] the synthesis step is extended to time delay systems, but important results for the practical application are still missing. Also, the analysis step is not developed in the literature and results have not been compared with existing tuning methods.

In [15] an analytical solution to find the Hurwitz stable PID parameters for a first order system plus time delay is published; in [11] formulas to calculate some stability boundaries in the dead time for certain classes of quasipolynomials are given. However, both approaches are limited to very restricted cases.

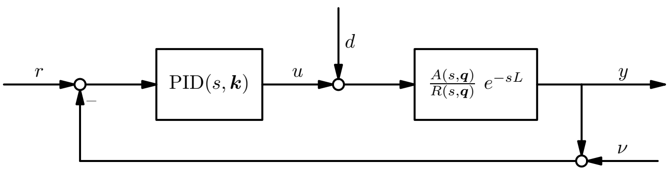

Figure 1: Single loop with PID controller and time delay system

2 Problem formulation

Consider a single loop containing a PID controller and a linear time delay system (see Fig. 1), given by the transfer functions

$$\mathrm{PID}(s,\boldsymbol{k}) = \frac{K_I + K_P s + K_D s^2}{s\,(1+T_R s)}, \tag{1}$$

$$G(s,L,\boldsymbol{q}) = \frac{A(s,\boldsymbol{q})}{R(s,\boldsymbol{q})}\, e^{-sL}, \tag{2}$$

where $\boldsymbol{k} = (K_I, K_P, K_D)^T$ are the controller parameters. (T_R ensures feasibility of the controller and filters noise ν; it is assumed to be fixed prior to the controller design, e.g. by adding a non dominant pole to $R(s,\boldsymbol{q})$). The unknown but constant plant parameters are the dead time $L > 0$ and the parameters in the vector $\boldsymbol{q}$. They lie in an *operation domain*

$$Q = \{\ (L,\boldsymbol{q})^T \mid L \in [L^-; L^+],\ q_i \in [q_i^-; q_i^+]\ \}, \tag{3}$$

where q_i^- and q_i^+ are specified as the lower and upper limits of parameter q_i in $\boldsymbol{q}$ (analog L^- and L^+).

The problem of designing a robust PID controller is to find a set of controller parameters $\boldsymbol{k} = \boldsymbol{k}^*$, that meets the specification for all values of $(L,\boldsymbol{q})^T \in Q$. Specifications are assumed in the form of *Hurwitz stability* (all roots of the characteristic function are in the open left half plane (LHP)) and *σ-stability* (all roots have a real part smaller then a real number σ_0).

The characteristic function of the loop in Fig. 1

$$P(s,\boldsymbol{k},L,\boldsymbol{q}) = (K_I + K_P s + K_D s^2)\, A(s,\boldsymbol{q}) + \underbrace{s\,(1+T_R s)\, R(s,\boldsymbol{q})}_{B(s,\boldsymbol{q})}\, e^{sL}, \tag{4}$$

with polynomials

$$A(s,\boldsymbol{q}) = a_0(\boldsymbol{q}) + a_1(\boldsymbol{q})\, s + \ldots + a_m(\boldsymbol{q})\, s^m, \tag{5}$$

$$B(s,\boldsymbol{q}) = b_0(\boldsymbol{q}) + b_1(\boldsymbol{q})\, s + \ldots + b_n(\boldsymbol{q})\, s^n, \tag{6}$$

with $a_m(\boldsymbol{q}) \neq 0$, $b_n(\boldsymbol{q}) \neq 0$ belongs to the class of *quasipolynomials* [5, 13] due to the dead time. (Note that $b_0(\boldsymbol{q}) = 0$ for basic case of a PID controller (1). However, later a $b_0(\boldsymbol{q}) \neq 0$ may appear through transformations, see section 7.)

The *principal term condition* [13] requires for Hurwitz stability that in the case of PID control ($K_D \neq 0$) the degrees fulfil $n \geq m+2$. In the sequel we treat only this case (i.e. we assume a proper $A(s,\boldsymbol{q})/R(s,\boldsymbol{q})$ for $T_R \neq 0$.)

3 Parameter space approach

The parameter space approach is used to solve the problem in two main steps. In the *controller synthesis* step, we compute the stable (either Hurwitz or σ-stable) region in the space of controller parameters $\boldsymbol{k}$ for several representatives $(L^*, \boldsymbol{q}^*)^T$ out of Q (usually the vertices). A candidate for a robust controller $\boldsymbol{k}^*$ is chosen from the intersection of stable regions.

This controller satisfies the specification for the representatives. The second step, the *control loop analysis*, is applied to test the robust stability for the continuum of all values in Q. Now we compute the stable region in the space of plant parameters $(L, \boldsymbol{q})^T$ with fixed controller $\boldsymbol{k}^*$. If Q lies entirely in the stable region, then a solution of the problem is found.

The calculation of a Hurwitz stable region in a parameter space (either $\boldsymbol{k}$ or $(L,\boldsymbol{q})^T$) is based on the fact that the roots of the quasipolynomial (4) with continuous coefficient functions $a_i(\boldsymbol{q}), b_i(\boldsymbol{q})$ do not jump when the parameters are changed continuously. Thus, a stable quasipolynomial, whose roots all lie in the LHP, becomes unstable if and only if at least one root crosses the imaginary axis. The parameter values of the root crossings form the *stability boundaries* in the parameter space, which can be classified into three cases: the *real root boundary (RRB)*, where a root crosses the imaginary axes at the origin (substitute $s = 0$ in the quasipolynomial), the *infinite root boundary (IRB)*, where a root leaves the LHP at infinity (set $|s| \to \infty$) and the *complex root boundary (CRB)*, where a pair of conjugate complex roots crosses the imaginary axes (substitute $s = \mathrm{j}\omega$ and sweep over all real $\omega > 0$). These stability boundaries separate different regions in the parameter space. To classify a region as Hurwitz stable it suffices to prove stability for one inner test point (e.g. by the Nyquist criterion).

4 Controller design algorithm

The proposed controller design procedure is summarized in the following steps:

1. Specify the maximum real part σ from closed-loop settling time requirements.

2. Compute the σ-stable regions in controller parameter space for representatives (usually the vertices) of the Q-domain.

3. Determine the intersection of the σ-stable regions.

4. Choose a candidate controller out of the intersection.

5. Compute the σ-stable region in plant parameter space for the candidate controller.

6. If the Q-domain lies entirely in the σ-stable region, then the problem is solved.

σ-stability can be reduced to the Hurwitz case by the substitution $s = v + \sigma_0$ which leads to a transformation in parameters and polynomials (see section 7). So Hurwitz stability is considered first in the next paragraphs.

5 Controller synthesis

For each fixed representative $(L^*, \boldsymbol{q}^*)^T$ the Hurwitz stability boundaries of (4) in the $\boldsymbol{k}$-space are determined. The RRB turns out to be simply a straight line given by the equation

$$P(0,\boldsymbol{k}) = K_I\, A(0) + B(0) = 0 \Leftrightarrow K_I = -\frac{b_0}{a_0}. \qquad (7)$$

(In the basic case we have $b_0 = 0$ and the RRB is $K_I = 0$.)

More theoretical difficulties arise when calculating the IRB. Quasipolynomials possess an infinite number of roots, which can not be calculated analytically in the general case. However, the asymptotic location of roots far from the origin is well known [5]. It turns out that infinite root boundaries only exist, if the degree equation $n = m + 2$ is fulfilled (in case of $K_D \neq 0$). These are two straight lines

$$K_D = \pm \frac{b_n}{a_m}. \qquad (8)$$

The calculation of the CRB starts analog to the delay free case of polynomials [1]. The root condition $P(\mathrm{j}\omega, \boldsymbol{k}) = 0$ can be separated into a system of two equations for real and imaginary part

$$\begin{pmatrix} R_P(\omega,\boldsymbol{k}) \\ I_P(\omega,\boldsymbol{k}) \end{pmatrix} = \begin{pmatrix} R_A & -\omega^2 R_A \\ I_A & -\omega^2 I_A \end{pmatrix} \begin{pmatrix} K_I \\ K_D \end{pmatrix} + \\ + \begin{pmatrix} R_{\tilde{B}} - K_P \omega I_A \\ I_{\tilde{B}} + K_P \omega R_A \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad (9)$$

where $\tilde{B}(\mathrm{j}\omega) = B(\mathrm{j}\omega)\, e^{\mathrm{j}\omega L^*}$ and R, I denote the real and imaginary parts of A, $\tilde{B}$ and P at $(\mathrm{j}\omega)$.

Clearly, the matrix multiplying $(K_I, K_D)^T$ is singular. Thus, the key idea is to fix $K_P = K_P^*$ and to evaluate the CRB in the (K_D, K_I)-plane. A solution of (9) exists and only exists for the real zeros ω_{gi} of

$$\begin{aligned} g(\omega) &= \det \begin{pmatrix} R_A & R_{\tilde{B}} - K_P^* \omega I_A \\ I_A & I_{\tilde{B}} + K_P^* \omega R_A \end{pmatrix} \\ &= \omega\, K_P^* \,(R_A^2 + I_A^2) + R_A I_{\tilde{B}} - I_A R_{\tilde{B}}. \end{aligned} \qquad (10)$$

The zeros of $g(\omega)$ are called *singular frequencies*. For each i appears a straight line as CRB in the (K_D, K_I)-plane, ruled by the equation

$$K_I = \omega_{gi}^2\, K_D + K_I^0(\omega_g), \qquad (11)$$

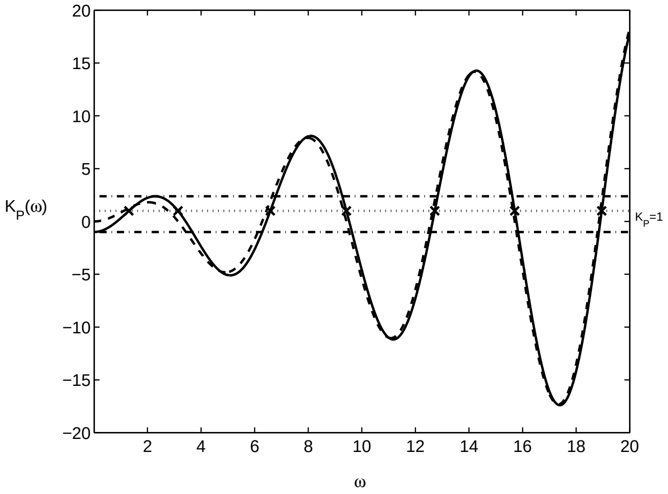

Figure 2: Function $K_P(\omega)$ (solid), its limit function (dashed), singular frequencies for $K_P^* = 1$ (x-marks) and stabilizing K_P-interval (dash-dotted) of $G_1(s)$.

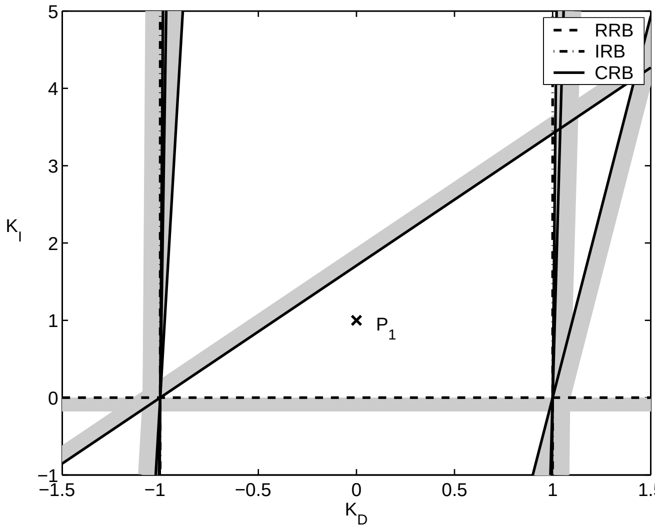

Figure 3: Stability boundaries in (K_D, K_I)-plane for $K_P^* = 1$ of $G_1(s)$. The side of the lines with more unstable poles is shaded.

where $K_I^0(\omega_g)$ can be easily determined from the first or second row of (9).

Thus, the stability boundaries RRB, IRB and CRB are straight lines in the (K_D, K_I)-plane and partition the plane into convex polygons. (Additionally, for each boundary line the side can be determined which possesses the lower number of stable poles, see [3].)

The singular frequencies may be determined by a graph of

$$K_P(\omega) = \frac{-R_A I_{\tilde{B}} + I_A R_{\tilde{B}}}{\omega(R_A^2 + I_A^2)}. \quad (12)$$

Graphically, the singular frequencies for a fixed K_P^* are the abscissa values of the intersections between the $K_P(\omega)$-plot and the $(K_P = K_P^*)$-line. Due to the dead time the number of singular frequencies is infinite. Algorithms for the automatic calculation of the singular frequencies can be found in [8].

The function $K_P(\omega)$ and the resulting boundaries in the (K_D, K_I)-plane for a fixed $K_P^* = 1$ are demonstrated in Fig. 2 and Fig. 3 for the example system (with ideal PID controller $T_R = 0$)

$$G_1(s) = \frac{1}{s+1}\, e^{-s}. \quad (13)$$

Stability checks of test points prove that the polygon around P_1 is stable (and it is the only one). Two questions arise: How many singular frequencies have to be evaluated and which K_P-values lead to a stable polygon?

1. For high frequencies, function (12) tends to a much simpler limit function of the form

$$K_P(\omega) \overset{\omega\to\infty}{\longrightarrow} \alpha\, \omega^\beta\, \mathrm{trig}(\omega L^*), \quad (14)$$

where α and β are real constants ($\beta \geq 1$) and trig is either the $\sin$ or $\cos$ function. It can be shown, that singular frequencies which lie in a frequency range where (14) and (12) are practically identical result in CRB lines which do not contribute to boundaries of the stable polygon. So, it suffices to compute and evaluate only a finite number of low singular frequencies, given by the comparison of (14) and (12).

2. The question of determining the interval of K_P which leads to a stable polygon is analytically solved only for a first order system plus time delay [15]. Following work hypothesis for the general system (2) is stated:
Work hypothesis. *Provided that there is a stabilizing PID controller, a Hurwitz stable polygon exists in the (K_D, K_I)-plane, if K_P^* lies in the interval of K_P that produces the highest possible number of singular frequencies.*
The result of the hypothesis for $G_1(s)$ is depicted in Fig. 2.

The entire stable region in ***k***-space can be computed by gridding K_P in its stabilizing interval and extracting the stable polygon for each gridded K_P^* (see Fig. 4).

6 Control loop analysis

Now we determine the Hurwitz stable region in plant parameter space by mapping the stability boundaries of the quasipolynomial

$$P(s, L, q) = D(s, q) + B(s, q)\, e^{sL}, \quad (15)$$

where

$$\begin{aligned} D(s, q) &= (K_I^* + K_P^*\, s + K_D^*\, s^2)\, A(s, \boldsymbol{q}) \\ &= d_0(q) + d_1(q)\, s + \ldots + d_{m'}(q)\, s^{m'} \end{aligned} \quad (16)$$

The case of mapping the boundaries to a plane of L and one additional parameter q is treated. If there are more uncertain parameters, then they have to be gridded.

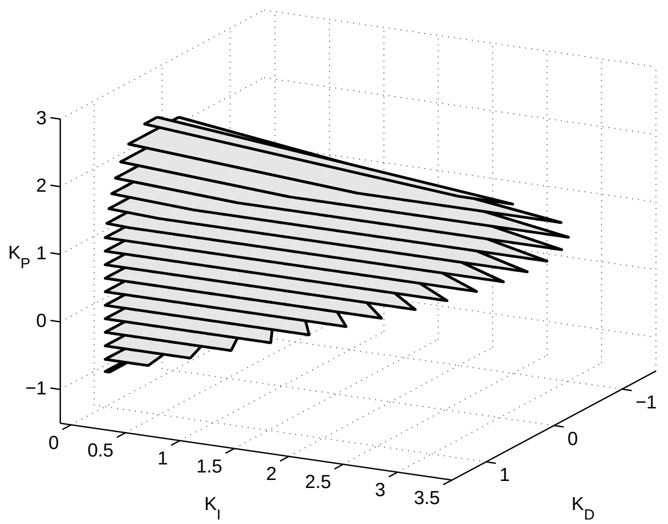

Figure 4: Hurwitz stable controller parameters $\boldsymbol{k}$ of $G_1(s)$.

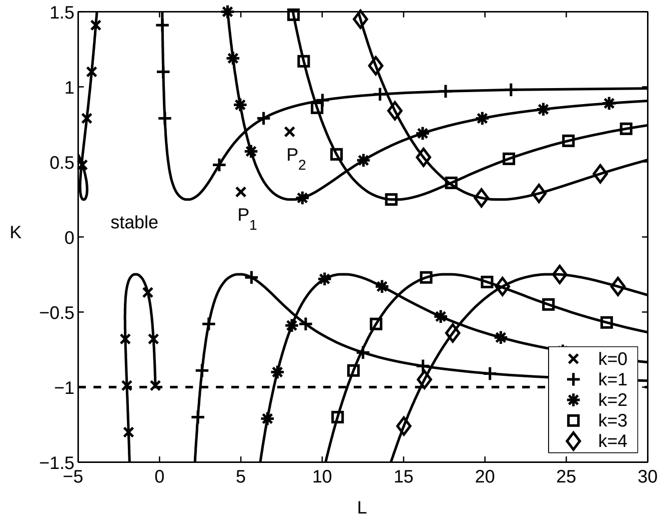

Figure 5: Stability boundaries RRB (dashed) and CRB (solid, $k = 0, .., 4$) of $P_2(s, L, K)$.

The RRB are lines $q = q^*$, where q^* are the solutions of the equation

$$d_0(q) + b_0(q) = 0. \tag{17}$$

If the degrees of $B(s, q)$ and $D(s, q)$ fulfill $n = m'$, then the solutions of the following equation lead to IRB

$$d_{m'}(q) \pm b_n(q) = 0. \tag{18}$$

The CRB condition $P(\mathrm{j}\omega, L, q) = 0$ can be split into a modulus and a phase equation

$$\left| \frac{D(\mathrm{j}\omega, q)}{B(\mathrm{j}\omega, q)} \right| = 1, \tag{19}$$

$$L(\omega, q, k) = \frac{1}{\omega} \left(\arg(D(\mathrm{j}\omega, q)) - \arg(B(\mathrm{j}\omega, q)) + \right. \\ \left. + (2k-1)\pi \right), \quad k \in \mathbb{Z}. \tag{20}$$

The modulus equation does not depend on L and can be transformed to a polynomial in ω^2. Thus, the equation can be solved by finding the roots of a polynomial, but it is especially convenient if (19) can be solved symbolically for q (e.g. if q is the dc-gain). With (20) the corresponding values for L can be added to plot the CRB curves. The phase equation carries the uncertainty k (only $k \geq 0$ generate nonnegative L), so there is an infinite number of CRB. However, as k grows, the curves move due to (20) to higher dead times.

Fig. 5 shows the boundaries and the stable region for the example quasipolynomial

$$P_2(s, L, K) = K + (s^2 + 0.25\, s + 1)\, e^{sL}. \tag{21}$$

In [11] this example serves for the calculation of some dead time stability limits (corresponding to $k = 1$), which agree with these presented here. However, Fig. 5 presents the stable region completely for $L \lessapprox 22$.

7 PID design using σ-stability

The concept of σ-stability can be used to speed up the transient responses robustly. Concerning the synthesis step, this case can be reduced to the Hurwitz case by substituting $s = v + \sigma_0$. Following transformations result

$$\begin{aligned} K_I' &= K_I + K_P\sigma_0 + K_D{\sigma_0}^2, \\ K_P' &= K_P + 2K_D\sigma_0, \\ K_D' &= K_D, \\ A'(v) &= A(v + \sigma_0), \\ B'(v) &= e^{\sigma_0 L^*} B(v + \sigma_0). \end{aligned} \tag{22}$$

Following the parameter space approach presented in section 3, the smallest possible value for σ_0 is determined for a given system (2) and an operation domain (3), s. t. the intersection of the stable regions belonging to the vertices of Q is not empty. This σ_0 is approximated by an iterative approach: Beginning with zero, σ_0 is stepwise reduced and the function $K_P(\omega)$ is plotted for each vertex, until the work hypothesis reveals that there is no interval of K_P that stabilizes simultaneously all vertices. With the last σ_0 having such a interval, the stable $\boldsymbol{k}$-regions are computed for all vertices, and a controller $\boldsymbol{k}^*$ is taken out of the intersecting region.

The analysis step can be reduced to he Hurwitz case, if the second parameter q enters in form of a dc-gain into the quasipolynomial. In that case, the transformations are

$$\begin{aligned} q'' &= q\, e^{-\sigma_0 L}, \quad & D''(v) &= D(v + \sigma_0), \\ L'' &= L, & B''(v) &= B(v + \sigma_0). \end{aligned} \tag{23}$$

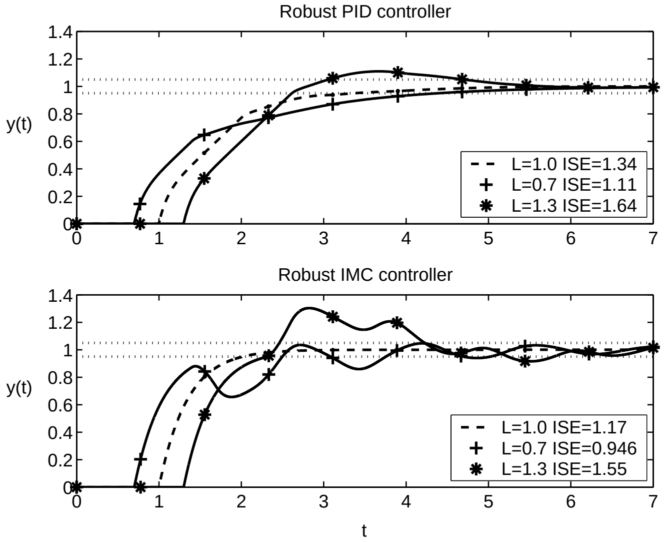

Figure 6: Step responses of PID and IMC controlled loop of $G_3(s, L)$ for nominal, maximal and minimal L.

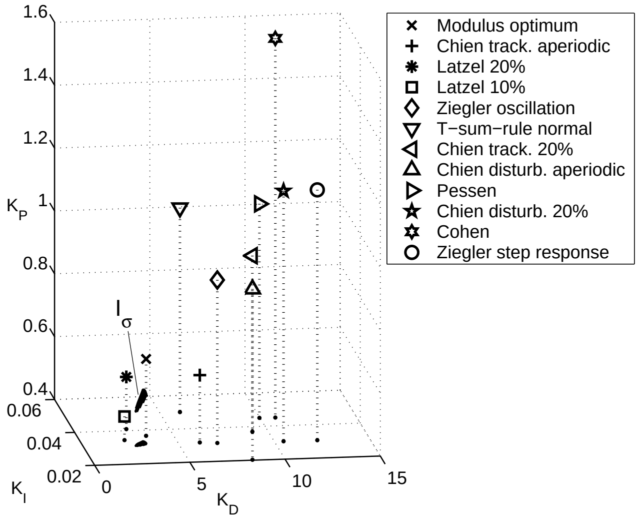

Figure 7: Intersection (I_σ) of the σ-stable regions ($\sigma_0 = -0.05$) of the vertices of Q_4 for $G_4(s, L, K)$ and tuning rule controllers. The tuning rules are sorted by the settling time after a reference step for the nominal plant, see [14, 4].

8 Application to literature examples

We apply the proposed method to design PID controllers for example systems of the literature and compare the results with other PID tuning methods or control concepts. Whenever possible, we respect the following comparison laws: Copy the plant model and the controller with all parameters. Prefer robust control methods and use the same operation domain. Apply the same criteria to evaluate the results. Take only literature examples whose results can be reconstructed.

First the comparison is illustrated by the following system and operation domain

$$G_3(s) = \frac{1}{1+s}\, e^{-sL}, \quad Q_3: \ L \in [0.7\,; 1.3]. \tag{24}$$

which serve as an example in [10] to develop an *Internal Model Control (IMC)* controller.

The robust PID controller parameters are $K_I = 0.58$, $K_P = 0.82$, $K_D = 0.26$ with $T_R = 0.1$ and $\sigma_0 = -0.8$. For a comparison, in Fig. 6 the step responses for nominal, maximal and minimal dead times are shown for the robust PID and IMC controller. Clearly, by using the PID controller the overshoot, settling time and oscillations are reduced, while the Integral Square Error (ISE) is slightly higher.

The second example,

$$G_4(s, L, K) = \frac{K}{(1+5s)^3}\, e^{-sL},$$
$$Q_4: \ L \in [12\,; 18], \ K \in [0.9\,1.1], \tag{25}$$

is used to compare the proposed method with tuning rules that are applied to the system in [14, 4]. Notice the intersections of the σ-stable regions and the tuning rule controllers in Fig. 7. (The tuning rules are applied to the nominal plant $L = 15$, $K = 1$). Clearly, the the tuning rules leading to the lowest settling time lie close to the intersection region. In [14, 4] it is stated that it depends heavily on the plant which tuning rule produces the fastest settling time. However, the proposed method finds a fast controller universally for all systems in the class (2) and guarantees robustness in the whole operating domain.

Additionally, in [8] the proposed method is applied to a great variety of systems (up to fourth order, stable and unstable, with real and complex poles, with zeros) in different parameter dependencies, which serve as benchmark problems for single loop control strategies in the literature (IMC [10], tuning rules etc. [14, 4, 9] and genetic optimization [7]). In all considered cases we achieve superior or at least similar results: The overshoot of the robust PID controller is low, the transient responses are robustly fast, there are only small oscillations and the responses for different operation points resemble largely.

9 Conclusions

The parameter space approach offers convincing results in the synthesis of robust PID controllers for time delay systems. The developed tuning method is systematic, universal and transparent and leads to superior or similar results than literature examples. Exact stability (Hurwitz or σ-stability) regions can be determined in the space of controller and plant parameters while treating the dead time without approximation.

The development of an interactive graphical software package based on the stated algorithm seems very promising to be a helpful tool in daily engineer's work. So the engineer would be able to re-tune the great amount of existing PID loops at low cost in industry.

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