FUZZY PREDICTIVE CONTROL STRATEGIES AND ITS APPLICATION TO A LABORATORY TANK

Doris Sáez[1], Esteban Kemerer[2]

[1]Departamento de Ingeniería Eléctrica, Universidad de Chile
Tupper 2007, Santiago, Chile, Phone: 56-2-6784091 Fax: 56-2-6720162 E-mail: dsaez@ing.uchile.cl
[2]Departamento de Ingeniería de Automatización y Control Industrial, Universidad Nacional de Quilmes
Av Calchaquí 5800 - 1888 Florencio Varela, Buenos Aires, Argentina
Phone: 54-11-42757714. E-mail: esteban@unq.edu.ar

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Abstract

In this work, different predictive control strategies based on linear and non-linear models are described. Fuzzy models are considered in order to represent the non-linearlities of the process. A laboratory tank is used for testing and comparing the different control strategies proposed.

1 Introduction

The essence of model based predictive control (MBPC) is the optimisation of the future process behaviour with respect to the future values of the process manipulated variables. The use of non-linear models in MPC is motivated by the need of improving the quality of the prediction of the inputs and outputs [1].

During the last years many works have emerged regarding non-linear fuzzy predictive control. Skrjanc [13] describes a predictive controller based on Takagi&Sugeno fuzzy models. The algorithm uses a GPC objective function and fuzzy models for the predictions. Using fuzzy logic also solves the optimization problem. The proposed controller uses an adaptive algorithm to solve the process parameter change problems or disturbances, by adjusting the membership function gains of the control action increment.

Cipriano [4] describes a GPC controller based on Takagi&Sugeno fuzzy models. In this work, a linear GPC controller is derived for each rule of the fuzzy model. Therefore, the fuzzy controller includes the same premises as the fuzzy process model and the consequences are given by the resulting control action. The main disadvantage of the proposed control algorithms is that they require the parameters tuning for the different predictive controllers of each rule, however they have an easy and fast implementation.

Roubus [12] proposes a fuzzy predictive controller based on the Takagi&Sugeno fuzzy model linearization. At every sampling time, a linear model is derived by evaluating the fuzzy model premises or the satisfaction degrees. Then, a linear predictive controller is designed for the resulting linear model and, in the next sampling time, the linear model is updated. This method is applicable to multivariable systems and a minor computing effort is required to calculate the control actions. Kim [10] points out, in a similar work, that the prediction with the literalized fuzzy model is not necessarily the optimal prediction. Huaguang [11] describes a similar controller that also includes a stability analysis of the closed-loop system.

Espinosa [5] proposes a new fuzzy predictive control algorithm. This algorithm uses a fuzzy model prediction. In this case, the free response is obtained by simulating the fuzzy model with the constant future inputs equals to the last input u(t-1). On the other hand, the forced response is obtained by simulating the fuzzy model at the present sampling time. Hence, the predictive control analytical solution is similar to the one obtained with the GPC algorithm.

Espinosa [6] extends the predictive control algorithm based on fuzzy models to multivariable systems. Hadjili [8] and Espinosa [7] describe a similar predictive controller. Then, fuzzy predictor uses the constant satisfaction degrees for the N future sampling times. The analytical solution of the GPC is considered for the resulting predictions.

In Babuska [3] works a similar multi-step predictor is proposed. First, the fuzzy model is liberalized at the present sampling time. Next, the resulting control action is used to predict y(t+1) and again the non-linear model is liberalized around the last operational point. This procedure is repeated until t+N, that is to say, it is used for the prediction horizon. This prediction is more precise and it is useful for longer prediction horizons. However, this method implies more computing effort.

Nounou [11] formulates a new fuzzy predictive algorithm by proposing the control action that is calculated for the lineal rule with the highest satisfaction degree at the present sampling time. This algorithm is favorably compared to Roubos [12] algorithms.

In this work, the fuzzy predictive control strategies are first described. Then, the application to level control of a laboratory tank is shown including a comparative analysis of the proposed fuzzy predictive controllers. Finally, the conclusions are presented.

2 Fuzzy predictive control strategies

Non linear predictive control designates an integrated approach of control methods that uses a non-linear process model to obtain the control signal by optimising an objective function, even with constraints [2].

2.1 Predictive control

In the model based predictive control area the most popular objective function used to calculate the control action is the quadratic one. The general expression for such an objective function is [1]:

$$J = \sum_{j=N_1}^{N_y} \delta(j)\left[w(t+j) - \hat{y}(t+j/t)\right] + \sum_{i=1}^{N_u} \lambda(i)\left[\Delta u(t+i-1)\right]^2 \quad (1)$$

where $\delta(j)$ and $\lambda(i)$ are coefficients that weight the future behaviour, $w(t+j)$ is the future reference trajectory, N_1 and N_y are the minimum and maximum prediction horizons and N_u is the control horizon. Summarizing, the control action is obtained minimising an objective function such as Equation (1). To do this, the prediction of the control variables is calculated as a function of past values of the inputs and outputs and of future control actions.

In this work, the non-linear fuzzy dynamic models are considered for the design of non-linear predictive control strategies.

2.2 Takagi & Sugeno fuzzy models

Fuzzy models have been used for the identification of non-linear systems [14]. This paper considers the following Takagi&Sugeno [15] fuzzy model:

$$\begin{aligned}
&\text{if } y(t-1) \text{ is } A_1^i \text{ and} \cdots \text{and } y(t-na) \text{ is } A_{na}^i \text{ and} \\
&u(t-nk-1) \text{ is } A_{na+1}^i \text{ and} \cdots \text{ and } u(t-nb-nk) \text{ is } A_{na+nb}^i \qquad (2) \\
&\text{then } y_i(t) = p_0^i + p_1^i y(t-1) + p_2^i y(t-2) + \ldots \\
&+ p_{na}^i y(t-na) + p_{na+1}^i u(t-nk-1) + \ldots + p_{na+nb}^i u(t-nb-nk)
\end{aligned}$$

where A_i^r is the fuzzy set of variable i of rule r, p_j^i is consequence parameter of rule i associated with variable j and y_i is the output of rule i. The output of the fuzzy model is:

$$y(t) = \sum_{i=1}^{Nr} w_i y_i(t) \qquad (3)$$

where Nr is the rules number and w_i is the normalized activation degree of rule i. The premise parameters can be obtained based on the process knowledge or using the fuzzy clustering method [14]. Next, the consequence parameters are obtained using the Takagi & Sugeno method based on least squares [15].

2.3 Fuzzy predictive control

In this work, a fuzzy predictive controller based on Takagi-and-Sugeno fuzzy models is considered where a linear predictive controller is derived for each rule of the fuzzy model. Therefore, the fuzzy controller includes the same premises as the fuzzy process model (see Equation (2)) and the consequences are given by the resulting control action. That is,

$$\begin{aligned}
&\text{if } y(t-1) \text{ is } A_1^i \text{ and} \cdots \text{and } y(t-na) \text{ is } A_{na}^i \text{ and} \\
&u(t-nk-1) \text{ is } A_{na+1}^i \text{and} \cdots \text{ and } u(t-nb-nk) \text{ is } A_{na+nb}^i \qquad (4) \\
&\text{then} \\
&\quad \Delta u_i(t) = f_i\left(\Delta u(t-1), \ldots, y(t), y(t-1), \ldots\right)
\end{aligned}$$

where f_i denotes the predictive control law for the rule i. The proposed fuzzy predictive controller has an easy and fast implementation.

3 Application to level control of a laboratory tank

3.1 Tank description

As shown Figure 1, a laboratory tank is considered, located at Automatization Laboratory, Universidad Nacional de Quilmes, Argentina. The control problem is to follow level set-point changes by adjusting the flow rate of liquid entering the tank. The position of the inferior valve is considered as a disturbance that regulates the exhaust water. It is important to remark that as the valves behaviour is clearly non-linear, the complete process becomes non-linear.

Figure 1. The laboratory tank

3.2 Tank level modelling

In this work, two experimental data sets are obtained. The first data set is generated, when a pseudo white noise is applied to the input variable (input valve position) and the output response (tank level) values are saved. Also, in order to produce different operating points, the inferior valve position is randomly changed.

In the second data set, closed loop identification is used in order to generate different data sub set for four operating zones, changing the inferior valve position to different fixed positions (25%, 50%, 75%, 100%).

Next, different models are proposed for representing and controlling the laboratory tank level.

3.2.1 Linear models

Using the first experimental data set, the following Autorgresive with eXagenuoes variable (ARX) model is obtained:

$$y(t) = 1.59y(t-1) - 0.58y(t-2) + 0.003u(t-1) + e(t) \quad (5)$$

where y(t) is the tank level and u(t) is the input valve position.

3.2.2 Takagi & Sugeno fuzzy models based on empirical data

The premise parameters of the Takagi & Sugeno models are obtained using the fuzzy clustering and the consequence parameters are determined by the least squared method.

Then, using the first experimental data set (open loop identification), the following fuzzy model is obtained:

$$\begin{aligned} &R_i : \text{If } \Delta y(t-1) \text{ is } A_i \text{ and } u(t-1) \text{ is } B_i \text{ and } u(t-2) \text{ is } C_i \\ &\quad \text{then} \qquad (6) \\ &y_i(t) = p_0^i + p_1^i y(t-1) + p_2^i y(t-2) + p_3^i u(t-1) + p_4^i u(t-2) \end{aligned}$$

where y(t) is the tank level and u(t) correspond to the input flow rate. In Figure 2, the membership functions of the premises are presented. In Table 1, the consequence parameters are shown.

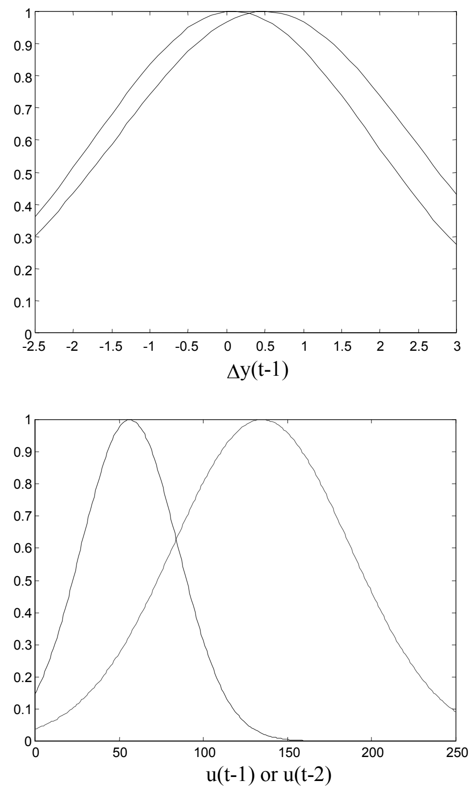

Figure 2. Membership functions

Rule i	p_0^i	p_1^i	p_2^i	p_3^i	p_4^i
1	-1.044	1.583	-0.583	-0.006	0.015
2	-1.168	1.562	-0.562	-0.005	0.011

Table 1. Consequence parameters

3.2.3 Takagi & Sugeno fuzzy models based on process knowledge and empirical data

As it was mentioned before, a second experimental data set (closed loop identification) is obtained for four inferior valve positions (25 %, 50 %, 75%, 100 %).

In this case, four linear models are determined for each data subset that corresponds to the consequence linear models of a fuzzy model as shown in Table 2. The exhaust valve position is considered as the premise input variable because it determinates the different operating conditions. The membership functions are shown in Figure 3.

% inferior valve position	Rule i	p_1^i	p_2^i	p_3^i
25	1	1.504	-0.503	0.008
50	2	1.422	-0.425	0.013
75	3	1.363	-0.366	0.012
100	4	1.5677	-0.569	0.009

Table 2. Consequence parameters

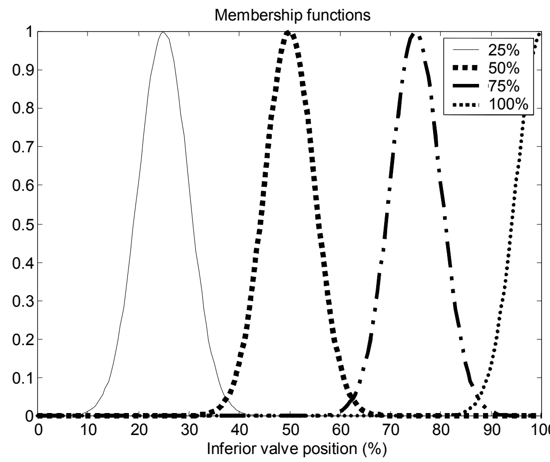

Figure 3. Membership functions for the inferior valve position

Therefore, the fuzzy model is given by:

$$\begin{aligned} &R_i : \text{If } v(t) \text{ is } A_i \text{ then} \\ &\quad y_i(t) = p_1^i y(t-1) + p_2^i y(t-2) + p_3^i u(t-1) \end{aligned} \qquad (7)$$

3.3 Tank level control strategies

Next, the different control strategies are tested using the experimental tank.

3.3.1 Evaluation basis

In order to compare the different control strategies the following GPC index will be used:

$$J(t) = \sum_{k=1}^{N_2} (\hat{y}(t+k) - r(t+k))^2 + \lambda \sum_{k=1}^{N_u} (\Delta u(t+k-1))^2 \quad (8)$$

where y(t+k) is k ahead prediction of the tank level, r(t+k) is the tank level reference, and Δu(t) is the input valve position increment. $N_2 = N_U = 35$, given by the stabilising time.

Also, Figure 4 shown the different tests that are used to evaluate and compare the different control strategies. Test 1 corresponds to the disturbance given by the inferior valve position changes while the tank level reference holds constant (150). Test 2 represents reference changes in the tank level while the inferior valve position holds constant (50 %).

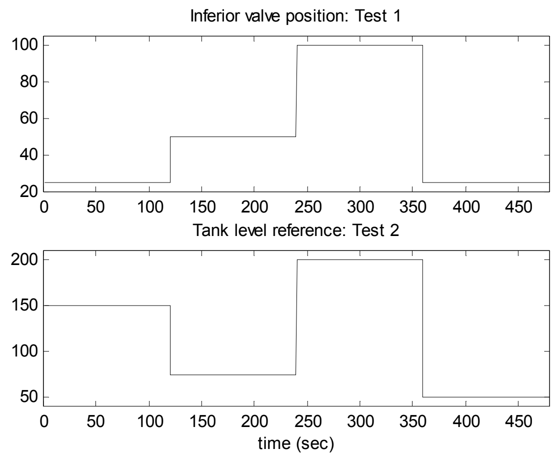

Figure 4. Tests 1 and 2.

Next, the different controllers applied to the laboratory tank level control are described.

3.3.2 Linear predictive control

The predictive controller is designed in order to minimise the same index defined in Equation (9). Then, the control action is given by:

$$\begin{aligned}\Delta u(t) &= -0.064\Delta u(t-1) - 22.33y(t) \\ &+ 34.47y(t-1) - 12.56y(t-2) + 0.42r(t)\end{aligned} \tag{9}$$

where $\Delta u(t)$ is the input valve position increment, $y(t)$ is the tank level and $r(t)$ is the tank level set-point.

3.3.3 Fuzzy predictive controller 1

Using the Takagi & Sugeno fuzzy model based on empirical data defined in Equation (6), the following fuzzy predictive controller is derived (see section 2.3):

$$\begin{aligned}R_i &: \text{If } \Delta y(t-1) \text{ is } A_i \text{ and } u(t-1) \text{ is } B_i \text{ and } u(t-2) \text{ is } C_i \\ &\text{then } \Delta u_i(t) = p_1^i \Delta u(t-1) + p_2^i \Delta u(t-2) \\ &+ p_3^i y(t) + p_4^i y(t-1) + p_5^i y(t-2) + p_6^i r(t)\end{aligned} \tag{10}$$

Table 3 presents the consequence parameters. The membership functions are the same defined in Figure 2.

Rule i	p_1^i	p_2^i	p_3^i	p_4^i	p_5^i	p_6^i
1	-0.13	-0.26	-17.94	27.42	-9.89	0.41
2	-0.11	-0.22	-20.34	30.81	-10.89	0.42

Table 3. Consequence parameters

3.3.4 Fuzzy predictive controller 2

Using the fuzzy model based on the process knowledge and empirical data (Equation (7)), the following fuzzy predictive controller is derived (see Section 2.3)

$$\begin{aligned}R_i &: \text{If } v(t) \text{ is } A_i \text{ then } \Delta u(t) = p_1^i \Delta u(t-1) \\ &+ p_2^i y(t) + p_3^i y(t-1) + p_4^i y(t-2) + p_5^i r(t)\end{aligned} \tag{11}$$

Table 4 presents the consequence parameters. The membership functions are the same defined in Figure 3.

Rule i	p_1^i	p_2^i	p_3^i	p_4^i	p_5^i
1	0.12	-15.24	22.08	-7.25	0.41
2	0.14	-10.89	14.84	-4.35	0.41
3	0.13	-10.88	14.23	-3.76	0.41
4	0.13	-15.42	23.27	-8.25	0.41

Table 4. Consequence parameters

3.3.5 Comparative analysis

In Tables 5 and 6 the results of the control strategies described before are compared for the two tests, including the mean value of the objective function defined in Equation (8) and the improvement (%) compared with the process with a linear predictive controller. Also, the standard deviations of the controlled and manipulated variables are presented.

Control Strategy	Mean J	% Impr.	Std(y)	Std(u)
Linear predictive controller	$4.093\text{x}10^6$	-	2.119	35.806
Fuzzy predict. contr. 1	$4.034\text{x}10^6$	1.44 %	2.339	34.794
Fuzzy predict. contr. 2	$3.993\text{x}10^6$	2.44%	2.0680	32.909

Table 5. Index Test 1

Control Strategy	Mean J	% Impr.	Std(y)	Std(u)
Linear predictive controller	$3.390\text{x}10^6$		52.415	78.092
Fuzzy predict. contr. 1	$3.272\text{x}10^6$	3.49%	51.193	64.660
Fuzzy predict. contr. 2	$3.255\text{x}10^6$	3.98%	51.787	73.208

Table 6. Index Test 2

It is clear from Table 5 that the fuzzy predictive control based on process knowledge and empirical data (Fuzzy Predictive Controller 2) achieves the best results (2.44% regulation improvement), followed by the fuzzy predictive control based on empirical data (Fuzzy Predictive Controller 1). These results are due to the fact that Test 1 corresponds to the disturbance given by the inferior valve position changes, and the Fuzzy Predictive Controller 2 uses this variable as input for its fuzzy rules.

Table 6 shows a significant improvement (3.49-3.98%) with fuzzy predictive controllers. In this case, the best results are obtained when the tank process is excited by different tank level references.

Figures 5 and 6 show the controlled and manipulated variables for Test 1 and Test 2 respectively. The different control strategies are shown in the graphic with different colours. Linear predictive controller is given by green line, Fuzzy Predictive Controller 1 is red line, and Fuzzy Predictive Controller 2 is light blue line.

It can be seen in the figures that the manipulated variable obtained with the Fuzzy Predictive Controller 2 has less variability, confirming the information obtained from the variance analysis presented on Tables 5 and 6.

4 Conclusions

In this paper, two fuzzy predictive control strategies based on Takagi & Sugeno models are described. Also, an application to a tank level control is presented. The results show a better behaviour using the fuzzy predictive controllers than linear predictive controllers. This fact confirms that fuzzy controllers are a good and suitable solution to control non linear systems. Finally, it was experimentally confirmed that the fuzzy predictive controllers have an easy and fast implementation.

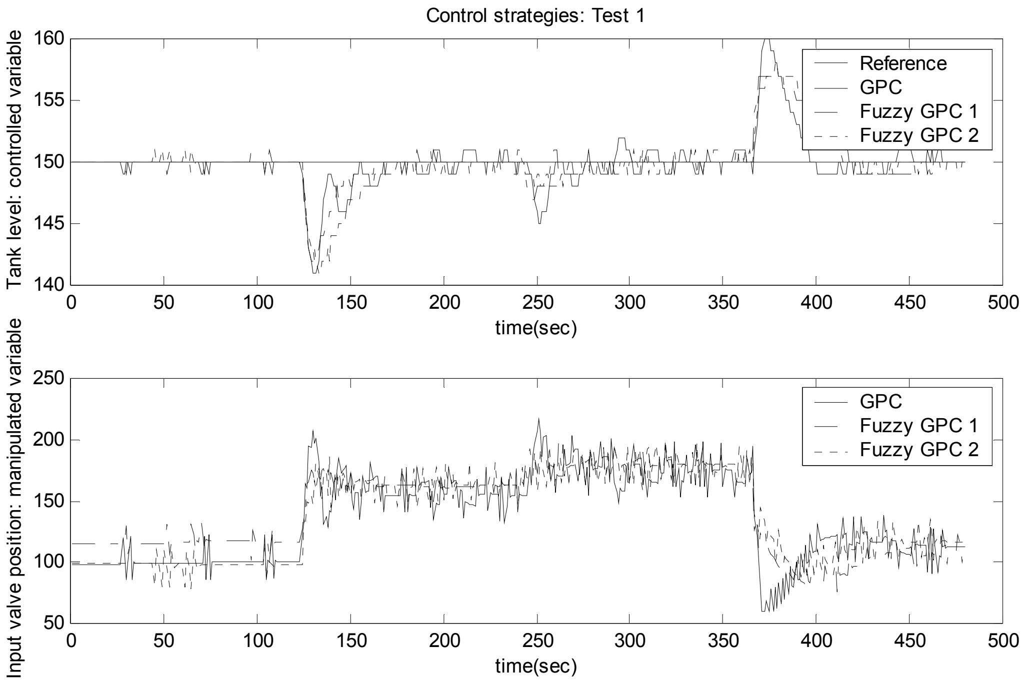

Figure 5. Control strategies: Test 1

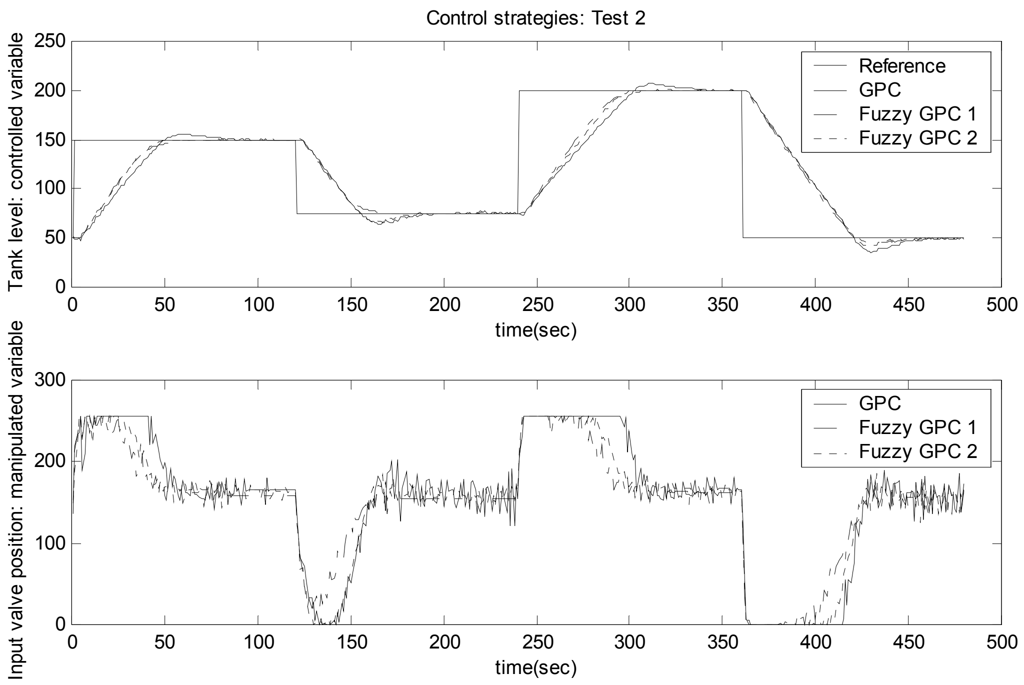

Figure 6. Control strategies: Test 2

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