AUXILIARY SIGNAL DESIGN FOR FAILURE DETECTION IN UNCERTAIN SAMPLED-DATA SYSTEMS

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Abstract

In this paper the normal and the failed behaviors of a sampled-data system are modeled using two distinct uncertain models. A proper auxiliary signal is an input signal for which the behaviors of the two systems do not intersect making guaranteed failure detection possible. Algorithms for the design of optimal proper auxiliary signals are developed.

1 Introduction

The active approach to failure detection consists in acting upon the system using a test signal called an *auxiliary signal* in order to detect abnormal behaviors which would otherwise remain undetected during normal operation. The use of extra input signals specifically in the context of failure detection has been introduced by Zhang [11] and later developed by Kerestecioğlu and Zarrop [4, 5, 6].

In this paper, we study in particular the problem of active failure detection in continuous-time dynamical systems with sampled observations. This work follows previous works where we have considered multiple models to represent the behaviors of normal and failed systems and used a deterministic set membership approach to seek guaranteed detectability (see for example [7, 9, 8, 2]). Space prohibits a full discussion of these works; this can be found in [3]. Many of the ideas we use here come from these studies. What is new in this paper is the hybrid nature of the model (continuous-time dynamics with discrete-time observations). These models, which are called sampled-data systems, are often encountered in practice where a physical process is connected to a digital controller/detector.

The outline of the paper is as follows. In Section 2, we specify precisely the class of uncertain models we consider, and we formulate the design of the auxiliary signal problem as an optimization problem. We give a solution to this optimization problem in Section 3.

2 Problem formulation

We consider the situation where there exist two uncertain models denoted Model 0 and Model 1 representing respectively the normal and the failed behavior of the system. We assume that the system has input v which can be used by the failure detection mechanism over a period of time called the *test period* and an output y which is available to the detector during this test period and used for online detection.

In the sampled-data framework we are considering in this paper, the input v is piece-wise constant so, over a finite horizon, it can be represented by a finite-dimensional vector $\mathbf{v}$. And the values of the output y are only available at sampling times.

The problem is to construct a signal $\mathbf{v}$, the auxiliary signal, in such a way that

$$\mathcal{A}_0(\mathbf{v}) \cap \mathcal{A}_1(\mathbf{v}) = \emptyset \tag{1}$$

where $\mathcal{A}_0(\mathbf{v})$ represents the set of outputs $\mathbf{y}$ consistent with Model 0 and $\mathcal{A}_1(\mathbf{v})$ represents the set of outputs $\mathbf{y}$ consistent with Model 1. An auxiliary signal $\mathbf{v}$ for which this condition is satisfied is called *proper*.

It would be fairly easy to find proper auxiliary signals of very large size. Such signals would in general not be of interest in practice. That is why among all proper auxiliary signals, we look for those which are optimal with respect to certain cost criterion. Often we use the energy of the signal as the cost, but other criteria can be considered as well.

The problem we consider is that of finding a constructive method for the computation of the optimal proper auxiliary signal $\mathbf{v}$.

2.1 Uncertain model

Consider the following uncertain model over the test period $[0, t_n]$

$$
\begin{aligned}
\dot{x} &= (A + M\Delta G)x + (B + M\Delta H)v, &(2)\\
y(j) &= Cx(t_j) + N\mu(j) &(3)\\
&\overline{\sigma}(\Delta(t)) \le 1, &(4)\\
&(x(0) - x_0)^T P_0^{-1}(x(0) - x_0) < 1, &(5)\\
&\sum_{j=0}^{k} |\mu(j)|^2 < 1 &(6)
\end{aligned}
$$

for $k \le n$. Here Δ is a matrix time-varying uncertainty and μ is a series of unknown vectors. This type of model has been studied for example in [10].

The input v is piecewise constant, i.e., for $j \in [0, n-1]$,

$$v(t) = v_j \text{ if } t_j \le t < t_{j+1}. \quad (7)$$

This corresponds to the usual blocking following the D/A conversion stage where the digital controller acts on the continuous time plant. $y(j)$ is the sampled value of the output at time t_j produced by A/D conversion.

We reformulate this uncertain system as follows

$$
\begin{aligned}
\dot{x} &= Ax + Bv + M\nu, &(8)\\
z &= Gx + Hv, &(9)\\
y(j) &= Cx(t_j) + N\mu(j) &(10)
\end{aligned}
$$

where ν and z are called respectively the noise input and noise output. We assume that (A, M) is controllable and that N has full row rank. Letting

$$\nu = \Delta z \quad (11)$$

and by introducing some conservatism we obtain the following constraint on the noises (see Chapter 6 of [10]).

$$x(0)^T P_0^{-1} x(0) + \int_0^{t_k} |\nu|^2 - |z|^2 \, ds + \sum_{j=0}^{k} |\mu(j)|^2 < 2. \quad (12)$$

The models we consider for the purpose of failure detection are of the type (8)-(10), (12). In particular, we consider that we have one such model for the normal system and another, for the failed system. We say that observations $y(0), \cdots, y(k)$ are consistent with the model if there exists ν, $x(0)$ and $\mu(j)$ such that (8)-(10) and (12) are satisfied. The role of the auxiliary signal is to make sure that no observation is consistent with both the model of the normal system and the model of the failed system.

Note that clearly every observation $y(0), \cdots, y(k)$ consistent with the original uncertain model (2)-(6) is also consistent with the model (8)-(10), (12). Thus an auxiliary signal separating two models of the type (8)-(10), (12), also separates their underlying uncertain models.

The consistency of $y(0), \cdots, y(k)$ can be tested online using a causal filter; see for example Chapter 6 of [10].

2.2 Multi-model optimization formulation

Consider two models of the type (8)-(10), (12), one representing normal behavior ($i = 0$) and the other, failed behavior ($i = 1$):

$$
\begin{aligned}
\dot{x}_i &= A_i x_i + B_i v + M_i \nu_i, &(13)\\
z_i &= G_i x_i + H_i v, &(14)\\
y(j) &= C_i x(t_j) + N_i \mu_i(j) &(15)
\end{aligned}
$$

with

$$\mathcal{S}_i = x_i(0)^T P_{i,0}^{-1} x_i(0) + \int_0^{t_k} |\nu_i|^2 - |z_i|^2 \, ds + \sum_{j=0}^{k} |\mu_i(j)|^2 < 2. \quad (16)$$

Clearly, the two models have different noises and states, but v and y must be the same in the two models. Let

$$\sigma(\mathbf{v}_k, k) = \min_{\substack{\nu_0, \nu_1, y\\ x_0, x_1}} \max(\mathcal{S}_0, \mathcal{S}_1) \quad (17)$$

where the vector $\mathbf{v}_k$ is defined by

$$\mathbf{v}_k = \begin{pmatrix} v_0 \\ \vdots \\ v_{k-1} \end{pmatrix} \quad (18)$$

and the v_j's are defined in (7). Then, thanks to the fact that N_i's have full row rank, the non-existence of a solution to (13), (14), (15) and (16) is equivalent to:

$$\sigma(\mathbf{v}_k, k) \ge 2. \quad (19)$$

Any $\mathbf{v}_k$ satisfying this condition is proper. An optimal auxiliary signal is a signal in the set of proper auxiliary signals minimizing a quadratic cost $q(\mathbf{v}_k)$. Often the cost is just the energy of the signal, i.e.,

$$q(\mathbf{v}_k) = \sum_{j=0}^{k-1} |v_j|^2. \quad (20)$$

But general positive quadratic cost functions can also be considered.

3 Optimal proper auxiliary signal design

To solve the optimization problem (17), we express $\sigma(\mathbf{v}_k, k)$ as follows:

$$\sigma(\mathbf{v}_k, k) = \max_{\beta\in[0,1]} \phi_\beta(\mathbf{v}_k, k) \tag{1}$$

where

$$\phi_\beta(\mathbf{v}_k, k) = \min_{\substack{\nu_0,\nu_1,y\\ x_0,x_1}} \beta\mathcal{S}_0 + (1-\beta)\mathcal{S}_1 \tag{2}$$

subject to (13)-(15), $i = 0, 1$. Thus for a given k, the optimal $\mathbf{v}_k$ is obtained by solving the following optimization problem:

$$\min_{\mathbf{v}_k} q(\mathbf{v}_k), \quad \text{subject to } \max_{\beta\in[0,1]} \phi_\beta(\mathbf{v}_k, k) \geq 2. \tag{3}$$

Using the fact that both ϕ_β and q are quadratic functions (ϕ_β is the solution of a quadratic cost optimization problem subject to linear constraints), we obtain the following key result.

Theorem 3.1 *Let*

$$\lambda^* = \max_{\substack{\mathbf{v}_k,\, k<n\\ \beta\in[0,1]}} \frac{\phi_\beta(\mathbf{v}_k, k)}{2q(\mathbf{v}_k)} \tag{4}$$

and suppose $\mathbf{v}^*_{k^*}$ realizes the max. Then

$$\mathbf{v}^* = \frac{1}{\sqrt{\lambda^* q(\mathbf{v}^*_{k^*})}} \mathbf{v}^*_{k^*} \tag{5}$$

defines an optimal proper auxiliary signal over the test period $[0, t_n]$.

Note that the resulting $v^*(t)$ entering the continuous time system will be as follows: for $0 \leq t < t_{k^*}$,

$$v^*(t) = \mathbf{v}^*(j), \quad \text{if } t_j \leq t < t_{j+1}. \tag{6}$$

In case k^* is less than n, the auxiliary signal does not make use of the whole test interval.

Clearly the larger λ^* is, the smaller the cost q is. So λ^* measures how easy it is to separate the two models. That is why

$$\gamma = \sqrt{\lambda^*} \tag{7}$$

is called the *separability index* of the system.

After combining the two models and eliminating y from (15), we can use the following notations

$$x = \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}, \nu = \begin{pmatrix} \nu_0 \\ \nu_1 \end{pmatrix}, z = \begin{pmatrix} z_0 \\ z_1 \end{pmatrix}, \mu(j) = \begin{pmatrix} \mu_0(j) \\ \mu_1(j) \end{pmatrix},$$

$$A = \begin{pmatrix} A_0 & 0 \\ 0 & A_1 \end{pmatrix}, B = \begin{pmatrix} B_0 \\ B_1 \end{pmatrix}, C = \begin{pmatrix} C_0 & -C_1 \end{pmatrix},$$

$$M = \begin{pmatrix} M_0 & 0 \\ 0 & M_1 \end{pmatrix}, N = \begin{pmatrix} N_0 & -N_1 \end{pmatrix}, G = \begin{pmatrix} G_0 & 0 \\ 0 & G_1 \end{pmatrix},$$

$$H = \begin{pmatrix} H_0 \\ H_1 \end{pmatrix}, P_\beta^{-1} = \begin{pmatrix} \beta P_{0,0}^{-1} & 0 \\ 0 & (1-\beta)P_{1,0}^{-1} \end{pmatrix},$$

$$Q_\beta = \begin{pmatrix} \beta I & 0 \\ 0 & (1-\beta)I \end{pmatrix}, R_\beta = \begin{pmatrix} \beta I & 0 \\ 0 & (1-\beta)I \end{pmatrix},$$

to express ϕ_β as follows

$$\phi_\beta(\mathbf{v}_k, k) = \min_{x,\nu,z,\mu_j} x(0)^T P_\beta^{-1} x(0) + \int_0^{t_k} |\nu|^2_{Q_\beta} - |z|^2_{R_\beta}\, ds + \sum_{j=0}^{k} |\mu_j|^2 \tag{8}$$

subject to

$$\begin{aligned} \dot{x} &= Ax + Bv + M\nu, && (9)\\ z &= Gx + Hv, && (10)\\ 0 &= Cx(t_j) + N\mu(j). && (11) \end{aligned}$$

We solve this optimization problem in two steps.

Lemma 3.1 *The optimization problem (8) can be expressed as follows:*

$$\phi_\beta(\mathbf{v}_k, k) = \min_{x_j,\mu(j)} V_k(x_0, x_1, \ldots, x_k) + \sum_{j=0}^{k} |\mu(j)|^2 \tag{12}$$

subject to

$$0 = Cx_j + N\mu(j) \tag{13}$$

for $0 \leq j \leq k$, *where*

$$V_k(x_0, x_1, \ldots, x_k) = \min_{x,\nu,z} x(0)^T P_\beta^{-1} x(0) + \int_0^{t_k} |\nu|^2_{Q_\beta} - |z|^2_{R_\beta}\, ds \tag{14}$$

subject to (9), (10) and

$$x_j = x(t_j), \quad 0 \leq j \leq k. \tag{15}$$

The optimization problem (14) is a well-posed LQ problem. That is because thanks to controllability of (A, M), for all x_j, there exists ν such that (15) is satisfied. The solution to this problem is obtained using the Lagrangian method.

Lemma 3.2 *The solution to the optimization problem (14) can be obtained by solving the following multi-point boundary value problem*

$$\begin{aligned} \dot{x} &= Ax - MQ_\beta^{-1}M^T\lambda + Bv && (16)\\ \dot{\lambda} &= G^T R_\beta^{-1} Gx - A^T\lambda + G^T R_\beta^{-1} Hv && (17) \end{aligned}$$

with boundary conditions $x(t_i) = x_i$. The optimal ν then satisfies

$$\nu = -Q_\beta^{-1} M^T \lambda \tag{18}$$

Conditioning on $x(t_i)$ implies that we can solve the problem separately over each interval $[t_j, t_{j+1}[$. Noting in addition that over $[t_j, t_{j+1}[$, $v(t) = v_j$ is constant, we can explicitly solve the boundary value problem.

Theorem 3.2 *There exists matrices $\mathcal{J}_\beta(i)$ such that*

$$V_k(x_0, x_1, \ldots, x_k) = x_0^T P_\beta^{-1} x_0 + \sum_{i=0}^{k-1} \psi_i(x_{i+1}, x_i, v_i) \quad (19)$$

where

$$\psi_i(x_{i+1}, x_i, v_i) = \begin{pmatrix} x_{i+1}^T & x_i^T & v_i^T \end{pmatrix} \mathcal{J}_\beta(i) \begin{pmatrix} x_{i+1} \\ x_i \\ v_i \end{pmatrix}. \quad (20)$$

If the system matrices are time-invariant and t_i's form a regular grid (i.e., $t_{i+1} - t_i$ is constant), then $\mathcal{J}_\beta(i) = \mathcal{J}_\beta$ for all i.

Proof Let

$$\xi(t) = \begin{pmatrix} x(t_j + t) \\ \lambda(t_j + t) \end{pmatrix}. \quad (21)$$

Then it is straightforward to show that

$$\xi(\tau) = A_d \xi(0) + B_d v_j \quad (22)$$

where $\tau = t_{j+1} - t_j$. Matrices A_d and B_d are readily computed from system matrices; they depend on β. For example if the system matrices are time-invariant, then

$$A_d = \exp(\mathcal{A}_\beta \tau) \quad (23)$$

where

$$\mathcal{A}_\beta = \begin{pmatrix} A & -MQ_\beta^{-1}M^T \\ G^T R_\beta^{-1} G & -A^T \end{pmatrix}. \quad (24)$$

In the time varying case, A_d and B_d are obtained by a straightforward integration.

In (22), $\xi(0)$ and $\xi(\tau)$ are partially known (their first vector components are x_j and x_{j+1} which are given). So, from this equation we can completely characterize $\xi(0)$ (and $\xi(\tau)$). We obtain

$$\xi(0) = S x_{j+1} + T x_j + W v_j \quad (25)$$

for some matrices S, T and W.

Noting that

$$\psi_j(x_{j+1}, x_j, v_j) = \int_{t_j}^{t_{j+1}} |\nu(s)|^2_{Q_\beta} - |Gx(s) + Hv_j|^2_{R_\beta} \, ds$$

and thanks to (18) and the fact that

$$\begin{pmatrix} x(s) \\ \lambda(s) \end{pmatrix} = \xi(s - t_j) = \Psi(s - t_j)\xi(0) + \Phi(s - t_j) v_j$$

where Ψ and Φ depend only on system matrices and $\xi(0)$ is given in (25), we find that $\psi_i(x_{i+1}, x_i, v_i)$ is a quadratic function. $\square$

Now that we have the solution to the inner optimization problem, we need to solve

$$\phi_\beta(\mathbf{v}_k, k) = \min_{x_j, \mu(j)} x_0^T P_\beta^{-1} x_0 + \sum_{j=0}^{k-1} \psi_j(x_{j+1}, x_j, v_j) + |\mu(j)|^2 \quad (26)$$

subject to

$$0 = Cx_j + N\mu(j). \quad (27)$$

The minimization over μ is straightforward and gives

$$\mu(j) = N^T (NN^T)^{-1} C x_j. \quad (28)$$

Thus

$$\phi_\beta(\mathbf{v}_k, k) = \min_x x_0^T P_\beta^{-1} x_0 + \sum_{j=0}^{k-1} \psi_j(x_{j+1}, x_j, v_j) + x_j^T C^T (NN^T)^{-1} C x_j. \quad (29)$$

We can express this optimization problem as a large static optimization problem

$$\phi_\beta(\mathbf{v}, k) = \min_{\mathbf{x}} \begin{pmatrix} \mathbf{x}^T & \mathbf{v}^T \end{pmatrix} \begin{pmatrix} \mathcal{X}_\beta(k) & \mathcal{Y}_\beta(k) \\ \mathcal{Y}_\beta(k)^T & \mathcal{Z}_\beta(k) \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{v} \end{pmatrix} \quad (30)$$

where

$$\mathcal{X}_\beta(k) = \begin{pmatrix} X_\beta(0) & J_4^T & & & \\ J_4 & X_\beta(1) & & & \\ & & \ldots & & \\ & & & X_\beta(k-1) & J_4^T \\ & & & J_4 & X_\beta(k) \end{pmatrix} \quad (31)$$

where

$$\begin{aligned} X_\beta(0) &= P_\beta^{-1} + J_2 + C^T (NN^T)^{-1} C \quad (32) \\ X_\beta(i) &= J_1 + J_2 + C^T (NN^T)^{-1} C \quad (33) \\ X_\beta(k) &= J_1 + C^T (NN^T)^{-1} C \quad (34) \end{aligned}$$

for $0 < i < k$ and where

$$\begin{pmatrix} J_1 & J_4 & J_5 \\ J_4^T & J_2 & J_6 \\ J_5^T & J_6^T & J_3 \end{pmatrix} = \mathcal{J}_\beta(i) \quad (35)$$

(note that for simplicity of notations, we do not indicate explicitly the possible dependence of the J's on i and β).

$$\mathcal{Y}_\beta(k) = \begin{pmatrix} J_6 & & & \\ J_5 & J_6 & & \\ & J_5 & \ldots & \\ & & \ldots & J_6 \\ & & & J_5 \end{pmatrix} \quad (36)$$

and

$$\mathcal{Z}_\beta(k) = \mathrm{Diag}(J_3). \quad (37)$$

Assuming $\mathcal{X}_\beta(k) > 0$, we have that the minimum in (30) exists and that the solution is given in terms of the Schur's complement of the matrix defining the quadratic form in (30):

Lemma 3.3 *Suppose $\mathcal{X}_\beta(k) > 0$, then*

$$\phi_\beta(\mathbf{v}, k) = \mathbf{v}^T(\mathcal{Z}_\beta(k) - \mathcal{Y}_\beta(k)^T\mathcal{X}_\beta(k)^{-1}\mathcal{Y}_\beta(k))\mathbf{v}. \quad (38)$$

The problem is that $\mathcal{X}_\beta(k)$ can be very large making the direct construction of the Schur's complement impractical. Fortunately $\mathcal{X}_\beta(k)$ has a band structure which can be used to recursively test its positivity and construct the Schur's complement of the large matrix.

Lemma 3.4 *The matrix $\mathcal{X}_\beta(k)$ defined in (31) is positive definite if and only if $\Lambda_\beta(j)$, for $j = 0, \dots, k$, is positive definite where*

$$\Lambda_\beta(j+1) = X_\beta(j+1) - J_4\Lambda_\beta(j)^{-1}J_4^T \quad (39)$$

with $\Lambda_\beta(0) = X_\beta(0)$, and the $X_\beta(j)$'s are defined in (32)-(34).

If $\mathcal{X}_\beta(k)$ is not positive definite for any β, there exists no proper auxiliary signal of length k because ϕ_β is $-\infty$ and cannot satisfy the constraint in (3).

Once we know that the minimization problem has a finite solution, we can compute it recursively as follows.

Lemma 3.5 *Let*

$$\Delta_\beta(k) = \mathcal{Z}_\beta(k) - \mathcal{Y}_\beta(k)^T\mathcal{X}_\beta(k)^{-1}\mathcal{Y}_\beta(k) \quad (40)$$

where $\mathcal{X}_\beta(k)$, $\mathcal{Y}_\beta(k)$ and $\mathcal{Z}_\beta(k)$ are defined respectively in (31), (36) and (37). Then $\Delta_\beta(k)$ is obtained from the following recursive formulae

$$\begin{aligned}\Delta_\beta(j+1) &= \begin{pmatrix}\Delta_\beta(j) & 0\\ 0 & J_3\end{pmatrix} - \Gamma(j)^T\Lambda_\beta(j)^{-1}\Gamma(j)\\ \Gamma(j+1) &= \begin{pmatrix}0 & J_5\end{pmatrix} - J_4\Lambda_\beta(j)^{-1}\begin{pmatrix}\Gamma(j) & J_6\end{pmatrix}\end{aligned}$$

with $\Omega(0) = [\,]$ and $\Gamma(0) = [\,]$.

Since q is a positive quadratic function, for some positive-definite matrix $\mathcal{Q}(k)$, we have

$$q(\mathbf{v}_k) = \mathbf{v}_k^T\mathcal{Q}(k)\mathbf{v}_k. \quad (41)$$

For example, if q represents the energy of $\mathbf{v}_k$, then $\mathcal{Q}_k = I$.

Lemma 3.6 *Let*

$$\lambda_\beta(k) = \frac{1}{2}\max_{\mathbf{v}_k}\frac{\phi_\beta(\mathbf{v}_k, k)}{q(\mathbf{v}_k)}. \quad (42)$$

Then $\lambda_\beta(k)$ is the largest value of λ for which

$$\Sigma_\beta(k) = \Delta_\beta(k) - 2\lambda\mathcal{Q}_k \quad (43)$$

is singular.

Thus the computation of $\lambda_\beta(k)$ amounts to solving a generalized eigenvalue problem for which reliable computer programs exists. Note also that $\Delta_\beta(k)$ is constructed recursively, so $\lambda_\beta(k)$ is obtained by a recursive formula.

The value of λ^* defined in Theorem 3.1 (and thus the separability index $\gamma = \sqrt{\lambda^*}$) can now be computed as follows

$$\lambda^* = \max_{\beta, k\leq n}\lambda_\beta(k). \quad (44)$$

Lemma 3.7 *Let $\beta = \beta^*$ and $k = k^*$ yield the maximum in (44). Then $\gamma = \sqrt{\lambda^*}$ is the separability index of the system and an optimal proper auxiliary signal is given by (5) where $\mathbf{v}_{k^*}^*$ is any vector in the null-space of $\Sigma_{\beta^*}(k^*)$.*

4 Conclusion

We have presented a constructive solution to the problem of optimal proper auxiliary signal design for a class of uncertain sampled-data systems. The proposed solution can easily be implemented in standard scientific software packages such as Scilab [1] and Matlab.

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References

[1] C. Bunks, J. P. Chancelier, F. Delebecque, C. Gomez (ed.), M. Goursat, R. Nikoukhah, and S. Steer. *Engineering and Scientific Computing with Scilab*. Birkhauser, Basel, 1999.

[2] S. L. Campbell, K. Horton, R. Nikoukhah, and F. Delebecque. Auxiliary signal design for rapid multimodel identification using optimization. *Automatica*, 38:1313–1325, 2002.

[3] S. L. Campbell and R. Nikoukhah. *Auxiliary Signal Design for Failure Detection*. Princeton University Press, To appear in 2004.

[4] F. Kerestecioglu and M. B. Zarrop. Optimal input design for change detection in dynamical systems. In *European Control Conf.*, pages 321–326, 1991.

[5] F. Kerestecioğlu. *Change Detection and Input Design in Dynamical Systems*. Research Studies Press, 1993.

[6] F. Kerestecioğlu and M. B. Zarrop. Input design for detection of abrupt changes in dynamical systems. *Int. J. Contr.*, 59:1063–1084, 1994.

[7] R. Nikoukhah. Guaranteed active failure detection and isolation for linear dynamical systems. *Automatica*, 34(11):1345–1358, 1998.

[8] R. Nikoukhah, S. L. Campbell, and F. Delebecque. Detection signal design for failure detection: a robust approach. *Int. J Adaptive control and Signal Processing*, 14:701–724, 2000.

[9] R. Nikoukhah, S.L. Campbell, K. Horton, and F. Delebecque. Auxiliary signal design for robust multi-model identification. *IEEE Trans. Automat. Contr.*, 47:158–163, 2002.

[10] I. R. Petersen and A. V. Savkin. *Robust Kalman Filtering for Signals and Systems with Large Uncertainties*. Advances in Design and Control. Birkhauser, 1999.

[11] X. J. Zhang. *Auxiliary Signal Design in Fault Detection and Diagnosis*. Springer-Verlag, Heidelberg, 1989.